ELEMENTS

OF

NATURAL PHILOSOPHY.

BY

Sir JOHN LESLIE, Esq.

PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF EDINBURGH, AND CORRESPONDING MEMBER OF THE ROYAL INSTITUTE OF FRANCE.

VOLUME FIRST,

INCLUDING

MECHANICS AND HYDROSTATICS.

SECOND EDITION, CORRECTED AND ENLARGED.

EDINBURGH:

PUBLISHED BY

OLIVER & BOYD, TWEEDDALE COURT, AND GEO. B. WHITTAKER, LONDON.

1829.

EDINBURGH:
PRINTED BY JAMES WALKER.

ADVERTISEMENT.

This volume is the first of a work designed to exhibit a Comprehensive View of the Principles of Natural Philosophy. Though compiled chiefly for the use of my Students, it is written, I hope, with sufficient expansion, to suit the taste of the public at large. I have endeavoured to render it as elementary as possible, without departing from the accuracy of science. No previous instruction is required, except an acquaintance with the simplest rudiments of Geometry and Algebra; but I would earnestly recommend the study of Geometrical Analysis to all those who aspire to the higher attainments.

The present volume may be considered by itself as in a great measure complete; and

two more will conclude my plan. I intend to annex copious illustrations, containing the more difficult demonstrations, historical notices and references to authors, with a correct set of the most useful tables. But I must defer all these additions till the next volume shall appear.

I have in this Edition carefully revised the whole, made several important additions, and given a copious table of contents.

I had designed the Second Volume of this Work to appear at the same time; but have since thought it better to wait for the results of a series of experiments projected on the Constitution and Power of Steam. I trust, however, in being enabled very soon to discharge my promise to the public.

COATES, FIFESHIRE,
28th October 1828.

CONTENTS.

INTRODUCTION.

I. SOMATOLOGY.

II. STATICS.

MECHANICS.

III. HYDROSTATICS.

VII. HYDRODYNAMICS.

INTRODUCTION.

OUR curiosity is first awakened by the changes incessantly passing around us. But experience soon testifies the constancy and regularity of this varied spectacle. The vast movements of the Universe are found all to consist in the repetition of similar events. It may for ever elude the power of human ingenuity, to discover the more secret springs which connect the links of the extended indissoluble chain. Yet, since the most complex phænomenon is always the result of a very few principles, the objects of Science are attained by distinguishing and classing those elementary facts. Men will seldom rest satisfied, however, with such moderate advances; and have often, in their hasty and rash attempts to penetrate the arcana of Nature, suffered severe disappointment. The pro-

per business of philosophical inquiry, is to study carefully the appearances that successively emerge, and trace their mutual relations.

All our knowledge of external objects being derived through the medium of the senses, there are only two ways of investigating physical facts,—by *Observation* or *Experiment*. Observation is confined to the close investigation and attentive examination of the phænomena which arise in the course of Nature; but Experiment consists in a sort of artificial selection and combination of circumstances, for the purpose of searching minutely after the different results.

The range of Observation is limited by the position of the spectator, who can seldom expect to follow Nature through her winding and intricate paths. Those observations are of the most value which include the relations of time and space, and derive greater nicety from their comprising a multiplied recurrence of the same events. Hence Astronomy has attained much higher degree of perfection than the other physical sciences.

Experiment is a more efficient mean than Observation, for exploring the secrets of Nature. It requires no constant fatigue of watching, but comes in a great measure under the control of the inquirer, who may often at will either hasten or delay the expected event. Though the peculiar boast of modern times, yet the method of proceeding by Experiment was not wholly unknown to the ancients, who seem to have concealed their notions of it under the veil of allegory. PROTEUS * signified the mutable and changing forms of material objects; and the inquisitive philosopher was counselled by the Poets to watch that slippery Dæmon when slumbering on the shore, to bind him and compel the reluctant captive to reveal his secrets. This gives a lively picture of the cautious but intrepid advances of the skilful experi-

* ——————— novit namque omnia vates,
Quæ sint, quæ fuerint, quæ mox ventura trahantur.
* * * * * * *
Hic tibi, nate, prius vinclis capiundus, ut omnem
Expediat morbi causam eventusque secundet.
Nam sine vi non ulla dabit præcepta, neque illum
Orando flectes: vim duram et vincula capto
Tende; doli circum hæc demum frangentur inanes.
GEORG. IV. 392.-402.

menter. He tries to confine the working of Nature—he endeavours to distinguish the several principles of action—he seeks to concentrate the predominant agent—and labours to exclude as much as possible every disturbing influence. By all these united precautions, a conclusion is obtained nearly unmixed and not confused, as in the ordinary train of circumstances, by a variety of intermingled effects. The operation of each distinct cause is hence severally developed.

The main object, therefore, in all physical inquiries, is, by an Analytical Process, to separate the various effects which Nature has blended together. The history of Astronomy, from the earliest times, affords the finest examples of such successful induction. The true Philosopher labours to reduce the number of principles or ultimate facts. In proportion as his views expand, he will find the relations so disclosed, converging invariably to a common centre. But he should avoid indulging to excess the taste for simplification. In remounting to the source of all power, it is evident that he must at last reach some impassable limit. Prudence will direct him when he ought

to stop, or await the discovery of some new instrument, to assist his ulterior inquiries.

Every extension of the powers of calculation, every addition to our stock of philosophical apparatus, every improvement in the mode of their construction,—is a prelude to the farther advances of physical science. Even slight alterations in the practice of the arts have sometimes led to the most important theoretical conclusions. Several discoveries in science are sometimes invidiously referred to mere fortuitous incidents. But the mixture of chance in this pursuit should not detract from the real merit of the invention. Such occurrences would pass unheeded by the bulk of men; and it is the eye of genius alone that can seize every casual glimpse, and descry the chain of consequences. Bodies had in all ages been viewed with indifference falling to the ground; yet the accidental sight of an apple, as it dropped from the tree, was sufficient to awaken, in a pensive mood, that lofty train of reflections, which conducted Newton to the system of Universal Attraction.

After new facts have been disclosed, either by strict observation, or delicate and careful experiment, the process of Synthetical Deduction should commence. But to pursue a train of conclusions successfully, requires the exercise of sound judgment, and the application of vigorous and instructed intellect, under the guidance of a sober and cautious logic. The most important instrument in forwarding this operation is Geometry, to which, indeed, we are indebted for whatever is most valuable in Natural Philosophy. By this powerful aid the nobler branches of science have in modern times been carried to a sublime elevation.

But the most rigorous application of Mathematical reasoning will not always succeed in unveiling the secrets of Nature. The philosophical inquirer must often content himself at first with seeking merely to approximate the truth. Analogy, in such cases, may serve as a tolerable guide to assist his steps. Some gifted individuals advance, indeed, with a sagacity which seldom fails. They shape their path according as the light successively breaks in; and here again Geometry lends its penetrating aid.

Nor should Hypotheses themselves, however liable to abuse, be excluded absolutely from Natural Philosophy. They will often suggest new modes of research, and in this way produce beneficial effects. They may always serve as preludes of inquiry; but they are generally too fallacious and enticing, to be allowed with safety to gain any durable possession of the mind.

The gift of a lively fancy is an important requisite to every physical observer. This faculty has accordingly appeared conspicuous in all the great discoverers. The imagination of the Philosopher differs from that of the Poet, only because it calls forth less vivid images; but it is equally creative, and equally sentient to the flitting scenes of Nature. It supplies the inquirer with fresh expedients, enables him to multiply the points of attack, and sheds lustre over the scenes which he contemplates. But imagination requires to be restrained, by the exercise of a sound judgment. The Philosopher pushes his researches with ardour, yet is cautious and deliberate in drawing his conclusions. His attention is arrested by the appearance of anomalous facts.

While he doubts and pauses, he derives courage, amidst all his perplexity, from the near prospect of starting some new principle. The detection of a single error is a sure step to actual discovery.

Such is the only successful mode of interrogating Nature. But Philosophers, in all ages, have betrayed extreme impatience, in submitting to a plan of investigation, apparently so timid, slow, and laborious. It was more agreeable to human indolence and presumption, at once to frame hypotheses which might, in imagination at least, connect the mass of preconceived opinions. But such rash attempts lulled curiosity asleep, and fatally arrested the progress of genuine science. Hence that glimmering twilight which so long overspread the world.

Science first dawned in the genial climes of the East; but its warming rays were soon absorbed in the cheerless fog of despotism. The body of knowledge which had been created by the efforts of unfettered genius, became the exclusive property of the order of priesthood, who rendered it an engine of power, subservient to the purposes of a gloomy and

debasing superstition. The discoveries of happier times were entombed in silence and darkness.

A MORE auspicious morning at length arose. Greece, though but a spot on the surface of our globe, began her career of glory, and gave early tokens of those eternal benefits she was destined to confer on the human race. Her sages gleaned instruction by visiting foreign lands, and the seats of ancient renown. They gathered the dying embers of science, and rekindled them by the breath of their genius. But, quickly emerging from a state of pupillage, they displayed the riches of a lively fancy, and all the resources of a fertile invention.

THALES, the founder of the Ionic Sect, having spent a large patrimony and many years of his life in foreign travel, transplanted into Greece the Science of the Egyptian Priests. His successors, ANAXIMANDER and ANAXIMANES, taught, with a few modifications, the same doctrines. Their knowledge appears to have been superficial, yet was it not the less aspiring. They indulged in cosmological systems, which pretend to explain the origin

and formation of all things. Such bold speculations flattered human vanity, and charmed the imagination by a glittering semblance of truth. Those early sages held every substance whatever to be composed of four distinct elements, Earth, Water, Air and Fire, merely combined in various proportions. Earth and Water they viewed as naturally ponderous and inert, while they fancied Air and Fire, endued with elastic virtue, to possess lightness and activity. While the earthy matter settled towards the centre of the universe, and the aqueous fluid rolled along the surface of the globe; the Air and Fire, or Æther, soared aloft, the former occupying the whole region below the moon, and the latter streaming through the boundless extent of space. The same pure lambent fluid, being collected into globular masses, formed the groupes of stars. But portions of this divine essence descended, to communicate the vital sparks, and animate the subjects below. Such visions, so like the picturing of fantastic dreams, were yet firmly believed in former ages; they became afterwards incorporated with the vulgar creed; and still deeply tincture the language of Poetry.

Anaxagoras, who continued the Ionian line, rose to higher eminence. He relinquished a splendid fortune, and dedicated the best portion of his life, to the pursuit of knowledge, under the Priests of Thebes and the Magi of Persia; and, on his return to Greece, he was induced, from the dread of foreign conquest, to transfer the School of Ionia to Athens. His eloquence and learning made a powerful impression in that intellectual city. But he drew upon himself the odium of the illiberal, by broaching some opinions at variance with the received tenets, and anticipating obscurely the modern discoveries.

The island of Samos had now the honour of giving birth to Pythagoras, the first sage who took the modest but auspicious name of Philosopher, or *Lover of Wisdom.* Nature bestowed on him her choicest gifts, and assiduous cultivation heightened all those qualities which command the minds of men. Pythagoras sacrificed his ample fortune, and devoted the vigour of his days, to the acquisition of knowledge. He was admitted into the sacred college of Memphis; he resided some time in Phœ-

nicia, visited Persia, and pursued his eastern journey to the banks of the Indus. After an absence of near thirty years, he returned home, fraught with various information, and was viewed with awe and reverence by his assembled countrymen at the Olympic Games. He began to give instructions in his native island, but soon removed to the Grecian colony planted at Tarentum on the coast of Calabria. In that voluptuous city, he collected numerous pupils, and founded the powerful Italian Sect, which subsisted after his death during the course of several centuries.

Not to shock the prejudices of his countrymen, Pythagoras judged it prudent to separate the doctrines he taught into two distinct sorts—the *Exoteric* and the *Esoteric*. The former consisted in discourses addressed to the people in the temples and other places of public resort, and calculated to reform their habits and dispositions; the latter comprised those recondite principles communicated only to the very few disciples who, after a very long and severe probation, were esteemed fit and safe depositaries of such momentous truths. He strenuously urged the

study of Mathematics, especially those parts which are concerned about number and proportion. His imagination being full of numerical relations, he founded the theory of Music, which he cultivated both as an art and a science. But he transferred his ideas of music to the harmony of the celestial motions. Rising to the sublime conception of the true system of the Universe, he seems to have veiled that noble discovery in a fine allegory. Under the symbol of APOLLO playing on the lyre, he taught his chosen disciples, that all the planets, including our Earth, are inhabited worlds, which revolve about the Sun as their common centre; and he farther maintained, that those bodies, while they circle round the great luminary, perform a most harmonious concert, though such ravishing and heavenly sounds are lost to our gross ears, and drowned amidst the jarring noise which prevails here below.

THE most distinguished of the successors of PYTHAGORAS was EMPEDOCLES, born at Agrigentum in Sicily. He conceived all bodies to consist of infinitely small corpuscles in a state of perpetual mo-

tion, held together by the inherent force of matter, yet kept separate by the antagonism of its action. He thus made an important step in physical science, by introducing the play of two opposite principles, termed in his figurative language *friendship* and *strife*, very similar to the forces of attraction and repulsion, which perform such important service in the philosophy of modern times.

Xenophanes transplanted most of the tenets of the Pythagoreans into the small sect which he founded at the town of Elea in Campania. He has the merit of being the first to advance the important principle in Geology, that the exterior crust of our globe was once in a fluid state, and that the fossil shells and other marine exuviæ discovered in the bowels of the earth, and on the tops of the highest mountains, had, at some remote period, been formed under the waters of the ocean.

Leucippus, who sprung from that school, asserted not only the doctrine of atoms, but maintained the existence of a separate *plenum*, and anticipated the influence of centrifugal force. These doctrines

were extended and improved by his disciple DEMOCRITUS, who flourished during the time of the Peloponnesian war, and may be considered as one of the first geniuses that Greece, so rich in talent, ever produced. To procure the higher degrees of knowledge, he visited Egypt, conversed with the Magi of Babylon, and appears to have pursued his journey eastwards as far as the Ganges. The expence incurred in these distant travels consumed the whole of his patrimonial inheritance, and DEMOCRITUS, on his return, was content to occupy a small garden near Abdera, where he passed a prolonged life of parsimony and seclusion, wholly intent on making experiments, and investigating the operations of Nature. He rectified many of the prevailing errors in physics; proved the existence of a *plenum* to be incompatible with local motion; rejected the notion of *levity*, that was attributed to the elements of Air and Fire; maintained that the weight of bodies is always proportional to their mass or quantity of matter, and that they would hence fall in *vacuo* with equal celerities; and he likewise entertained tolerably correct general views concerning the properties of Heat and Light. These tenets are expounded in

the elegant poem of LUCRETIUS, who gratefully and fervently styles his master in philosophy, *pater et rerum inventor*.

PHILOSOPHICAL sects had multiplied over Greece when SOCRATES arose, to give a more useful impulsion to the human mind. Disgusted with the miserable sophisms and illusory dreams of pretenders in science, he justly thought the first step towards real knowledge was to discover the extent of our ignorance. His aim was not so much the elaborate education of a few individuals, as the diffusion of solid instruction among the body of Athenian youth. Averse from idle speculations, he recommended that kind of philosophy alone which is grounded on fact and experiment. He was accustomed to pour unmeasured contempt on every species of hypocrisy and pretension. But his intrepid conduct and high celebrity created a host of secret enemies, and the venerable sage at last fell a martyr to the cause of truth and virtue.

PLATO, the most illustrious of all the disciples of SOCRATES, after the murder of his revered master,

withdrew from the polluted city, and visited the seat of the Pythagoreans at Tarentum, where he was initiated in the whole *arcana* of the Italic School. He next placed himself under the tuition of THEODORUS, an eminent mathematician of Cyrene in Lybia. With a mind thus prepared, he travelled in disguise over the whole of Egypt, carefully examining every object, and gleaning all the information he could reach. Being prevented by war from proceeding farther, he returned home, and purchased the Grove of Academus, in the immediate vicinity of Athens; in which sequestered spot, he delivered, under the shade of spreading oriental planes, his eloquent and impressive lessons, to youths of the first distinction, who flocked from all parts, attracted by the fame of his sublime genius.

The philosophy of PLATO was deeply imbued with the mysticism of the Italic School. Yet to him we are deeply indebted for the beautiful method of Geometrical Analysis, which furnished so powerful an instrument to direct the process of investigation. This invention, in the hands of his successors in the Academy, was continually extending the boundaries

of Science. It led immediately to the discovery of the Conic Sections, which, though cultivated for ages merely as a fine speculation, suggested at last to GALILEO the laws of motion, and aided KEPLER in his search after the true orbits of the planets. Unfortunately, the soaring disposition of the Academics spurned the study of material objects, and was apt to exhale in airy sublimation.

ARISTOTLE, whose authority so long maintained the most despotic sway in the learned world, was born at Stagira, on the confines of Thrace and Macedon, 385 years before the Christian æra. He studied twenty years under the direction of PLATO, on whose death he retired to Mysia and Mytelene, till he was summoned by PHILIP of Macedon, to superintend the education of the young Prince. When ALEXANDER marched against Persia, his preceptor went to Athens, and opened the Peripatetic School at a place called the Lyceum. But ARISTOTLE kept a regular correspondence with the Macedonian hero, who gave the most liberal encouragement to his projected work on the History of Animals. The philosopher continued only twelve years

more at Athens, and apprehending the tempest of a persecution, prudently retired to Chalcis, where he spent the last two years of his life.

The very numerous writings of ARISTOTLE were suffered to rot in a damp cellar, for an hundred years after his death, and seem never to have been much esteemed by the Greek and Roman authors. No philosopher, either in ancient or modern times, has taken such a wide range of disquisition; and yet his universal genius was marked by soundness of judgment, precision of thought, and singular acuteness, bordering occasionally in subtlety. Though ambitious to maintain the character of an original thinker, he endeavoured to ground all his philosophy on the close observation of facts. Restraining the flights of imagination, he confined his researches to the circumstances of real life, and the actual constitution of the universe. His conclusions were therefore of most value, whenever he could obtain accurate information, on which to exercise his penetrating sagacity. The Natural History of ARISTOTLE must be deemed a wonderful production, for the time in which it was composed. He was the found-

er of Comparative Anatomy; and according to CUVIER, a most competent judge, the divisions which he adopted are still the best that could be made. He was the first who distinguished nerves from tendons; he noticed the auditory and optic nerves of the mole; he traced the optic and olfactory nerves in fishes; and described with great accuracy the process of the incubation of eggs, and the developement of the chick. His Meteorology sparkles with fine remarks and just conclusions; and even his Physics, and the book *De Cœlo*, contain, amidst much idle and extravagant speculation, some interesting doctrines, which might deserve to be separated and abridged. His peculiar tenets likewise deserve notice, merely for their influence in the History of Philosophy. We regret that ARISTOTLE should have suffered his balanced mind to be drawn aside, by the love of subtlety and the illusion of general hypothesis, from the strict investigation of facts. If the Peripatetics had cultivated Geometrical Analysis with the same ardour as the followers of PLATO, they would infallibly have made great and successful progress in Natural Philosophy.

An event now took place, which mightily contributed to the permanent extension of Mathematical Science in all its branches. After the premature death of Alexander, his conquests had been shared among the companions of his arms. Egypt fell by lot to Ptolemy, who, selecting for his residence the city of Alexandria, occupied by a Grecian colony, embellished it, and rendered it a distinguished seat of learning. A magnificent edifice was erected, styled the *Museum*, in which men of science, invited from all countries, were liberally entertained at the public expense, and provided with books and instruments for the prosecution of their studies. It was farther aggrandized by the munificence of his successors, who founded a vast library, and raised a spacious and well-furnished observatory. This royal establishment survived all the changes of fortune during nine hundred years, and conferred incalculable benefits on the human race. A succession of the ablest mathematicians shed lustre on the first three centuries of the Alexandrian School. Euclid digested the Elements of Geometry into a System, excellent at that period, though certainly not adapted to the present state of the Science. Apollonius

extended the Conic Sections, and improved Geometrical Analysis, in which he was likewise followed by PAPPUS, and by DIOCLES and NICOMEDES, who invented some of the higher curves. DIOPHANTUS applied a similar investigation to arithmetical problems, and the few symbols he used may be considered as anticipations of that perfect system of characters, or pictured language, which modern Algebra exhibits.

MEANWHILE our knowledge of the surface of the globe was extended by the adventures of a bolder navigation. The Indian Sea had been explored by the small squadron of NEARCHUS, who attended the expedition of ALEXANDER into the East. But the republic of Marseilles, a Grecian colony settled in the south of France, had the merit of fitting out the first voyage of discovery. EUTHYMENES sailed towards the equator, while PYTHIAS, an able astronomer, shaped his course towards the north, entered the Baltic, discovered Thulè, and remarked the features of the circumpolar climate. He noticed the phenomena of tides, which are unknown along the shores of the Mediterranean; and on his return, he

made an accurate observation of the obliquity of the ecliptic, which was then 23° 48′, conformable to the deductions of profound theory.

SICILY had the honour of giving birth to the most inventive genius of all antiquity. ARCHIMEDES showed from infancy a passion for science, and having received what instruction his native city of Syracuse could afford, he visited Alexandria, and other seminaries abroad. After his return, he devoted himself entirely to the charms of abstract study, and pursued his deep researches with the most ardent and intense application. He gave unlimited extent to the notation of numbers, and founded the method of indivisibles, which led him to the finest discoveries in Geometry. He assigned the quadrature of the parabola, approximated to that of the circle, and disclosed the fine relations which subsist between the cylinder and its inscribed cone and sphere. But ARCHIMEDES deserves to be regarded as the first who really studied Natural Philosophy in the right way. His advances were splendid and triumphant. He detected the fundamental principles of Mechanics and Hydrostatics, and illumined those

branches of science by the torch of Geometry. He pointed out the centre of gravity, and determined its position in a variety of figures; and he unfolded the properties of floating bodies, and thus traced the rudiments of naval architecture. He likewise reduced those principles into practice, and constructed such powerful engines as enabled the valour of his countrymen to resist for three years the whole efforts of a Roman squadron and besieging army. But perseverance and military discipline at last prevailed, and one fatal night was Syracuse involved in the horrors of assault. Amidst the general confusion and carnage, an infuriated soldier entered the apartment of ARCHIMEDES, and, regardless of his calm occupation, massacred him on the spot, in the 75th year of the Philosopher's age, and 212 before CHRIST.

GEOMETRICAL science had now acquired some form and consistency, and astronomy was extending its domains. ARISTARCHUS of Samos devised an ingenious method of estimating the relative distances of the sun and moon; and though, with his imperfect instruments, he could obtain only vague results,

yet were they sufficient to expand immensely our conceptions of the solar system. ERASTOTHENES observed, with precision, the obliquity of the ecliptic, and determined the circumference of the earth, by measuring the intercepted arc of the meridian between Alexandria and Syene in Upper Egypt. But HIPPARCHUS was a genius of much higher order. He found the exact length of the year, ascertained the distance of the moon, and approximated to that of the sun; he distinguished the unequal intervals between the equinoxes, and traced the precession of those points. This fine discovery suggested to him the scheme of ascertaining and registering the positions of the principal fixed stars. But he transferred the same method to terrestrial observations, and was the first who defined the places on our globe by their latitude and longitude. Without rejecting the inveterate axiom of antiquity, that an uniform and circular motion was alone befitting celestial bodies, he sought to explain the apparent inequalities by the ingenious hypotheses of *Eccentrics* and *Epicycles*, which, being multiplied in the sequel, fatally overloaded astronomical science.

PTOLEMY, who resided at Alexandria after Egypt had become a Roman province, under the Emperors ADRIAN and ANTONINUS, was one of the best and most indefatigable observers that ever lived. Less original than HIPPARCHUS, he laboured with equal zeal to promote astronomy. He not only improved every part of the science, but digested the multifarious data into one great system. He discovered the lunar evection, and celestial refraction. He likewise composed a general treatise of Geography, and applied the theory of Projections, which he had invented, to the construction of maps. The study of spherical trigonometry was begun by HIPPARCHUS, extended by THEODOSIUS and MENELAUS, but reduced to a practical form by PTOLEMY.

OTHER philosophers of the Alexandrian School applied themselves to Mechanics. CTESEBIUS improved the clepsydra, invented the pump, and constructed an engine for discharging arrows by the force of condensed air. HERO not only formed the crane, but contrived machines which acted from the variable elasticity of air, as affected by the alternation of heat and cold; a principle which in after times led

GALILEO and SANCTORIO to the construction of our Thermometers.

BUT the genius of Greece, which had been sinking under oppression, at length evaporated in polemical disputes. The Romans were now masters of the world; and perhaps no people deserved less the favour of fortune or the gratitude of posterity. In the whole range of their existence, they never made a single step towards the advancement of science. All the knowledge attained by them in arts or philosophy, had been derived immediately from the Greeks. Their education was entirely practical, and calculated only to form orators, statesmen and warriors.

ANOTHER race of men became lords of the ascendent. The Arabians, impelled by the enthusiasm of a new religion, spread the terror of their arms in all directions. They subdued Egypt, Syria, and Persia, and compressed the limits of the Eastern Empire. In the West, they occupied Spain, and penetrating into the heart of France, they threatened to extinguish the Christian name. But the

fervour of their zeal soon abated, and their schemes of ambition were at length absorbed in the arts of peace. The Arabians grew passionately attached to the science of the Greeks. They carefully collected all the philosophical writings of that wonderful people, and caused them to be translated into their own language. A succession of enlightened princes encouraged those efforts with unbounded munificence. The Caliphs stored their palaces with public libraries, and adorned them with splendid observatories.

The Arabians cultivated more especially Geometry and Astronomy; but they likewise studied Botany and Chemistry. Less prone than the Greeks to speculation, they directed their chief efforts to practical science. They soon became most skilful calculators, and accurate observers. They substituted the *Sines* instead of *Chords* in trigonometry; and farther improved this important branch of science by the introduction of *Tangents*, which, in allusion to the art of Dialling, they termed *Shadows*. But the greatest benefit which the Arabians rendered to mankind, was the communication of the decimal no-

tation of numbers. With this beautiful, though very simple contrivance, which they styled *Indian*, though it be still unknown or unpractised in any part of lower India, they seem not to have become acquainted before the end of the tenth century of our æra. The use of the ten digits in arithmetic having passed over to the Moorish kingdom of Spain, was thence transfused through the Christian nations of Europe about the beginning of the fourteenth century, though it was not generally adopted till near two hundred years afterwards.

Though the Arabians had little claim to the character of original invention, they were assiduous in collecting useful information from all parts. They practised Brewing and Distillation, arts totally unknown to the Greeks and Romans, but probably derived from the experience of the Tartarian hordes. They invented other chemical processes, and gave names, which are still preserved, to vessels of certain forms.

Mental darkness, during this period, brooded over the fairest regions of the Christian world.

The wrecks of knowledge lay buried in the convents, while the more active spirits expended all their energy in violent sports or savage depredations. Yet talent was not extinct in the middle ages, though it unhappily ran to waste. Superstition encouraged pilgrimages over Europe, and every convent opened its hospitable gates indiscriminately to the weary traveller. Rome was still the centre of the Christian commonwealth, and multitudes resorted from all parts to the metropolitan city. The refinement of Italy was, by this continual intercourse, partially reflected to the remotest corners of Europe.

The Crusades, undertaken against the Saracens from the twelfth to the fourteenth centuries, though stimulated by the wildest fanaticism and the maddening passion for military exploits, may yet be regarded as the main cause of the renovation of the human intellect. Those expensive armaments, exhausting the fortunes of the haughty chiefs, contributed to lighten the chains of feudal tyranny; and, by giving a wider circulation to wealth, they gradually raised into consideration that middle class of men who constitute the bulwark of a free state. The

Crusaders themselves, during the suspension of hostilities, could not fail, from their intercourse with the Saracens, who surpassed them so greatly in knowledge and refinement, to receive much important information both in arts and science. To this origin many of the subsequent improvements, which advanced the progress of society in Europe, may be distinctly traced.

But native genius was not inactive during the middle ages. Some of the most valuable arts arose in that benighted period. The curious process of converting cotton into paper was invented about the beginning of the eleventh century. Three centuries afterwards, linen rags were manufactured into a still better material, which, serving as a cheap and convenient substitute for parchment or vellum, has greatly promoted the practice of writing. The use of letters was about this time farther assisted, by the introduction of Spectacles, which Salvino degli Armati first constructed at Florence about the year 1285. Small spheres of crystal or glass had been employed by the ancient engravers of gems, to aid their sight; but the transition from these globes to

mere convex lenses, though apparently trivial, led to the most important consequences.

THOUGH the attractive power of the magnet was known to the Greeks, they remained ignorant of its more wonderful property, of pointing towards the North. This directive power had perhaps been discovered by the Chinese, and some intimation of it seems to have been brought by the Crusaders from the East. The magnet hence acquired the name of *loadstone* or *leading-stone,* the first compass in Europe having been made near the close of the thirteenth century at Amalphi, near Genoa. This noble invention give a prodigious spring to navigation and commercial enterprise.

THE eastern nations had been long acquainted with the deflagrating property of nitre or saltpetre; but when this wonderful substance was imported into Europe by the Crusaders, it was confounded, from its external appearance, with the natron or soda brought from Egypt. More than two centuries elapsed before its vast explosive force was observed. This most important discovery was probably made

by SCHWARTZ, a German monk, about the year 1382: It has extended the empire of man over Nature, by the gift of a new and tremendous power, which, though commonly diverted to the work of destruction, has yet rendered wars less rancorous and sanguinary than before.

BUT a nobler trophy distinguished the same period. The Romans had, for various purposes, used metal stamps; the Chinese employed carved blocks of wood for impression; but the modern art of printing, by means of combined moveable types, was invented about the middle of the fifteenth century. The ingenuity and perseverance of GUTTENBERG and SCHOEFFER, encouraged by the wealth of FAUST, a burgess of Mentz, have conferred by far the greatest benefit ever bestowed on the human race. In the short space of thirty years, this invaluable art was carried to its highest perfection.

SCHOEFFER likewise began the art of engraving, and executed in 1491, for ARNDES, burgomaster of Lubec, a series of the figures of plants and animals, on wooden blocks. This emulation of genius ac-

corded with the state of society in Europe. The taste for knowledge had been fast advancing. The Romish clergy were anxious to promote learning, as the sure means of aggrandizing their order. In all the convents and cathedrals, they had opened schools; and part of the ample revenues of the Church was dedicated to the gratuitous education of youth. Other schools were established for communicating the higher degrees of instruction, which was greatly facilitated, by the adoption of the Latin language as the medium of intercourse over Christendom. These seminaries, when they became so enlarged as to include all the branches of liberal knowledge, were termed *General Studies*; but afterwards, when they had obtained the sanction of Papal bulls, and the protection of legal privileges, they assumed the title of *Universities*. An apprenticeship of seven years, copied afterwards in the mechanical trades, was required to complete the course of education, consisting of the *Trivium*, followed by the *Quadrivium*. The tenets of ARISTOTLE were expounded with indefatigable diligence; but those opinions, drawn from the Arabic, had been miserably corrupted by the severe process of a double translation.

The vigour of genius which, if better directed, could have soared to sublime discovery, was consumed in idle disputations and unprofitable subtleties.

But a brighter prospect began to open. Petrarch resorted to the pure fountains of ancient learning, and his warm enthusiasm and eloquent exhortations made a very deep impression on the minds of his contemporaries. The study of the Greek language was gradually introduced into Italy by the frequent embassies dispatched from Constantinople, to implore the interposition of the Roman See against the formidable encroachments of the Turks. The munificent patronage of the Medici Family created and diffused a taste for liberal knowledge. Those princely merchants spared no cost in collecting the dispersed manuscripts, and invited scholars, with liberal salaries, from all parts of the Levant, to teach their countrymen the refined language of ancient Greece. The final capture of Constantinople by the Turks, in the year 1453, occasioned a general dispersion of the men of letters, who transported to Italy the perishing wrecks of Greek philosophy. The new art of printing happily at this crisis preserved them from extinction.

Every thing conspired to excite a general fermentation. The diligence of the press quickly multiplied the monuments of ancient literature; but it required the labour of a century, to digest and correct those precious remains. The veneration paid to such unrivalled compositions, repressed for a time the working of native curiosity. Religious controversies too, though ultimately productive of the greatest benefits to society, distracted for a long course of years the proper exercise of the human faculties. At length the genius of invention burst forth with renovated powers.

This day-spring of reason may be dated from the middle of the sixteenth century; since which epoch, the tide of discovery has flowed in a rapid and majestic stream. Philosophy and the arts have advanced together, reflecting mutual lights. Little more than two hundred years have yet elapsed; but it has been a period of extreme activity, investing our species in a blaze of intellectual glory.

The age of science succeeded to the age of erudition. The study of the ancient classics had infused

some portion of taste and vigour; but men soon began to feel their own strength, and hastened to exert it. The bolder spirits, bursting the trammels of authority, ventured to question inveterate opinions, and to explore with a fearless eye the wide fields of human knowledge. COPERNICUS partly restored the true system of the world. PÜRBACH and MÜLLER abbreviated the calculations of the Astronomer, by their signal improvements in Trigonometry. The famous painter LEONARDO DA VINCI led the way to sound philosophical inquiry; and not only urged the necessity of having recourse in Physics to experiment, but directed it successfully to the Composition of Forces. UBALDI, and more especially STEVINUS, extended the principles of Mechanics and Hydrostatics. The fine genius of GALILEO detected and applied the laws of motion; and re-invented and constructed the Telescope, which had just been found out in North Holland. This truly wonderful instrument he directed to the heavens, and thus marked the varying phases of the planets, and discovered the harmony of new worlds.

THE bold exuberant imagination of KEPLER,

working on the register of the accurate observations of TYCHO BRAHE, and employing the most intense labour in computing and combining them, at last drew aside the veil, and disclosed to view those eternal laws which govern the revolutions of the heavenly bodies. A powerful auxiliary was yet wanting to the calculator, and our great countryman NAPIER rendered himself immortal, by the sublime discovery of Logarithms.

THE Alchemists, however extravagant in their pretensions, had constantly promoted experimental science. BAPTISTA PORTA, not only collected all the curious facts he met with in his travels, but founded on his return to Naples an association of individuals for the purpose of exploring Nature. This obscure society was the parent of all the institutions and academies afterwards established in Italy, for the prosecution of physical inquiries. The example quickly spread over Europe.

AT length the light of science penetrated into England; and the seventeenth century commenced with the successful labours of DR GILBERT of Col-

chester, whose high merit has not yet received the due meed of praise. His treatise on the Magnet was a model of the application of philosophical analysis; it soberly reduced the various facts to a few leading principles; and threw occasional gleams on other branches of science. GILBERT not only established Terrestrial Magnetism, but laid the foundation of Electricity *.

* It is pleasing to find the discoveries of GILBERT, mentioned with applause by some of his ablest successors. The following passage is extracted from the English draft of the inaugural speech recited in Latin by Mr (afterwards the famous Sir CHRISTOPHER) WREN, on his installation to the Professorship of Astronomy in Gresham College, in the year 1657.

——" And now began the first happy appearance of liberty to philosophy, oppressed by the tyranny of the Greek and Roman monarchies."——" Among the honourable assertors of this liberty, I must reckon GILBERT, who having found an admirable correspondence between his Terella, and his great magnet of the earth, thought this way to determine his great question, and spent his studies and estate upon this inquiry; by which *obiter* he found out many admirable magnetical experiments. This man would I have adored, not only as the sole inventor of magnetics, a new science to be added to the bulk of learning, but as *the father of the new philosophy*, CARTESIUS being but a builder on his experiments. This person I should have commended to posterity in a statue, that the deserved marble of HARVEY might not stand to future ages without a marble companion of his own profession. He kept correspondence with the *Lyncei Academici* at Rome, especially with FRANCISCUS SAGREDUS, one of the interlocutors in the dialogues of GALILÆUS, who laboured to prove the motion of the earth negatively, by taking off objections, but GILBERT positively; the

KEPLER had reduced the ordinary principles of Optics into a systematic form. SNELLIUS soon after discovered the law of Incidence and Refraction, which DES CARTES simplified, and employed in the explication of other properties of Light, and the brilliant phænomena of the Rainbow. But the same penetrating genius, by applying Algebra to Geometry, effected a memorable revolution in mathematical investigation, attended with the most important consequences.

ITALY had enriched the art of observation by the Thermometer, though this instrument was not brought to perfection till more than a century afterwards. But the same country produced likewise the Barometer, which TORRICELLI invented after the death of his master GALILEO. OTTO GÜERICKE in

---

one hath given us an exact account of the motion of gravity upon the earth; the other of the more secret and more obscure motion of attraction and magnetical direction in the earth; the one I must reverence for giving occasion to KEPLER, (as he himself confesses,) of introducing magnets into the motions of the heavens, and consequently of building the elliptical astronomy; the other of his perfecting the great invention of telescopes to confirm this astronomy; so that if the one be the BRUTUS of liberty restored to philosophy, certainly the other must be the COLLATINUS."

Germany, pursuing a different route, discovered the construction of the Air-Pump, and employed it in the investigation of various important phænomena. Both these instruments concurred to establish the existence of Atmospheric Pressure, and to refute the most inveterate errors that infected the ordinary creed of Philosophy. The doctrines of ARISTOTLE had frequently been attacked in detail, and they no longer inspired the same confidence and veneration. But it was reserved for the profound and daring genius of DES CARTES to demolish completely that imposing fabric. Original and inventive, he soared above the influence of prejudice, and ranged freely through the treasures of knowledge. Unfortunately he would not stoop to the slow procedure of physical analysis, but suffered himself to be hurried away by the ambition of erecting a grand system. The Cartesian principles, afterwards variously modified, maintained their ascendency over the greatest part of Europe during the space of near a century. So far they no doubt obstructed the course of genuine science; but, at the same time, they certainly extinguished for ever the scholastic wrangling, while they bore the seeds of their own destruction.

The Higher Geometry—that instrument of sublime discovery—was now advancing with a rapid progression. Cavalleri had invented his Method of Indivisibles; Wallis produced his Arithmetic of Infinites; James Gregory and Mercator discovered the doctrine of Series; and Barrow and Roberval, in their method of drawing tangents, traced the rudiments of the Differential Calculus. The theory of the collision of bodies first given by Des Cartes, had been corrected and completed by Huygens, Wallis and Wren. Mechanics were likewise enriched by many contrivances of the ingenious Dr Hooke. Huygens, who greatly surpassed him in mathematical science, having investigated the properties of oscillating bodies, applied them most skilfully to regulate the movements of clocks and watches, by connecting their train with a pendulum or a spring. But this able Philosopher, pursuing his analysis still farther, founded the prolific theory of Centrifugal Force. By the help of Geometry, he likewise discovered the law which connects the density of the strata of our atmosphere with their elevation.

SCIENCE was about to take a higher flight, when NEWTON arose, and bore away the palm of triumph. This immortal genius compressed the empirical laws of KEPLER into the single principle of *Attraction*; and descending again from that principle, he deduced, by a synthetical involution, the great phænomena of the universe. These conclusions were in general most felicitous; but where the powers of his calculus proved insufficient, he approximated to the results by some tentative process, guided by that sagacity in which he was never excelled. The same penetration that ranged through the celestial spaces could define the figure of our earth, and calculate the tides of the ocean. The properties of water and air—the motion of currents, and the propagation of sound—were equally brought under the dominion of Geometry. But the analytical procedure, directed to the decomposition of Light, in a series of conclusive experiments, disclosed still greater wonders, and

"Untwisted all the shining robe of day."

NEWTON's fine researches in Optics, as they began, so, after a long interruption, they closed his scientific labours.

NEWTON and LEIBNITZ had separately discovered, about the same time, the method of Fluxions and Fluents, or the differential and integral Calculus. The former stated the principles with more logical strictness, but the latter adopted a far preferable notation. This superior algorithm has chiefly contributed to the prodigious extension which the Higher Analysis received on the Continent. NEWTON himself made but inconsiderable advances in integration; and the progress afterwards achieved in England by TAYLOR, COTES, and MACLAURIN, however respectable, will scarcely bear a comparison with the towering ascent of the BERNOUILLIS, of the great EULER, and of D'ALEMBERT or CLAIRAULT.

THE system of mechanical philosophy was now seated on a firm foundation, though many parts of the structure still remained incomplete.—RÖMER had proved that light travels with a certain prodigious velocity, and BRADLEY very ingeniously applied this discovery to explain the Aberration of the Fixed Stars, which the delicacy of a Zenith Sector, constructed by GRAHAM, had enabled him to detect. The Newtonian doctrine still experienced some opposition on

the Continent, from the prior ascendency of the Cartesian tenets. But the mensuration of a Degree of the Meridian, performed within the Arctic Circle, and under the Equator, between the years 1736 and 1742, affording results entirely conformable to the Theory of Attraction, finally decided the victory. The Integral Calculus, now so vastly expanded, was directed to the solution of the more arduous questions which NEWTON had either not solved, or had only sketched. The conclusions hence deduced were found exactly to harmonize with observation. The greatest mathematicians in Europe have since exerted their skill in improving the more delicate parts of the theory. GAUSS has struck out new paths of investigation. The recent calculations of LAGRANGE and LAPLACE have brought to light various important and unexpected conclusions. All the anomalies in the heavens are now found to be periodical. Practical Astronomy has hence acquired remarkable precision; and the improvements of the Lunar Theory have wonderfully hastened the advancement of Navigation. The various observations of HERSCHEL, and the late discoveries of

ful hands of DAVY, BERZELIUS and others, the Voltaic Pile has displayed the most astonishing results.

MAGNETISM has also, within these few years, been advancing to maturity. The various circumstances which affect the declination and depression of the Needle are at length ascertained with some degree of precision. Empirical laws have hence been framed, that seem to indicate the changes of magnetic influence which are going forward in different parts of the surface of the earth. But the connecting principles, which would harmonize the whole, remain still unknown.

THE analogy between Magnetism and Electricity had long been suspected; but the very curious experiment of OERSTED has now put the question beyond all doubt. The Galvanic action, variously combined with the force of Magnetism, is incessantly bringing into view the most beautiful and surprising facts. The unequal distribution of heat, and the rapid circulation of certain metals, are also found to affect materially the needle. No conclusive explication has yet been offered; but every thing seems

to betoken our near approach to some grand and pervading discovery.

BUT amidst all this splendour, it must however be confessed, that science has declined from the severe majesty which distinguished the age of mechanical philosophy. Patient induction, though much commended, has very few followers at present; and the passion for hypotheses appears to have again obtained ascendancy in the learned world. Vague and fanciful images are but too often substituted for close reasoning. The more popular branches of physics have absolutely grown rank with metaphorical expression. We may therefore conclude with the judicious admonition of the finest genius of the seventeenth century. "When the weakness of men," says the acute PASCAL, "is unable to find out the true causes of phænomena, they are apt to employ their subtlety in substituting imaginary ones, which they express by specious names that fill the ear without satisfying the judgment. It is thus that the *sympathy* and *antipathy* of natural bodies are asserted

to be their efficient and unequivocal causes of several effects, as if inanimate substances were really capable of sympathy and antipathy. The same thing may be said of *antiperistasis*, and various other chimerical causes, which afford only a vain relief to the avidity of men to know hidden truths, and which, far from discovering them, serve only to conceal the ignorance of those who invent such explications, and nourish it in their followers."

ELEMENTS

OF

NATURAL PHILOSOPHY.

Natural Philosophy is the science that unfolds those general principles which connect the events of the material world. It assumes as a basis the constancy and permanence of the actual state of things.

The appearances which present themselves to our observation, are called Phænomena; and the common relations which pervade these phænomena, are termed Laws.

The business of the natural philosopher is to remount patiently from effects to causes, till he approach the Fountain of all power and intelligence; and from this eminence to again descend, and trace the lengthened chain of consequences. This double mode of procedure corresponds to the *Analysis* and *Synthesis* of the Ancient Greek Geometry.

The analysis or investigation of physical facts is conducted, either by *Observation*, or by *Experiment*. *Observation* is the close inspection and attentive examination of those phænomena which arise successively in the course of nature: *Experiment*, as the term implies, consists in a sort of trial or artificial selection and combination of circumstances, for the purpose of searching after the remote results. The main object of the philosopher is to separate always the various effects which are blended together in the ordinary concurrence of events.

The primary facts being once detected from close observation or delicate experiment, the synthetical deduction can be safely pursued, by the exercise of a sober and cautious logic. But the most important instrument, in forwarding this process of re-combination, is Geometry, to which indeed we are indebted for whatever is most valuable in Physical Science.

The most satisfactory mode of proceeding in the exposition of the phænomena, is to consider Bodies as (1.) in a state of *Rest*, and (2.) in a state of *Motion*. The essential properties which belong to each distinct body form a branch of science that may be termed Somatology, but which has hitherto been styled incorrectly *Corpuscular Philosophy*. The mutual action of bodies, which produces their equilibrium or quiescence, constitutes Statics. Those properties, again, which bodies display while in a state of motion, form the branch of science called

PHORONOMICS. But since all motion arises from the application of force, this department has been more generally termed DYNAMICS. The principles of Dynamics, applied to the revolutions of the heavenly bodies, uprear the sublime science of PHYSICAL ASTRONOMY. The same principles, directed to the process of art, elucidate the construction of machinery, and constitute the Theory of MECHANICS. The principles of *Statics* applied to liquids explain the conditions of their equilibrium, and form HYDROSTATICS. The principles of *Dynamics*, extended likewise to liquids, unfold the properties of their motions, and compose the branch of science termed HYDRAULICS, or more properly HYDRODYNAMICS. The same principles, extended to a far more subtle fluid, constitute PHOTONOMICS, or the Theory of Light, which, in reference to vision merely, has been long designated by the less perfect name of OPTICS. Nearly allied to Photonomics, is the science of PYRONOMICS, which treats of the properties of Heat. When Pyronomics embrace likewise the affections of Humidity, they comprise the science of *Meteorology*, and explain the character and condition of Climate. The principles of Dynamics are applied, besides, though as yet in a very limited extent, to MAGNETISM and ELECTRICITY ; and as these interesting branches of science shall advance to perfection, they will assuredly come more within the range of calculation.

A Course of Natural Philosophy may hence be rightly distributed under twelve distinct heads.

1. Somatology, which includes the exposition of the general Properties of Bodies that are essential to their separate existence.

2. Statics, which explains the Equilibrium of bodies, as resulting from their mutual action, or from combined pressure and divellence.

3. Phoronomics, or Dynamics, which explores the Laws of Motion, and traces the flux of changes produced by the application of *Force*.

4. Physical Astronomy, which is the extension of *Dynamics* to develope the great Phænomena of the Heavens. It explains the motions and figures of the Planets, and deduces the various consequent effects.

5. Mechanics, in which the principles of *Dynamics* descend to improve the vulgar arts, and to explain the composition and arrangement of the various Machines contrived to assist the labour of man.

6. Hydrostatics, which consists in the application of the principles of *Statics* to explain the Equilibrium of Liquids or of Fluids in general: It treats likewise of the construction of Works depending on such properties.

7. Hydrodynamics, or Hydraulics, which consists in applying *Dynamics* to the Motion of Liquids: It consequently investigates the construc-

tion and performance of the various Engines employed to raise Water, or which are driven by the impulsion of that Fluid.

8. PNEUMATICS, which includes the application of *Statics* and *Dynamics* to Air and other Gaseous Fluids: It explains the constitution, the operations, and general phænomena of our Atmosphere.

9. PHOTONOMICS, which treats of the properties and operations of Light.

10. PYRONOMICS, which explores the properties and operations of Heat.

11. MAGNETISM, which investigates the properties of the Loadstone, and their application to the suspended Needle.

12. ELECTRICITY, which explains all the brilliant phænomena derived from those first produced by the rubbing of Amber.

Such appears to be the systematic arrangement of those subdivisions; but it will admit of being conveniently modified. I shall therefore dispose them more nearly in the order of difficulty.

I. SOMATOLOGY *

Comprehends our knowledge of Bodies, or External Substances.

The properties of body are detected by the senses—either from immediate observation—or through the application of experiment and the aid of instruments. The more obvious properties are revealed to us merely by sight or touch; but the penetration of the telescope has enabled us to survey vast systems of worlds dispersed through the remotest heavens, while the opposite power of the microscope has brought within our view, from the very verge of existence, a miniature creation of organised beings. Again, the most careless observer can hardly fail to perceive that Air is a compressible fluid, while it requires a very delicate experiment to discover the same property in Water.

The properties of body are either essential and permanent,—or they are contingent and susceptible of change or variation.

Body is essentially,

1. Extended.
2. Figured.
3. Impenetrable.
4. Divisible.
5. Porous.
6. Contractile or distensible.

* From Σῶμα, a *Body*.

It is contingently,

1. Moveable. 2. Ponderable.

Of these, the two first properties essential to bodies,—EXTENSION and FIGURE,—belong equally to Space, and hence constitute the foundation of Geometry.

III. IMPENETRABILITY forms the third discriminating feature of body. This principle is commonly regarded as an axiom, but it is a truth only derived from early and invariable experience. It rests on this incontrovertible fact, that no two bodies can occupy the same space in the same precise instant of time. Had the case indeed been otherwise, each body might be successively absorbed into the substance of another, till the whole frame of the universe, collapsing into a point, were lost in the vortex of annihilation.

But although the most palpable observation attests sufficiently the impossibility of the mutual compenetration of bodies, yet this property may be farther illustrated and confirmed by a few simple experiments.

1. A vessel being filled to the brim with water, if any solid, incapable of solution in that liquid, be plunged in it, a portion of the water will overflow, which is exactly equal to the bulk of the wood or metal immersed.

2. If a cylinder be gradually pressed downwards into a glass cylinder partly filled with water, the liquid will rise proportionally, till the space mounted over by its surface shall be equal to the portion of the cylinder introduced.

These two experiments show that water opposes the entrance of a solid substance, and retires on all sides to give room for its advance. Simple as the fact now appears, it was first distinctly noticed by Archimedes, who made it the ground of his Hydrostatical Theory.

The same truth is evinced by other experiments.

3. If a cork be rammed hard into the neck of a phial full of water, the phial will burst, while its neck remains entire.

4. Bladders filled with water and disposed upon a table will support very large weights placed on a board that has been laid over them.

5. The same experiment will succeed equally with bladders blown full of air.

6. But the disposition of the air to resist all penetration is made conspicuous in another way. Let a large and very tall glass vessel be nearly filled with water, on the surface of which a lighted taper is set to float; if over this glass, a smaller cylindrical vessel, likewise of glass, be inverted and pressed downwards, the contained air maintaining its place, the internal body of the water will descend, while the rest will rise up at the sides, and the taper will appear

for some seconds to burn, encompassed by the whole mass of liquid.

IV. DIVISIBILITY is another essential property of bodies. Every substance with which we are acquainted is capable of being separated into parts, and each of these again may be repeatedly subdivided. Nor has any limit ever been assigned to this process of continual subdivision, though it seems probable that, at some term, however distant, the resulting particles may lapse into simple *atoms*, incapable of any farther resolution.

The actual subdivision of bodies has, in many cases, been carried to a prodigious extent. A slip of ivory, of an inch in length, is frequently divided into an hundred equal parts, which are distinctly visible. But, by the application of a very fine screw, five thousand equidistant lines, in the space of a quarter of an inch, have been traced on a surface of steel or glass with the fine point of a diamond, producing delicate iridescent colours. Common writing paper has a thickness of about the 500th part of an inch; but the pellicle separated from ox-gut, and then doubled to form gold-beaters' skin, is six times thinner. A single averdupois pound of cotton has been spun into a thread 76 miles in length; and the same quantity of wool has been extended into a thread of 95 miles; the diameters of those threads being hence only the 350th and 400th parts of an inch.

The finest flaxen thread drawn by machinery at Leeds has only the 155th part of an inch in thickness, so that a single pound of it would reach $17\frac{1}{2}$ miles. At Dunfermline, yarn, of the 200th part of an inch in diameter, is now manufactured from the best foreign flax. But lately, an old woman, near Belfast, span Irish flax into yarn, a pound of which would extend to 30 miles, its diameter being only the 250th part of an inch. Yet this effort of skill and patience comes far short of the fineness of the thread sometimes used in Flanders, for the fabric of lace; a specimen of which, from St Quentin, but spun at Catillon sur Sambre, had only the 700th part of an inch in diameter, a pound of it being sufficient to extend to 342 miles.

But the ductility of some metals far exceeds that of any other substance. The gold-beaters begin their operations with a ribband an inch broad and 150 inches long, which had been reduced, by passing it through rollers, to about the 800th part of an inch in thickness. This ribband is cut into squares, which are disposed between leaves of vellum, and beat by a heavy hammer, till they acquire a breadth of more than three inches, and are thus extended to ten times their former surface. These are again quartered, and placed between the folds of gold-beaters' skin, and stretched out, by the operation of a lighter hammer, to the breadth of five inches. The same process is repeated, sometimes more than once, by a succession of lighter hammers; so that 376

grains of gold are thus finally extended into 2000 leaves, of 3.3 inches square, making in all 80 books, containing each of them 25 leaves. The metal is consequently reduced to the thinness of the 282,000th part of an inch, and every leaf weighs rather less than the fifth part of a grain. A particle of gold, not exceeding the 500,000th part of a grain, is hence distinctly visible to the naked eye.

Silver is likewise capable of being laminated, but will scarcely bear half the extension of gold, or the 150,000th part of an inch thick. Copper and tin have still inferior degrees of ductility, and cannot be beat thinner perhaps than the 20,000th part of an inch. These form what are called Dutch Leaf.

In the gilding of buttons, five grains of gold, which is applied as an amalgam with mercury, is allowed to each gross; so that the coating left must amount to the 110,000th part of an inch in thickness. If a piece of ivory or white satin be immersed in a nitro-muriate solution of gold, and then exposed to a current of hydrogen gas, it will become covered with a surface of gold hardly exceeding in thickness the ten millionth part of an inch.

The gilt wire used in embroidery, is formed by extending gold over a surface of silver. A silver rod, about two feet long and an inch and half in diameter, and therefore weighing nearly twenty pounds, is richly coated with about 800 grains of pure gold. In this country the lowest proportion allowed is 100

grains of gold to a pound of silver. This gilt rod is then drawn through a series of diminishing holes, till it has stretched to the vast length of 240 miles, when the gold has consequently become attenuated 800 times, each grain being capable of covering a surface of 9600 square inches. This wire is now flatted, the golden film suffering a farther extension, and having its thickness reduced to the four or five millionth part of an inch.

It has been asserted, that wires of pure gold can be drawn, of only the 4000th part of an inch in diameter. But Dr W. H. Wollaston, by an ingenious procedure, has lately advanced much farther. Taking a short cylinder of silver, about the third part of an inch in diameter, he drilled a fine hole through its axis, and inserted a wire of platinum only the 100th part of an inch thick. This silver mould was now drawn through the successive holes of a steel plate, till its diameter was brought to near the 1500th part of an inch, and consequently the internal wire, being diminished in the same proportion, was reduced to between the 4 and 5000th part of an inch. The compound wire was then dipped in warm nitric acid, which dissolved the silver, and left untouched its core or the wire of platinum. By passing the incrusted platinum through a greater number of holes, wires still finer were obtained, some of them only the 30,000th part of an inch in diameter. The tenacity of the metal, before reaching that limit, was even considerable; a platinum wire of the 18,000th

part of an inch in diameter supporting the weight of one grain and a third.

Such excessive fineness is hardly surpassed by the filamentous productions of nature. Human hair varies in thickness, from the 250th to the 600th part of an inch. The fibre of the coarsest wool is about the 500th part of an inch in diameter, and that of the finest only the 1500th part. The silk line, as spun by the worm, is about the 5000th part of an inch thick; but a spider's line is perhaps six times finer, or only the 30,000th part of an inch in diameter, insomuch, that a single pound of this attenuated substance might be sufficient to encompass our globe.

The red globules of the human blood have an irregular roundish shape, from the 2500th to the 3800th of an inch in diameter, with a dark central spot.

The trituration and levigation of powders, and the perennial abrasion and waste of the surface of solid bodies, occasion a disintegration of particles, almost exceeding the powers of computation. Emery, after it has been ground, is thrown into a vat filled with water, and the fineness of the powder is distinguished by the time of its subsidence. In very dry situations, the dust lodged near the corners and crevices of ancient buildings is, by the continual agitation of the air, made to give a glossy polish to the interior side of the pillars and the less prominent parts of those venerable remains.

So fine is the sand on the arid plains of Arabia, that it is carried sometimes 300 miles over the Mediterranean sea, by the sweeping and violent Sirocco. Very lately, the deck of a ship was covered with impalpable dust, while navigating the Atlantic, at the distance of 600 miles from the western coast of Africa. The rocks are peopled, along the shores of the Mediterranean, by the *pholas*, a testaceous and edible worm, which, though very soft, yet, by unwearied perseverance, works a cylindrical hole into the heart of the hardest stone. The marble steps of some of the great churches in Italy are worn by the incessant crawling of abject devotees; nay, the hands and feet of bronze statues are, in the lapse of ages, wasted away by the ardent kisses of innumerable pilgrims that resort to those shrines. What an evanescent pellicle of the metal must be abraded at each successive contact!

The solutions of certain saline bodies, and of other coloured substances, exhibit a prodigious subdivision and dissemination of matter. A single grain of the sulphate of copper, or blue vitriol, will communicate a fine azure tint to five gallons of water. In this case, the copper must be attenuated at least ten million times; yet each drop of the liquid may contain so many coloured particles, distinguishable by our unassisted vision. A still minuter portion of cochineal, dissolved in deliqueate potash, will strike a bright purple colour through an equal mass of water.

A small portion of iodine, heated within a sealed

flask, will fill the space with a beautiful purple steam, which in cooling condenses into minute acicular crystals; and the fumes of ammonia, however expanded, display their efficacy by the coloration of a bit of test paper, the moment it is introduced below the neck of the phial. But odours are capable of a much wider diffusion. A single grain of musk has been known to perfume a large room, for the space of twenty years. Consider how often, during that time, the air of the apartment must have been renewed, and have thus become charged with fresh odour! At the lowest computation, the musk had been subdivided into 320 quadrillions of particles, each of them capable of affecting the olfactory organs. The vast diffusion of odorous effluvia may be conceived from the fact, that a lump of *assa fœtida*, exposed to the open air, was found to have lost only a grain in the space of seven weeks. Yet, since dogs hunt by the scent alone, the effluvia emitted from the several species of animals, and even from different individuals of the same race, must be essentially distinct.

The vapour of pestilence conveys its poison in a still more subtle and attenuated form. The seeds of contagion are known to lurk for years in various absorbent substances, which, on exposure to the air, scatter death and consternation.

But the diffusion of the particles of light defies all powers of calculation. A small taper will, in a twinkling, illuminate the atmosphere to the distance of four miles; yet the luminous particles which fill

that wide concavity cannot amount to the 5000th part of a grain, which may be the whole consumption of the wax in light, in smoke, and ashes.

Animated matter likewise exhibits, in many instances, a wonderful subdivision. Between the Tropics, the small marine polypi, by the immensity of their combined numbers, speedily raise up clusters of coral reefs, so dangerous at present to the navigation of those seas, but which are destined at no very remote period, to form groupes of inhabited and cultivated islands. The milt of a codfish, when it begins to putrefy, has been computed to contain a billion of perfect insects; so that thousands of these living creatures could be lifted on the point of a needle. But the infusory animalcules display, in their structure and functions, the most transcendent attenuation of matter. The *vibrio undula*, found in duck-weed, is computed to be ten thousand million times smaller than a hemp seed. The *vibrio lineola* occurs in vegetable infusions, every drop containing myriads of those oblong points. The *monas gelatinosa*, discovered in ditch-water, appears in the field of a microscope a mere atom endued with life, millions of them playing like the sun-beams in a single drop of liquid.

v. Porosity is also an essential property of Bodies. It is not confined to the animal and vegetable compounds, which have an evident vascular structure,

but is found in every substance that we are able to explore. This may be shown—from inspection with the microscope,—by the passing of different fluids through solid bodies,—or by the transmission of light itself in all directions through their internal substance.

1. The porosity of wood is so remarkable, that air may be blown, by the action of the mouth and lungs, in a profuse stream, through a cylinder two feet long, of dry oak, beech, elm or birch. If a piece of wood, or a lump of marble, granite, or other compact stone, be plunged under water, and placed within the receiver of a pneumatic machine; on withdrawing the external pressure, the air which had been dispersed through their interior cavities, will issue from every point of their surface, and rise in a torrent of bubbles. A similar appearance is exhibited when such bodies are sunk under mercury, which in some cases becomes finely injected through all the ramifying pores, on restoring the pressure of the atmosphere. In like manner, mercury is forced through a piece of dry wood, and made to fall in the form of a fine divided shower.

If a few ounces of mercury be tied in a bag of sheep skin, it may be squeezed through the leather by the pressure of the hand, in numerous minute streamlets. This experiment illustrates the porosity of the human cuticle. From microscopic observations, it has been computed that the skin is perfo-

rated by a thousand holes in the length of an inch. If we estimate the whole surface of the body of a middle-sized man to be sixteen square feet, it must contain no fewer than 2304 million pores: and these pores are the mouths of so many excretory vessels, which perform that important function in the animal economy—*Insensible Perspiration.* The lungs discharge every minute 6 grains, and the surface of the skin from 3 to 20 grains; the average over the whole body being 15 grains of lymph, consisting of water, with a very minute admixture of salt, acetic acid, and a trace of iron. At this rate of transpiration, the loss would amount to $3\frac{3}{4}$ pounds troy, in the space of twenty-four hours. If we suppose this perspirable matter to consist of globules only about 30 times smaller than the red particles of blood, or about the 10,000th part of an inch in diameter, it would require a succession of three of them to issue from each orifice every minute.

The permeability of a solid body to any fluid, depends however on its peculiar structure and its relation to the fluid. A compact substance will sometimes oppose the entrance of thin fluid, while it gives free passage to a gross one. Thus, a cask, which could hold water, will permit oil to ooze through it; and a fresh humid bladder, which is air-tight, will yet, when pressed under water, imbibe a notable portion of that liquid. If a cylindrical piece of oak, ash, elm, or other hard wood, cut in the direction of its

fibres, be cemented to the end of a long glass tube, water will flow freely through it, in divided streamlets; but a soft cork inserted into a similar tube will effectually prevent all escape of the liquid. Mercury may be carried in a small cambric bag, which would not retain water for a moment. If a circular bottom of close-grained wood, divided by a fine slit from the 30th to the 100th part of an inch wide, be cemented to the end of a glass tube, it will support a column of mercury, from one to three or more inches high, the elevation being always proportional to the narrowness of the slit. Hence a cistern of box-wood is frequently used for portable barometers, the fine joints admitting the access and pressure of the air, but preventing the escape of the mercury. Yet a sufficient force might overcome this obstruction; and, in the same manner, the air which is confined in the common bellows under a moderate pressure, will, by a more violent action, be made to transpire copiously through the boards and the leather. The transmission of a fluid through a solid substance shows the existence of pores; but the resistance, in ordinary cases, to such a passage, is insufficient, therefore, to prove the contrary.

The permeability of translucent substances to the rays of light, in all directions, evinces the most extreme porosity. But this inference is not confined merely to the bodies usually termed diaphanous; for the gradation towards opacity advances by insensible

shades. The thin air itself is not perfectly translucid, nor will the densest metal absolutely bar all passage of light. The whole mass of our atmosphere, equal to the weight of a column of 34 feet of water, transmits, according to its comparative clearness, only from four-fifths to three-fourths of the perpendicular light, and consequently retains or absorbs from a fifth to a fourth part of the whole. But this absorption is greatly increased by the accumulation of the interposed medium. When the sun has approached within a degree of the horizon, and his rays now traverse a tract of air equal in weight to a column of 905 feet of water, only the 212th part of them can reach the surface of the earth.

By a peculiar application of my Photometer, I have found that half of the incident light, which might pass through a field of air of the ordinary density and 15½ miles extent, would penetrate only to the depth of 15 feet in the clearest sea-water, which is therefore about 5400 times less diaphanous than the ordinary atmospheric medium. But water of shallow lakes, though not apparently turbid, betrays a greater opacity, insomuch, that the perpendicular light is reduced one half, in descending only through the space of six, or even two feet. The same measure of absorption would take place in the passage of light through the thickness of two or three inches of the finest glass, which is consequently 500,000 times more opaque than an equal bulk of air, or

300 times more opaque than an equal weight or mass of this fluid.

But even gold is diaphanous. If a leaf of that metal, either pure or with only an 80th part of alloy, and therefore of a fine yellow lustre, but scarcely exceeding the 300,000th part of an inch in thickness, and inclosed between two thin plates of mica, be held immediately before the eye, and opposite to a window, it will transmit a soft green light, like the colour of the water of the sea, or of a clear lake of moderate depth. This glaucous tint is easily distinguished from the mere white light which passes through any visible holes or torn parts of the leaf. It is indeed the very colour which gold itself assumes, when poured liquid from the melting pot. A leaf of pale gold, or gold alloyed with about the 80th part of silver, transmits an azure colour; from which we may, with great probability, infer, that if silver could be reduced to a sufficient degree of thinness, it would discharge a purple light. These noble metals, therefore, act upon white light exactly like air or water, absorbing the red and orange rays which enter into its composition, but suffering the conjoined green and blue rays to effect their passage. If the yellow leaf were to transmit only the tenth part of the whole incident light, we should only conclude, that pure gold is 250,000 times less diaphanous than pellucid glass.

The inferior ductility of the other metals will not allow that extreme lamination, which would be requisite, in ordinary cases, to show the transmission of light. But their diaphanous quality may be inferred, from the peculiar tints with which they affect the transmitted rays, when they form the alloy of gold.

Other substances, which are commonly reckoned opaque, yet permit in various proportions the passage of light. The window of a small apartment being closed by a deal board, if a person within shut his eyes a few minutes to render them more sensible, he will, on opening them again, easily discern a faint glimmer issuing through the window. If this board be plained thinner, more light will successively penetrate, till the furniture of the room becomes visible, and perhaps a large print may be distinctly read.

Writing paper transmits about the third part of the whole incident light, and when oiled it often supplies the place of glass in the common work-shops. The addition of the oil does not, however, materially augment the diaphanous quality of the paper, but renders its internal structure more regular, and more assimilated to that of a liquid. The rays of light travel, without much obstruction, across several folds of paper, and even escape copiously through pasteboard.

Combining these various facts, it follows, that all bodies are permeable, though in extremely different degrees, to the afflux of light. They must therefore be widely perforated, and in every possible direction. The porosity of bodies is consequently so diffuse, that the bulk of their internal kernel, or of the ultimate obstacles which they present, may bear no sensible proportion to the space which they occupy.

VI. The last essential property which belongs to all bodies, is that of CONTRACTION OF DILATATION. Though absolute penetration is impossible, yet every substance, however dense or compact, can yet have its volume enlarged or diminished. This change of bulk is in some instances quite apparent, while, in other cases, to render it visible, requires either the agency of a vast force, or the application of some very delicate measure. The effect is produced on fluids, either by increasing or diminishing their ordinary external compression; but solid substances will have their volume contracted or distended by squeezing or pulling. A few experiments will confirm and illustrate the general proposition.

1. If a flaccid bladder be placed within the receiver of an air pump, it will gradually swell as the exhaustion advances, but, on restoring the external pressure, it will again shrink into its former dimensions.

2. If a tall flask, nearly filled with water, be inverted in a jar of water, and placed likewise under a pneumatic receiver, the air collected near the top of the flask will visibly expand as the operation of pumping proceeds, till it presses down the water, and begins to make its escape from below in the form of rarefied bubbles.

3. But air is easily compressed, by the opposite action of a syringe. In the vault or chamber of the air-gun, it is often condensed fifty or even eighty times. Nearly the half of that charge may be thrown into the pneumatic blow-pipe, from which, on partially opening the valve, it will again continue to flow for the space of a quarter of an hour.

Other gases are likewise notably contracted or dilated, by the increase or diminution of external pressure. But liquid substances themselves manifest a similar property, though in a much lower degree. If a large glass-ball, terminating in a long narrow and open stem divided into minute spaces corresponding to the millionth parts of the whole capacity, be filled with distilled water carefully purged of air, and introduced under the receiver of a pneumatic machine; as the exhaustion advances, the water will proportionally expand and rise near fifty divisions; but, on admitting the atmosphere again to compress the water, it will sink to its former place in the stem. The contraction which the water suffers, at every increase of pressure, exceeds not indeed the 20,000th

part of what air would undergo in like circumstances; but it is equally real, and evinces an inherent property of that liquid. Mercury treated in the same way shows a contraction three times less than water. Alcohol, ether, oils, and the various acids and saline solutions, are all condensed or expanded, though in different degrees, by the change of atmospheric pressure.

If the stem of the instrument now mentioned were made to screw to the ball at a wide aperture, fragments of solid bodies could be easily introduced, and the vacant space filled up with water; the contraction of this portion of water being deducted from the contraction of the mixture, would give the distinct condensation of the hard materials. In this way, the compressibility of the various stones and metals could be accurately examined. A series of experiments of this nature would essentially contribute to the improvement of the mechanical arts.

The contraction and distension produced by external or internal pressure on glass is quite visible, in a thermometer with a large bulb and very long tube. When the mercury stands near the top of the scale, it will immediately rise on reclining the tube, and will continue to flow till the thermometer has been reversed; but the mercury will again retreat, as the instrument is brought back to its vertical position. This experiment proves that the bulb has its capacity sensibly enlarged by the thrust of the mercurial column.

When long bars of wood, iron, or other metals are laid horizontally on supports, they bend downwards by their own weight, and this depression is increased by augmenting the incumbent pressure. See fig. 1. The upper fibres are therefore drawn into a narrower curve than those in the middle of the bar, while the under fibres are extended into a wider convexity: the particles of the former are thus contracted, and those of the latter distended. It is likewise obvious, that in this incurvation, the contraction or dilatation occasioned will be proportional to the thickness of the bar.

The various kinds of wood are far more compressible than water, and suffer, hence, a very considerable degree of condensation, on being let down to great depths in the ocean. Pieces of oak, ash or elm, plunged two or three hours in a calm sea, at the enormous depth of a thousand fathoms, and then drawn up, have been found to contain four-fifths of their weight of water, and to acquire such increase of density, as indicates a contraction of the wood into about half its previous volume. The specimens which have undergone this singular change, if thrown into a pail of water, will sink like a stone. Hence probably the reason why barks lost near the shore are afterwards discovered by their timbers breaking up and floating to the surface; while the ships which founder in the wide ocean, acquiring permanent density from the vast compression they sustain, remain

motionless at the bottom, and never rise again to disclose their fate.

But pieces of wood, even of the softer kinds, whether round or square, may be easily squeezed in the direction perpendicular to their fibres, by the action of a common vice. If allowed to stand only a few minutes under that compression, and then quickly thrown into a jar of water, they will sink and remain at the bottom. If the wood be kept much longer under the vice, it will take a *set*, and become constitutionally denser. Even cork may, by compression, be made to sink in water; but as its texture is nearly uniform, the force must be exerted on all sides. Into a thick and very strong glass cylinder, having a syringe adapted to it, introduce a large cork ball, and inject the air by smart and powerful strokes; the ball will gradually shrivel, till it has contracted even to less than one-third of its usual bulk; but, on allowing the charge of air to escape, the cork will speedily resume its former shape and dimensions. If the condenser be partly filled with water, on which the ball of cork is set to float, it will, under a like compression, though it has a minute portion of the liquid driven into its substance, shrink to nearly the same size as before, and soon fall to the bottom. Hence the success of the common experiment at sea, of letting down, in calm weather, to the depth of twenty or thirty fathoms, an empty corked bottle, and then drawing it up full of water, though the cork still remains in the neck.

The water is not in this case forced through the pores of the cork as generally supposed, but the cork itself, being condensed by the lateral pressure of the incumbent mass of liquid, allows it to enter by the sides and dislodge the air.

Bodies are also contracted or dilated, from the operation of some internal cause. Thus, dry air is visibly expanded by its union with moisture. If, in a warm room, the inside of a tall flask be wetted by a few drops of water, and the mouth inverted in a bason of water, the contained air, in proportion as it becomes humified, will discharge a copious stream of bubbles. On the other hand, a notable contraction of their joint volume is produced in the absorption of water by saw-dust, linen, or bibulous paper. The combination of equal measures of water and alcohol is accompanied by a contraction amounting to the fiftieth part of the whole bulk. A similar effect results from the solution of the sulphate of soda, and other highly soluble salts.

The alliage of different metals often betrays a large contraction. The power of tin to condense copper in the composition of bronzes was even remarked by the ancients, who combined these metals in various proportions, to form their knives, chisels or hatchets. Equal bulks of tin and copper are found to suffer a contraction amounting to not less than the fifteenth part of their whole volume.

A liquid, in joining any solid substance, commonly

occasions a general contraction; but the solid itself may yet by this accession expand with prodigious force. Thus, dry peas being rammed into a gun barrel, and their interstices filled with water, will in a short time burst the barrel. In like manner, if wedges of soft dry wood be driven into slits made with a saw in blocks of freestone or marble, and then have water poured upon them, these wedges will quickly swell and rend the rock. Such was the ancient mode of quarrying, before the explosive power of gunpowder came to be introduced. It is still practised in the art of cutting mill-stones in France, holes being bored at intervals in a line drawn across the block, and wooden plugs driven into them, and then wetted.

Hence, if the side of a thin piece of wood be moistened, it will bend backwards, the humidity insinuating itself into the soft parenchymatous matter between the fibres, and therefore enlarging the circle of flexure. The thinner the wood is sliced, the greater will evidently be the incurvation produced by the wetting of its convex surface. The fibres of hair and wool, by the unequal rubbing and moistening of their sides, are made to curl up, and to condense like a clue. On this property seems to be really founded the very important process of milling, fulling or felting, by which a raw web of woollen cloth is thickened, and its texture rendered firm and compact.

A leathern thong is extended by wetting, and so

are the filaments of hemp or flax when laid parallel. But the moisture spreads chiefly through the longitudinal interstices of those filaments, and consequently enlarges their thickness. As a hempen cord is shortened by twisting, so is it likewise contracted by wetting, the diameter of the coil being thereby increased, and the extension of the overlapping fibres hence proportionally curtailed. If a spunge dipped in hot water be drawn more than once along a well-spun rope, it will in dry weather occasion, during the space perhaps of an hour, a contraction amounting to the fifteenth or twentieth part of the whole length. This remarkable property has in some instances been employed as a very efficient mechanical agent.

The most powerful principle of internal expansion, is the introduction of Heat. This energy varies exceedingly, however, in different substances, but it can always be reduced to calculation, by comparing its effects with the opposite influence of external compression. Thus, the same absolute portions of heat communicated to cylinders of one inch diameter and height, of air, alcohol, water, mercury and copper, would enable those columns respectively to sustain the weights of 10, 12, 3 and 2 pounds.

The properties which may be regarded as only Contingent, and not Essential to the Constitution of Bodies, are MOBILITY and PONDEROSITY.

I. Every body at rest can be put in *motion*, and if no impediment intervenes, this change may be effected by the slightest external impression. Thus, the largest cannon-ball, suspended freely by a rod or chain from a lofty cieling, is visibly agitated by the horizontal stroke of a swan-shot, which has gained some velocity in its descent through the arc of a pendulum. In like manner, a ship of any burthen is, in calm weather and smooth water, gradually pulled along even by the exertions of a boy. A certain measure of force, indeed, is often required to commence or to maintain the motion; but this consideration is wholly extrinsic, and depends on the obstacles at first to be overcome, and on the resistance which is afterwards encountered. If the adhesion and intervention of other bodies were absolutely precluded, motion would be generated by the smallest pressure, and would continue with undiminished energy.

II. The other Contingent Property of Bodies is Ponderability. Every substance within our sphere of observation is found to possess *weight*, or *a disposition to gravitate towards the centre of the earth.* But to constitute gravity, it is not required that a body should invariably fall to the ground. Smoke ascends in the atmosphere, and a lump of lead rises in a tube of mercury, from the same cause that a pine tree, plunged into a lake, mounts again to the surface. Withdraw the air, the mercury, and the wa-

ter, which supported those comparatively lighter substances, and the smoke, the lead, and the timber, will immediately descend. Pour mercury over a smooth piece of cork applied to the bottom of a glass, and it will remain in the same situation, while an iron-ball can be set to float on the liquid metal. The order of Nature might here seem to be reversed. But since mercury does not insinuate itself through a very narrow interstice, it merely rests on the upper side, without pressing against the under side, of the cork. If *Levity*, however, as the Schoolmen asserted, had been a real property belonging to certain bodies, the smoke and the cork would, in every instance, have occupied the lower stations.

But the weight of a body is not the same in all places and situations. A lump of lead, which weighs a thousand pounds at the surface of our globe, would lose two pounds as indicated by a spiral spring, if carried to the top of a mountain four miles high; and, if it could be conveyed as deep into the bowels of the earth, it would lose one pound. The same mass transported from Edinburgh to the Pole would gain the addition of three pounds; but if taken to the Equator, it would suffer a loss of four pounds and a quarter.

The variable, and therefore contingent, weight of bodies, is only the gradation of that mutual and universal tendency, diminishing as the square of the distance, which retains the Moon in her orbit, and up-

holds the circulation of the whole system of Planets around the Sun. The gravitation even of small masses towards each other, such as balls of lead separated only by the interval of a few inches, has been detected by delicate experiment, and reduced to rigorous calculation. But when the approximation is closer, this force acquires a modified character, and passes into *Cohesion*. Thus, if two leaden bullets have a little portion of the surface of each pared clean, and be then pressed together with a slight twist, they will cohere firmly into one mass. In the same manner, gold or silver foliage and other ornaments, struck with a heavy hammer into the surface of polished iron or steel, become permanently united.

Within other limits, the tendency to mutual approach is changed into an opposite quality. Thus, drops of rain or dew run along the smooth and glossy surface of a cabbage leaf without spreading. If the dust of the *Lycopodium*, or Club-Fern, or even fine pounded rosin, be strewed on water contained in a glass, any smooth rounded piece of soft wood will float upon it, or may be immersed in the liquid, without being wetted, the powder preventing, by its repulsion, all contact of the water. A fine needle laid on the surface of water makes a dimple in which it swims. On the same principle, the slender limbs of insects, and the minute down which covers their wings, protects them from the penetration of humi-

dity. If the hand be rubbed with linseed oil, it may be plunged with impunity for a few seconds in boiling water, the oil repelling the water, and consequently checking the communication of heat. The application of palm-soap to the skin is still more effectual.

It thus appears that bodies are indefinitely porous, compressible without limits, and capable of assuming all varieties of form. How different is the constitution of ice, of water, and of steam? Examine the mutable aspects which mercury exhibits. Beginning at a low degree of cold, and ascending through the gradations of heat, we find it a friable solid, next a shining liquid, then a penetrating vapour, and lastly reduced to a fine red powder. A bright piece of any of the ductile metals passes successively into an earthy oxide and a pellucid glass. Charcoal is absolutely of the same nature as the diamond; yet what a contrast between the dingy appearance of the one and the dazzling lustre of the other? How variously are substances transformed by the operations of art? The skins of animals become changed into parchment and different kinds of leather, and its shreds into glue. The vegetable fibres are converted into matting, cordage, and linen cloth; and the rags of this again, reduced to a pulp, and manufactured into paper.

How diversified appear the compounds of the farinaceous substances! By a distinct operation, the

same grain produces gruel, bread, biscuit, starch, and a hard pellucid concretion resembling mother of pearl. But the plastic powers displayed in the process of vegetation and animal life infinitely surpass the resources of art. Many plants are fed by water and air alone, and consequently these fluids are capable of being transmuted into all the various products. In short, Nature exhibits only a chain of endless metamorphoses: The substance or material remains unchanged, but its form undergoes continual mutations.

The properties of bodies result from those of their component particles. At certain mutual distances they remain quiescent; but, at other distances, they show a disposition either to approach or to recede. Such opposite tendencies are commonly referred to the principles of *attraction* and *repulsion*. But all those diversified effects may be comprehended under a general law, which connects the mutual action of particles with their relative distance. In the language of modern analysis, the corpuscular energy is always some *function* of the distance; and it may be represented by an extended curve, of which the abscissæ mark the distances, and their ordinates express the corresponding forces. Fig. 2. exhibits this curve of primordial action; in which A denotes an action or ultimate particle, and B, C, D, E, &c., the successive positions of another particle, the perpendiculars CM, EN, GO, IP, LQ, &c., representing

their mutual action attractive between B and D, F and H, K and X, when above the axis AX, and repulsive between D and F, H and K, below it. The final branch of the curve must gradually assimilate itself to the law of universal gravitation. But the primary branch of the curve must, in like manner, continually approach AY, the perpendicular to the axis; and since no pressure or impulsion can ever accomplish the penetration of matter, it follows, from the principles of Dynamics, that the space included between the curve and that asymptote must be *infinite*.

Where the curve repeatedly crosses the axis, are so many quiescent positions, B, D, F, H, K, &c. in any one of which a particle would continue *in equilibrio*. But this equilibrium is of two kinds, the *stable* or the *instable*; the former easily recovering itself from any slight displacement, and the latter when once disturbed being irremediably dissolved. If the curve in its progress cross the axis from the side of repulsion to that of attraction, its intersection will evidently be a point of stability; for if a particle be pushed inwards, it will then be repelled back again; and if it be pulled outwards, it will experience an attractive force, which will recall it to its first position. But if the curve pass from attraction to repulsion, its intersection with the axis is a point of instable equilibrium; for, in proportion as a particle is pressed inwards, it will be pulled forcibly from its position; and if it be drawn outwards,

the repulsion, now conspiring, will bear it along with accumulating power. Thus, B, F and K, the transitions from repulsion to attraction, are points of stability; but D and H, the opposite transitions, are points of instability.

According as the ordinates, near the points of transition, increase less or more rapidly, the tendency of the particles to coalesce, or to separate, will be proportionally feeble or intense. If the curve cut the axis very obliquely, therefore, it will mark a limit of *languid* cohesion, as at the point F; but if it shoot nearly at right angles across the axis, as at B or K, it will indicate a limit of *powerful* cohesion.

Those atoms or ultimate particles have no sensible magnitude. But though the range of our conceptions may be unbounded, every thing in the material world appears to be distinct and determinate. Experience indeed informs us to what astonishing degree matter can be attenuated; but philosophy descries the existence of certain fixed or impassable limits at which the capability of farther subdivision utterly ceases. The primordial line of action is hence a physical, and not a mathematical, curve; or it is not strictly incurvated at every point, but proceeds by successive minute deflections, corresponding to the breadth of the elementary particles. Such a modification of the curve is represented by fig. 3.; being a serrated line, whose gradations answer to the successive stages of corpuscular action. Continuous

shades, indeed, exist only in our modes of conception; and Nature exhibits always individual objects, and advances by finite steps.

The material world is thus reducible to atoms, actuated by forces depending merely on their mutual distances. From such simple elements—the different arrangements of the particles—and their multiplied interior combinations, this sublime scene of the universe derives all its magnificence and splendour.

II. STATICS

Imports the stability resulting from the balance of connected bodies. It therefore explains the conditions that must determine any material system, among the several parts of which a mutual action is exerted, to maintain the state of quiescence.

Our ideas of *Power*, *Force*, *Action*, *Energy*, and other similar objects of contemplation, seem all to be derived from the muscular effort which we find, in our operations, required to precede or produce every external change. This feeling we spontaneously transfer to inanimate bodies themselves; and while the effects are the same, we associate likewise their origin with the same terms. *Force*, in its simplest form, may hence be represented by *weight*, whether this be made to act by *pressure* or by *traction*, or whether it *pushes* or *draws* the point of attachment. *Power* and *energy* are more complex conceptions, and may be regarded as the results or modifications of the application of force.

The fundamental principles of Statics, however few and simple, are to be discovered only by experiment, or an appeal to the actual constitution of Nature. But the primary relations being once detect-

ed, their various combinations, and the whole train of derivative properties, are easily traced by the help of Geometry.

To exclude all unnecessary complication, I shall consider, in this inquiry, the forces as directed to single particles or physical points. These forces may be conveniently measured by help of a spiral steel spring lodged in a thin cylindrical case, the traction or weight applied to it being indicated by the protrusion of a divided scale fixed to the remote end of the spiral. It will be sufficient to examine, 1. The equilibrium of *two* forces; and, 2. The equilibrium of *three* forces, in the limited case where two of these are equal.

1. Let (fig. 4.) the spring A, holding a little ball P, be suspended from a hook, and to the lower side of this ball append another similar spring B. If now a weight of 10 pounds be attached to the end of B, both the scales will descend till the ball P comes to rest, when they will stand opposite in the same vertical line, and each mark 10 divisions. If 10 pounds more be added, the scales will lengthen still farther in the same directions, and indicate the strain of 20 pounds. If, instead of hooking the ball P immediately to springs, a piece of small cord be fastened to each side of it, and then attached to those springs, the result will be precisely the same; the tension of any weight applied being still directed in the perpendicular, and indicated alike by both scales. The

same effects will take place, if; instead of weights, an exertion of animal power had been applied.

It hence follows, *that a body will remain at rest if it be urged by equal and opposite forces.* This truth is commonly assumed as intuitive; but if the proposition appears now so simple and natural as readily to command our assent, it is only because it accords with the information of our earliest experience. The untutored mind will hastily admit things that are most erroneous.

2. Let two similar springs be attached at the points A and B, (fig. 5.), situate in a horizontal line at the interval of twenty-four inches, and connect their ends by a silk thread capable of being lengthened at pleasure, and from the middle of which, at the point C, successive weights can be appended. It will soon be perceived that the direction CW of the weight will bisect the angle ACB, and occupy the same vertical plane. When the weight is increased from 2 to 4 pounds, the corresponding depression DC will increase likewise from 1 to 2 inches, while the strain at A and at B will still measure 12 pounds. But on farther augmenting the weight, though the quantity of depression will continue to follow the same ratio, yet the force of oblique traction will now begin to increase. If the weight be 10 pounds, the depression being 5 inches, the points A and B will be drawn in the directions AC and BC, each with a force equal to 13 pounds. Again,

if the weight applied at C amounted to 18 pounds, the depression DC would be 9 inches; but the strain at A and at B would reach 15 pounds. And, finally, on augmenting the weight to 32 pounds, and lengthening the thread, the depression will increase to 16 inches, while the strain of each spring will amount to 20 pounds.

It is easy to see, that since the angle ACB is always bisected, one-half of the weight at C is supported by the oblique traction of the spring A, while the other half is supported by the equal and similar traction of the spring B. But when half the weight was 5 pounds, and the depression DC 5 inches, the hypotenuse AC must have been 13 inches, while the strain in that direction measured 13 pounds. In like manner, when DC was 9 and 16 inches, the hypotenuse has been 15 and 20 inches, and the strain at A and at B indicated so many pounds. The force by which the point C is pulled in the direction CA, is to the weight by which it is drawn in the direction CW or DC, as AC to CD, or as radius to the cosine of the angle ACD. In general, therefore, it follows from this simple experiment, that *the oblique action of any force is proportional to the cosine of its inclination.*

Not to embarrass the investigation, I have thrown out of sight the weight of the spiral springs, which could be rendered very inconsiderable. But the experiment is greatly simplified, by substituting for

those springs, fine pulleys, over which a long thread is passed, holding an equal weight at each end; for it is readily perceived that the only effect of the pulley is to change the direction, without impairing its intensity, of any force applied at the circumference.

The principle now derived from observation will enable us to assign the conditions under which any two forces are balanced by a third force. Let a physical point A (fig. 6.) remain at rest, while it is drawn by three forces in the several directions AB, AC and AD, expounded or represented by the lengths of those lines. Produce DA till AE be equal to it; and the force AE, being thus equal and opposite to AD, must exert the same effect as the joint action of AB and AC. Through A, draw GAH at right angles to DE, meeting the parallels BG and CH. Wherefore the oblique action of the force AB on the point A is expressed by AG; and, for the same reason, the oblique action of AC is denoted by AH. But since the point A continues at rest, the equal forces AG and AH, by which it is urged, must likewise be exactly opposite. Consequently the lines AG and AH lie in the same plane, and therefore the force AD, or its extension AE, must always act in the plane of its balancing forces AB and AC. But the force AB draws the point A in the direction AE, with a force represented by AF; and the force AG draws it in the same direction, with a force expressed by AI. Wherefore

these two forces AF and AI are equivalent to the resulting force AE, and hence the segment FE must be equal to AI. Join EB and EC; and the triangles EBF and ACI, having the side FE equal to IA, the side FB or AG or AH equal to IC, and the contained angle EFB equal to the right angle AIC, are equal. Whence the angle FEB is equal to IAC, and these angles being alternate, the side BE is parallel to AC; but they are equal, and therefore AB and CE are likewise parallel. *Wherefore, if the lines expressing any two forces be formed into a parallelogram, its diagonal will expound the resulting force, or that which is equal and opposite, to the third or balancing force.*

This remarkable property is called the *Parallelogram of Forces*; and all the problems in Statics are reducible to the composition and resolution of forces, or to the finding of the sides of a parallelogram from its diagonal, and of the diagonal from the sides of the figure. But the solution is often simplified, by means of other derivative properties.

In the triangle ABE, it is evident from Plane Trigonometry, that AB : BE or AC : : sin AEB or CAE : sin BAE. *When two forces, therefore, balance any third force, they are inversely as the sines of the angles which they make with it.*

If AE (fig. 7.) be made radius, the perpendiculars EK and EL will evidently be the sines of the angles BAE and CAE. Wherefore, *When two*

forces balance a third force, they are inversely proportional to the perpendiculars let fall from any point in its extension upon the lines representing them.

From the point A, (fig. 7.) draw AM and AN perpendicular to AB and AC, and terminated by MON, a perpendicular to AE. The exterior angle MAE is equal to both the interior angles AMO and MOA; and taking away the right angles MAB and MOA, and there remains the angle BAE equal to AMO or AMN. In like manner, the angle CAE or AEB is proved to be equal to ANM. Wherefore the triangle MNA is similar to ABE, and AB : BE or AC : : AM : AN. Whence *three forces in equilibrio are proportional respectively to the several sides of a triangle drawn perpendicular to their directions.* A triangle so constructed is sometimes called the *Triangle of Forces.*

To find the force resulting from the combination of several forces acting upon a point in the same plane. Let the point A (fig. 8.) be drawn by the forces AB, AC, AD, and AE, extended all in one plane. Complete the parallelogram CABF, and the diagonal AF will exhibit the result of the forces AB and AC. Complete the parallelogram DAFG, and its diagonal AG will express the result of the three forces AB, AC, and AD. In like manner complete the parallelogram EAGH; and the diagonal AH will represent the force compounded of all the four

forces AB, AC, AD, and AE. But the construction may be simplified by drawing the lines BF, FG and GH equal and parallel to AC, AD and AE, and finally joining AH, which will express the resulting force.

If the directions of the forces AB, AC, and AD, (fig. 9.) lie in different planes, complete the parallelogram BACE, and the diagonal AE will express the result of AB and AC. Next, in the plane of AD and AE complete the parallelogram DAEF, and the diagonal AF will represent the compound force, which thus forms the diagonal of the parallelopiped.

If three connected points be urged by three balancing forces in the same plane, their directions will always tend to the same centre. Suppose the physical points A, B and C (fig. 10. and 11.) to be disposed into a triangle, either by threads or inflexible wires, and let them remain at rest, while they are pulled by forces acting in the directions AG, BH and CI, these lines when produced will meet in the same point O, either within or without the triangle. For from O, the concurrence of GA and HB, let fall the perpendiculars OD, OE and OF upon the sides of the triangle, and join OC. The force AG is held *in equilibrio* by the retraction of the threads in the directions AB and AC; whence the tension AB is to the tension AC, as OE to OD. For the same reason, the action of the force BH is

kept in check by the threads pulling the point B in the directions BA and BC; consequently the tension BA is to the tension BC as OF to OD. But since a general equilibrium obtains, the point B must be drawn with a force equal and opposite to that with which the point A is drawn in the direction AB. Wherefore, by equality of ratios, the tension AC is to the tension BC, as OF to OE. But these tensions must be equal to the opposite tensions CA and CB, by which the point C is pulled by its attached threads. The balancing force CI is, therefore, an extension of OC, and passes through the same centre O.

If the centre of concurrence O lies without the triangle, (fig. 11.), the points A and B must be connected by an inflexible rod; for the force AG is checked by a force proportional to OD, drawing the point A in the direction AC, and by another force proportional to OE, pushing it in the direction BA. If the point O coincides with the vertex C of the triangle, the forces AG and BH will be directed along the sides AC and BC, and no power will be exerted at the base, either to distend or contract the mutual distance of the points A and B.

It is obvious that the three balancing forces AG, BH, and CI will have the same intensity as if they originated in O, the centre of concurrence. If, therefore, KOL (fig. 12.) be drawn perpendicular to CO, and KM and LM perpendicular to BO and

AO, the sides KL, KM, and LM of the triangle thus formed will represent the forces CI, BH and AG.

Any oblique force AB (fig. 13.) may be decomposed into a force BC perpendicular to a given plane, and another force AC coincident with the plane. But such a force may likewise be resolved into three given forces at right angles to co-ordinate planes. Thus, let three planes mutually perpendicular pass through the point A (fig. 14.); the oblique force AB may be reduced to CB perpendicular to the plane AEC, and CA lying in that plane. Again, the force CA may be reduced into CE perpendicular to the plane AEF, and CD perpendicular to ADG. Wherefore the oblique force AB is decomposed into the perpendicular forces CB, CD, and CE.

Suppose it were required to find the figure which would be assumed by several connected inflexible lines attached by the ends to two fixed points, and having parallel forces applied at their several junctions. Let the lines AB, BC, CD, DE and EF, consisting either of threads, as in fig. 15., or of slender wires, as in fig. 16., tied or jointed at B, C, D, and E, and fastened to the points A and F, have the weights or parallel forces BG, CH, DI and EK applied to them, and pulling downwards or upwards. To maintain the equilibrium of these forces, it is evident that the compound or polygonal line

ABCDEF must take some determinate form. Thus, the point B (fig. 15.) is balanced by three forces, or by the weight BG, the strain BA, and the strain BC; and the point C again is kept at rest by the strain CB, the strain CD, and the weight CH. In like manner, the points D and E are each maintained in equipoise by the action of triple forces, composed of vertical weights and oblique strains. The same forces are exerted in fig. 16., only in opposite directions, the strains being there converted into thrusts.

But the strains BA and BC, (fig. 15.), balancing on both sides of the force BG, are as the sine of the angle CBG to the sine of the angle ABG; and for the same reason, the strains CB and CD, exerted at C, are as the sine of the angle DCH to the sine of the angle BCH. Now, the points B and C being both at rest, the strain BC must evidently be equal to the opposite strain CB, while the sine of CBG is equal to the sine of its supplemental angle BCH. Wherefore, by compounding the analogies, the strain BA is to the strain CD, as the sine of the angle DCH is to the sine of the angle ABG. In fig. 16. the same forces act as thrusts. Wherefore, generally, *at any point B, C, D and E, the strain or thrust is inversely as the sine of the angle which this makes with a vertical line, or directly as the secant of the angle which it forms with the horizon.*

It is evident that, in both figures, an oblique strain

or thrust AB, if reduced to the horizontal direction AL, will be diminished in the ratio of AB to AL, or of the radius to the cosine of the inclination LAB. Wherefore, since the strain or thrust at A is proportional to the secant of that angle, its action in the line of the horizon AF is always as the radius merely. The horizontal strain or thrust, as might be expected, is hence the same through the whole system of points.

Supposing the junctions to be multiplied indefinitely, the series of weights may be conceived as diffused over the chain, disposing it into a regular curve. If the base AL (fig. 17. and 18.) be therefore divided into equal parts, and the verticals MB, NC, OD, &c. drawn, the intercepted portions AB, BC, CD, &c. of the curve, being evidently *proportional to the secants of the angles of deflection from the horizon, must express the strains or thrusts exerted at the several points A, B, C, &c.*

By a similar decomposition of forces, it is easy to discover the relation of the weights necessary to form a given polygon. In fig. 15. and 16., the strain or thrust BA is to the weight acting at B, as the sine of the angle CBG is to the sine of the angle ABC; but if the polygon be supposed to be contained in a given arc of a circle while its sides are given, this angle ABC must have a given magnitude; consequently the strain BA is to the weight BG, as a constant quantity to the secant of the deflection at B.

But the strain at B was already shown to be proportional to that secant, and therefore *the weights at B must be as the square of the secant of deflection.* This elegant property can easily be exhibited experimentally.

Suppose the junctions to be indefinitely multiplied, the several weights being disposed in the arc AFL of a circle, (fig. 19. and 20.), which has O for its centre. Let this arc be distinguished into minute equal portions AB, BC, CD, &c. and having applied the tangent FQ, draw the secants OM, ON, OP, &c. It follows, from what has been demonstrated, that, in order to maintain an uniform incurvation, the weights of the portions FE, ED, DC, &c. of the circumference must be respectively proportional to OF^2, OM^2, ON^2, OP^2, &c. From C and P, draw CR and PS perpendicular to OB; from similar triangles, OC : OP : : CR : PS, and OF or OC : OP : : PS : PQ; whence, by composition, OC^2 or OF^2 : OP^2 : : CR or FM : PQ. If FM, therefore, denote the weight of an element of the circumference at F, PQ must express the weight of an equal portion at C. The weights of the successive portions FE, ED, DC, &c. are hence proportional to the segments FM, MN, NP, &c., or *to the differences of the tangents of the corresponding angles of deflection.* This proposition includes the whole *Theory of Arches*, whether suspended or incumbent.

Suppose two connected physical points A and B, (fig. 21.), which maintain invariably the same relative position, to be drawn by the forces AP and BP to the centre P: complete the parallelogram PAQB, and the diagonal PQ will evidently bisect AB in O; wherefore a force applied at O in the direction PO, and equal to 2PO, will balance the two forces AP and BP. But AB may be made the diagonal of any other parallelogram P′AQ′B; and consequently the middle point O is a centre at which the forces from A and B are held in equilibrium by another force tending to the same point P or P′. If these points be removed to an indefinite distance, the forces AP and BP will become equal and parallel, and the opposing force PQ will be double of either of them.

Conceive now three connected physical points A, B, and C, (fig. 22.), which constantly maintain their relative position, to be drawn towards a centre P in the same plane by the forces AP, BP, and CP. Construct the parallelograms PAQB and PQRC, and draw the diagonals AB, QC, PQ and PR. It is evident that PR will express the resulting force, and the problem is to assign in this line a point O, which may be independent of the variable position of P. The points K and L bisect the diagonals PQ and PR, which are the sides of the triangle PQC, and consequently the base PC is double of KL; but the converging lines PL and CK must cut each

other in the some ratio, and therefore PO = 2OL and CO = 2OK. Hence PL = 3OL, and PR = 6OL = 3PO; and CK = 3OK, or OK = $\frac{1}{3}$ CK. The force PR, which is triple of PO, balances, therefore, the three forces AP, BP and CP; but that force may be conceived to be attached at the centre O, which is determined directly from the points A, B and C—since K bisects AB, and KO is the third part of KC—this centre is consequently independent of the position of P. If P be removed to an indefinite distance, the several forces acting at A, B and C will become equal and parallel, and the counterbalancing force at O will be equal to their sum, and tend in the same direction.

In like manner, it may be shown, that if any number of atoms be attracted in the same plane by equal and parallel forces, a certain point may be found at which they will be balanced by an opposite aggregate force. Suppose four points A, B, C and D (fig. 23.) to be drawn towards a centre P by the forces AP, BP, CP and DP; it is evident, from the theory of their composition, that those tendencies will be counteracted by a single force in the intermediate direction PO. But, having let fall upon this the perpendiculars A*a*, B*b*, C*c* and D*d*; the force AP may be decomposed into *a*P and A*a*, the force BP into *b*P and B*b*, the force CP into *c*P and C*c*, and the force DP into *d*P and D*d*. Of these, the forces *a*P, *b*P, *c*P and *d*P, all in the same direction, must be coun-

terpoised by the force extending to P through O. The remaining forces aA, bB, cC and dD, being opposed to each other, must produce a mutual balance. Wherefore the perpendiculars Aa and Bb on the one side of PO must be equal to Cc and Dd, the perpendiculars on the other side. Bisect AB in H, and CD in I, and let fall the perpendiculars Hh and Ii. It is obvious, that $Aa + Bb = 2Hh$, and $Cc + Dd = 2Ii$; consequently $2Hh = 2Ii$, and $Hh = Ii$. Wherefore the oblique line HI must be bisected in the centre O. This point is hence derived merely from A, B, C and D, independent altogether of the position of P. Whatever be the place of P, the resulting force will pass through O; and if it be thrown to an indefinite distance, the several atoms will be urged in parallel directions, their forces $AP + BP + CP + DP$ being then equal to the aggregate force $aP + bP + cP + dP$ exerted at O.

If the particles have a permanent arrangement, though not in the same plane, the resulting force will in every position pass through the same individual centre. For suppose that, beyond those particles, a plane were drawn parallel to another touching the resulting force, it is evident that the sum of all the perpendiculars let fall upon it from each of the particles, divided by their number, will be equal to the mutual distance of the two planes. Hence the position of a plane passing through P in the direction

of the resulting force is given. But if P′ be assumed in another place, the position of a second plane passing through P′, and touching the direction of the resulting force, is likewise given. The intersection of these two planes is consequently a straight line given in position. Lastly, if the centre of the attractive forces be changed to P″, a third plane may be assigned, which shall indicate the direction of the resulting force. But this plane must cut the former line of intersection in a given point, which is the centre at which all the forces would be balanced. This centre therefore depends on the mutual arrangement of the particles, and has no reference whatever to the position of the variable concourse of those forces.

But it may be likewise proved, that, about the centre thus found, the system of particles will in every position maintain their equilibrium. For if any fourth plane be made to pass through the common intersection of the three given planes, the sum of the perpendiculars let fall upon it from the several particles on the one side must be equal to the sum of the perpendiculars let fall on the other side. To prove this, it will be sufficient to show, that of lines drawn at given angles to the three planes, the segments intercepted by any fourth plane are mutually balanced. Let (fig. 24.) AK, AL and AM be the three planes, meeting in the lines AB, AC and AD, and let them be cut obliquely by a fourth plane AMK. Conceive another plane AN passing through

AD, and meeting the plane AMK in K; if from any point P of the body or system of particles, the lines PF, PG and PH be drawn to the three fixed planes parallel to AD, AB and AC, their sums will be severally balanced. Produce GP to meet the plane AN in E, and a plane passing through GE will cut the planes AN, AL and AK in the lines EQ and GI, parallel to AD, and QI parallel to AB. Whence, employing the symbol $\int$ to denote summation, $\int$, PH $= \int$, GS $= 0$; but the triangle GSR is evidently given in species, and consequently PG has a given ratio to GS; wherefore $\int$, PG $= 0$, and since, from the general condition, $\int$, EG $= 0$, it follows that $\int$, PE $= 0$. The plane AN has thus the same property as AK, AL, and AM. Again, the oblique plane AO standing in the same relation to the planes AN, AK and AM, must likewise produce a balance in the summation of the lines drawn parallel to AB, AC and AD, the axes of the three given planes.

Every system of particles has therefore a certain, constant and individual point of equilibrium, which is called the *Centre of Gravity*. If that point be supported, the whole system must hence continue in a state of rest. To preserve this condition, it is necessary that an impediment should be opposed in the line of gravitation, or in the vertical drawn through the centre of gravity. The obstacle may be placed either above or below that centre, or the falling of

the body may be prevented either by suspension or by support. By the former, a permanent stability is procured; for if the system or body be drawn aside, the oblique action of gravity will pull it again into the vertical position. The lower the centre of gravity is placed, and the wider is the base, the firmer will the body stand. But if the support be not of sufficient breadth, the equilibrium will be precarious and unstable; since the moment the vertical line projects beyond the base, the body must totter and irretrievably tumble down.

In certain cases, however, a body resting upon a single point, may yet have a disposition to recover from any partial derangement, and to resume its vertical position. Thus, if the base be a plane, and the bottom of the body rounded, but such that the centre of gravity lies below the centre of curvature, the mass may rock backwards and forwards, but will soon regain its erect site. Let O (fig. 25.) be the centre of the incurvation at the end of the body, and G or g its centre of gravity lying in the axis AO. Conceive the body to be rolled on its horizontal plane from A to A′, the point which touched A will merge into a, and the axis will come into the position aO′. Now, if the centre of gravity G stood above O, it would evidently in the position G′ lean beyond the vertical A′O′, and the body would fall over; but if the centre of gravity were at g below O, it would still in changing to g', lie within the vertical A′O′,

and consequently the body would roll back to its first position.

Again, suppose the round end of the body to be implanted upon a circular base, (fig. 26.), while the contact shifts from the point A to A′, the radii AO and A′O will obviously converge to P, the centre of the arc AA′. Having drawn A′M parallel to AP to meet *a*O, if the centre of gravity should lie any where below M, its vertical will fall within the base AA′, and the body will, therefore, have a constant tendency to redress itself and recover its first position. But from the property of diverging and parallel lines, OP : AP : : *a*O : *a*M, or AP—AO : AP : : AO : *a*M, the height of what may be called the *metacentre*, or extreme vertical limit of the centre of gravity. If the base be convex, the first term of the analogy will be the sum, instead of the difference, of the radii AO and AP. On this principle seems to depend the curious phænomenon of *Rocking* or *Laggan Stones*, the natural joints of the columns being, in the course of ages, worn by the agitation of the wind into regular curved surfaces.

The centre of gravity of any body is determined, by dividing the sum of the distances of its constituent particles from any plane by the number of those particles. The surface of a triangle, its three sides, and its angular points, have all the same centre of gravity, which is situate at two-thirds of the length

of a straight line drawn from the vertex to bisect the base. The centre of gravity of a parabola lies at the distance of three-fifths of the axis from the vertex; that of a cone is placed at three-fourths of the length of the axis, and that of an hemisphere at five-eighths of the radius. A pyramid and its four terminating points have the same centre of gravity.

When the body consists of two parts, their common centre of gravity will be found, by dividing the distance between the centre of each reciprocally as their corresponding weights. The principle may be extended to bodies which are more complex. Thus, the centre of gravity of two portions being determined, they may be conceived to be united in that point and connected with a third portion, and the centre of the three portions found. All these again may be supposed collected in this last point and made to balance against a fourth portion, and their common centre of gravity computed. In this way, the process may, by successive steps, be carried to any extent, and whatever order is followed, the result will be always the same. Thus, in fig. 27., the centre of gravity of four points A, B, C and D in the same plane, is found by a progressive procedure. Join AB, and bisect it in K; conceive the points A and B to be collected at K, and trisect CK in L, the centre of these, and the third point C; lastly, suppose A, B and C to be condensed at L, cut off

OL, a fourth part of DL, and O will be the common centre of gravity of all the four points.

The same method is applicable either to surfaces or solids, the former being subdivided by triangles, and the latter decomposed into wedges. The centre of gravity of the larger portion of any figure may be conveniently found, by estimating from the defect. This centre is in a trapezium, deduced from the two triangles formed by producing its oblique sides, and in the frustum of a cone, from the cone itself and its upper portion. For the excess of the mass A above B is to B, as the mutual distance BA to the interval in this direction of the fragment beyond A.

Of irregular bodies, the centre of gravity may be discovered practically in various ways. If the body be poised, for instance, in two different positions on a sharp edge, the vertical drawn from the point of intersection will pass through the centre of gravity. Or, if a loose thread or string have its ends fastened to two distinct points of the body, and suspended in two different positions from a fixed pivot, the verticals let fall from this will cross in the centre of gravity.

Many singular and paradoxical appearances are dependent on the properties of the centre of gravity. Hence a ball may be suspended beyond the border of a table; hence an eccentric loaded cylinder will

roll partly up an inclined plane; and hence also a double cylinder will seem to advance along two rising and spread edges. Various toys are constructed on the same principles, which likewise direct the skill of the balancer.

It is a remarkable property of the centre of gravity of any number of points, that forces directed from them to this centre, and indicated by the several lines, will maintain a mutual equilibrium. Thus, if O (fig. 28.) be the centre of gravity of the points A, B, C, D, E, &c., the forces AO, BO, CO, DO and EO will all balance at O. For if a plane were supposed to pass through O, the several oblique forces OA, OB, OC, OD, OE, &c. might be reduced into forces perpendicular to that plane and other forces directed along it. But since an equilibrium obtains, the perpendiculars on the one side of the plane must be together equal to those on the other, and consequently O is the centre of gravity of all the points A, B, C, D, E, &c.

A still more distinguished property of the centre of gravity of any line or plane is, that the circumference traced by this about its boundary as an axis, drawn into the line or plane of revolution, is equivalent to the surface or solid thus described. In short, the space involved by circumvolution is the same, as if the line or plane had advanced directly forwards over the same extent of description. Hence the sur-

face and contents of solids of revolution are easily discovered.

But the most elegant property of the centre of gravity is, that, in any given plane, the sum of the squares of the distances of physical points from their centre of gravity is less than the aggregate of the squares of their distances from any point in the circumference of a circle described about that centre, by the square of the radius multiplied by the number of those points. It hence follows that the sum of the squares of the distances of points from their centre of gravity is always a *minimum*.

If three fixed points A, B and C (fig. 29.) in a horizontal plane be drawn by the forces AO, BO and CO to a remote centre O, the tension may be balanced, by triple the vertical force PO acting at an intermediate point P of the same plane. For this force PO being perpendicular to the plane, the forces expressed by the bases AP, BP and CP of the several triangles AOP, BOP and COP must be in equilibrium, and consequently P is the centre of gravity of the three points A, B and C. If we suppose m particles to be collected at A, and n particles at B, p particles at C, the resulting vertical must still pass through P, the common centre of gravity of the clusters or weights m, n and p at the points A, B and C. Wherefore a force $m \times$ AP must balance $n \times$ BP and $p \times$ CP, and it will thence follow that m is to n, as the area of the triangle BPC is to that

of APC, and that m is to p as the triangle APB is to BPC.

Conceive the centre O to be now removed to a vast distance, and the parallel forces directed to it will express the pressures which a horizontal plane resting upon the angles at A, B, C, would exert against those supports when a weight is laid at the point P. These pressures would be equally shared if the weight were incumbent on the centre of gravity of the triangle. In other positions the distribution of pressure would be unequal, depending on the relative proportions of the interior triangles APB, BPC and CPA.

Let ACB (fig. 30.) be a perfectly flexible chain, whose ends are attached at the points A and B. If we consider any portion CF of this chain, intercepted from the vertex or lowest point C, its weight must be supported by the oblique tensions exerted at C and F in the directions of CL and FL, the tangents at those points. The strains at C and F may therefore be conceived as acting at the common point L, to which likewise the action of the weight GL must be directed; or the vertical LG will pass through G, the centre of gravity of the arc CF, where all its efforts are united. Wherefore the strain at the lowest point C is to the weight of CF, as the sine of the angle GLF, or the cosine of FLM, is to the sine of FLM, that is, as radius to the tangent of the an-

gle FLM, which a line touching the curve at F makes with the horizon. Hence the tension at C being constant, the weight, and therefore the length, of any portion CF of the equable chain must be proportional to the tangent of the deflection of the curve at F; which is the geometrical definition of the CATENARY. (See *Geometry of Curve Lines*, p. 383.)

Again, from the same equilibrium of forces, about the point L, the strain LC is to the strain LF, as the sine of the angle FLG, or the cosine of FLM is to the sine of CLG or the radius, that is, as the radius to the secant of the angle FLM, which the curve makes with the horizon. The strain exerted at any point F is thus proportional to the secant of deflection, as was formerly shown.

If therefore the vertical line CO, or parameter of the catenary, represent the strain at the lowest point C, and the generating circle be described about the centre O; a straight line EH, drawn to touch this circle, will be parallel to the tangent FT applied to the curve, and equal to the length of the intercepted segment CF. Hence the tension exerted at any point F will be expressed by OE, which is the secant of deflection to a radius OC or the parameter: It thus exceeds the tension at the lowest point C by the abscisS CE. A tangent DI to the circle will hence be equal to the length of half the curve CB, and will indicate the flexure of its extremity B, while OD will denote the tension at that point.

1

1. Suppose the parameter of the Catenary, or the measure of tension at its lowest point C, to be equal to the depression DC. Since OC, being denoted by 1, it is evident that the tangent DI, which is equal to BC, half the curve, will be expressed by $\sqrt{3} =$ 1.7320508. Wherefore, by Prop. IV. of the *Catenary*, the hyperbolic logarithm of the ratio of OC to OC + CD + CB or of 1 to 3.7320508 will represent the ordinate DB, or half the width AB; consequently DB=1.3169578, or AB = 2.6339156, the entire length of the chain ACB being 3.4641016. In this case, the strain at the lowest point is expressed by 1, and at the ends A and B by 2.

2. Next, suppose that the strain at the lowest point C is equal to the whole weight of the chain. It is obvious that OC being still denoted by 1, DI is $\frac{1}{2}$, and OD $= \sqrt{\frac{5}{4}} = 1.1180340$, or CD = .1180340; whence DB = hyperbolic logarithm of 1.6180340 or .4812117, the whole span AB being .9624234, and the length of the chain itself 1. Here the width is 8.1538 times, and the extent of the curve 8.4724 times, the depression: The tension at C is 1, and at A or B it is 1.118034.

3. Again, suppose the tension at the lowest point to be double the weight of the chain. The parameter being expressed by unit as before, DI will

be $\frac{1}{4}$, and consequently OD = $\sqrt{\frac{17}{16}}$ = 1.0307764, or the depression CD = .0307764. The ordinate CB is therefore equal to the hyperbolic logarithm of 1.2807764 or .2474664, and the whole span AB = .4949328. Here then the span is 16.0816 times, and the extent of the chain 16.2462 times, the depression CD. The strain at the lowest point is 1, and that at both ends 1.0307764.

When the quantity of depression of the catenary is comparatively small, as happens in all practical cases, an easy approximation may be found. Thus, if the span AB be denoted by b, the depression DC by d, and the whole length ACB of the chain by l; it may be shown, from the general nature of curve lines, that the semiarc CB is very nearly equal to $DB+\frac{2DC^2}{3BD}$; and consequently $l=b+\frac{8d^2}{3b}$. But the semiarc is equal to the tangent DI, and its square is consequently equal to $OD^2-OC^2=DC(OD+OC)=$ CD.DN; or p denoting the parameter OC, it follows approximately, that $\frac{b^2}{4}+\frac{4d^2}{3b}=d\,(2p+d)$ $=2pd+d^2$, and therefore $p=\frac{b^2}{8d}+\frac{d}{6}$, or the strain at the lowest point. The strain at the extremities A and B will hence be expounded by $\frac{b^2}{8d}+\frac{7d}{6}$.

These expressions may be converted into others involving the length of the chain. Thus,

$b=l-\frac{8d^2}{3l}$, and OC or $p=\frac{l^2}{8d}-\frac{d}{2}$, and OD $=\frac{l^2}{8d}+\frac{d}{2}$, which last expressions hence represent the tensions at the lowest point C, and at both extremities A and B. As examples of approximation, some of the former instances may be resumed. Thus, let the strain at the lowest point be equal to the whole weight of the chain; it was then found that $l = 1$, and $d = .118034$. Wherefore $b = l - \frac{8d^2}{3l} = .9628$, true to every place of figures; and the strains at C, and at either extremity A or B, are respectively $\frac{l^2}{8d}-\frac{d}{2}$ and $\frac{l^2}{8d}+\frac{d}{2}$, or 1 and 1.118034, likewise the same as before. Again, when the strain at the lowest point is double of the weight of the chain; in which case, $l = .5$, and $d = .0307764$. Here $b = .4949483$, which approximates extremely near to the correct measure .4949328. The strains come out exactly as in the rigorous calculation. In practice, where the depression seldom exceeds the fourteenth part of the entire length of the chain, this very simple approximation will, therefore, be sufficiently accurate for every purpose. The engineer has only to recollect, that when the depression amounts to an eighth part of the length of the chain, the strain at each end is just equal to the whole of the weight sustained; and, in other cases, the quantity of depression will be inversely as the load.

Since three balancing forces AG, BH and CI (fig. 31.) tend always to the same centre, however oblique is the triangle ACB, this property must likewise hold in the extreme case, when the points A, B, and C may be considered as ranging in a straight line. Let ACB, therefore, be a rod or inflexible line, and suppose the extremities A and B to be drawn by forces tending to O, while an intermediate point C is sustained by a force in the direction CI. But this direction must proceed from O, the concurrent centre of the three forces. Draw CE and CF perpendicular to AO and BO, the directions of the other two forces; the force acting at A must be to the force AB, as CF to CE. If the point O be removed to an indefinite distance, the lines EA and BF will become parallel, and consequently CF is to CE, as CB is to CA. Wherefore, in the case of parallel forces *in equilibrio*, the force at A is to the force at B, as CB is to CA. Instead of supporting the point C by a force CI, it may be made to rest on a firm obstacle or *fulcrum*, which would exert an equal force upwards; the rod ACB would then become *a lever*, having the arms CA and CB, at the extremities of which the power and weight are applied. Hence the general principle of the lever, that *the power and weight are inversely as their distances from the fulcrum or point of suspension.*

Because the forces acting in the directions AO

and BO are inversely as the perpendiculars let fall from C upon them, the equilibrium will be maintained at whatever points in the lines AO and BO those forces be applied. Wherefore the same forces attached at the extremities N and P of the bent lever NCP, would likewise produce a mutual balance.

But the property of the straight lever, *that the power and weight are inversely as their distances from the point of suspension*, may be deduced from considerations of the most familiar kind. Let AB (fig. 32.) represent a cylinder of homogeneous matter, such as wood or metal: It could evidently be supported at the middle point O, where the stress will be the same if the whole weight had been concentrated in that point. But the cylinder might be conceived to consist of two distinct portions, AC and BC, which would be separately sustained at their middle points D and E. Wherefore, the weight of AC attached at D, and the weight of BC attached at E, would balance the inflexible line DE, if upheld at the centre O by the whole weight of AB. But $OD = AO - AD = \frac{1}{2}AB - \frac{1}{2}AC = \frac{1}{2}BC$, and $OE = OB - EB = \frac{1}{2}AB - \frac{1}{2}CB = \frac{1}{2}AC$; consequently OD is to OE, as BC to AC, or as the weight concentrated at E is to the weight concentrated at D. This in substance is the elegant demonstration of the property of the lever, given by Stevinus and Galileo.

The next object is, from the decomposition of forces, to assign the portion of any weight which is supported by the action of an inclined plane. Every plane, it should be remarked, is in some degree flexible. Under the point where pressure is applied, the fibres or connected particles bend into a concave surface, till their mutual approximation creates a repulsive energy sufficient to withstand the force directed to the centre of concavity, and therefore perpendicular to the plane. Let the horizontal and vertical lines AB and BC (fig. 33.) determine the oblique position of the plane AC, on which a weight is supposed to be placed at D. Let FD denote the measure or vertical pressure of that weight, and decompose this into HD perpendicular to the plane, and FH or ED parallel to it. But HD is the only part supported by the plane, and the other part DE would require to be sustained by a force acting along AC. The triangle DEF is obviously similar to ABC, and therefore DF is to DE, as AC to CB; that is, *the whole weight is to the force of descent on the oblique plane, as the length of the plane to its height, or as radius to the sine of its elevation.*

The true proportion of any weight to its tendency of descent along an inclined plane was first established by Stevinus, who derived it from a very simple but extraneous consideration. Suppose a chain or a chaplet composed of equal attached balls were thrown loosely over the triangle ABC, (fig. 34.) and the two

ends united a little below the base AB. It is evident this chaplet would remain at rest; for were it drawn partly round, each ball would always be succeeded by a similar one in the same situation; if it began of itself to move, therefore, it would move for ever, which is a proposition inadmissible. But the branch of the chaplet below the base, being suspended equally at both extremities, must evidently be in a state of equilibrium. Wherefore the remainder of the chaplet would by itself maintain a balance, or the part of it which hangs along the perpendicular must form an equipoise to the part that leans against the inclined plane; and consequently a weight placed on the plane, would be to the force which in the same direction sustains it, as the length, to the height, of the plane.

From the resolution of forces, may likewise be deduced the *Theory of the Pulley*. This elementary machine consists of a circle moveable about an axis, and having a very flexible cord applied in the groove of its circumference. Let a pulley, from whose centre D (fig. 35.) hangs a weight P, be supported by the bend of a cord which has its ends attached at A and B. It is evident that the tension of this cord at A may be considered as acting at any point in the direction of the tangent AC, while the tension at B is exerted in the direction of the tangent BC. These oblique forces may, therefore, be conceived to

unite their efforts at the point of concourse C. But the weight D, acting in a vertical line from the centre or axis of the pulley, must pass through the same point C. Wherefore, this weight is to the strain of the cord at A or B, as the sine of the angle ACB, is to the sine of its half ACE or BCE; or, having let fall the perpendicular AF upon BC, the weight P will be to the strain at A, as AF to AE. Hence, if AF were to represent the whole weight applied, AE and BE would express the portions supported by the joint action of the cord at A and B. The loss of force occasioned by this obliquity is, therefore, in the ratio of AB to AF, or as radius to the cosine of the angle ACE. But if the points A and B be brought nearer, and consequently the angle ACB diminished, AF will obviously approximate to AB; and when the opposite folds of the cord are parallel, they will each sustain exactly one half of the attached weight.

Hence, if the end of the cord be fastened at B, *the force at the other end pulling in the direction CB, and requisite to sustain the given weight P, is to that weight, as the sine of the angle ACE to the sine of its double ACB,* which approximates as these angles diminish, to the ratio of 1 to 2, the limit of parallel action.

III. PHORONOMICS

Explores the properties of Moving Bodies. It is often comprehended under *Dynamics*, which traces the series of changes arising from the application of Force. This most important branch of physical science is entirely of modern origin; and though it rests on the simplest principles, it has yet, by the aid of the higher analysis, been pushed to an amazing extent.

It would be superfluous to attempt any definition of motion, and quite futile to answer the sophisms that have been urged against its very existence. The Aristotelian distinction of motion into *Natural* and *Constrained* was rightly conceived; but, from want of close observation, it was inaccurately applied.

All motion is performed either *in a straight line* or *in a curve;* and it flows *uniformly*, or with an *accelerating* or *retarding* pace. The *velocity* or *celerity* of a moving body is the minute space which it describes in a given instant of time.

If any body be held in equipoise by the action of combined forces, it will continue at rest. Thus, a book laid upon the table will press down the surface, till the approximation of the succumbent particles

exerts a repulsive effort just equal to the weight of the book, and sufficient therefore to sustain it in the same position. All stability is hence produced, by the mere balance of opposing forces.

But if this equilibrium be deranged, the body will advance in the direction of the preponderating force. Supposing that, after having thus acquired motion, it be liberated from external influence, and abandoned to its mere impulsion, the object is now to ascertain experimentally, as the only basis of Phoronomics, what course and rate the body will pursue. Unfortunately these conditions are strictly unattainable in our place of the universe. The action of gravity is never one moment suspended, and no motion upon earth is exempt from the operation of some retarding causes. We can only approximate to the undisturbed effect, by neutralizing the constant influence of gravitation, and diminishing the obstruction created by friction and atmospherical resistance. If a body move along a perfectly horizontal plane, its weight being exerted vertically, will neither incite nor impede its progress. Let a ball be propelled along a spacious level floor, and its motion will, in every case, appear rectilineal. An ivory ball, rolled with a certain velocity on a Turkey carpet, will suffer a visible relaxation in its course; but, with the same impulsion, it will advance farther on a surface of baize, still farther on smooth oaken planks, and, on a sheet of pure ice, it will scarcely seem at all to

abate its celerity. A leaden ball, in similar circumstances, will maintain its motion through a much greater extent. The gradual diminution of velocity being thus proportioned to external impediments, must be attributed solely to their influence. We are entitled, therefore, to conclude, that any body, after having received an impulsion, would, if left wholly to itself, continue to move in a straight line, and with an uniform celerity. Every deviation from the rectilineal course betrays the influence of some extrinsic deflecting force; while the relaxation of motion, in any case, intimates the accumulated effect of incessant impediments.

If a sling be whirled and its pellet discharged, it will no longer describe a circle, but will shoot off in a tangent. The rectilineal is hence that motion which the missile naturally takes, its previous circulation having been merely constrained by the continued retraction exerted by the muscles of the arm. In all the circular movements of machinery, the deflection from a rectilineal course is maintained by an attractive or centripetal force, occasioned by the distension of the particles from the axis to the circumference.

When friction is greatly diminished, a piece of fine mechanism, set once in motion, will continue to circulate for a very long time. This may be effected, either by adapting friction-wheels, or by making the pivots turn in fine holes of agate or diamond. Under the exhausted receiver of an air-pump, the

motion is found to last much longer. In short, the more the friction is diminished and the resistance of the air abstracted, the nearer will the machine approach to perfection, and to the constant maintenance of its celerity. It is the impossibility of absolutely excluding those impediments on the surface of our globe, that must prevent any efforts of ingenuity from ever achieving a *perpetual motion*. But, in the celestial spaces, no such obstructions occur, and the revolution of the planetary system, with all its balanced cyclical variations, bears the stamp of ETERNITY.

If the boxes of Atwood's machine, (which turns smoothly upon friction wheels, (see fig. 35.) in which those wheels are, for the sake of clearness, however, omitted, and likewise the attached pendulum,) be equally loaded, and a small bar laid across one of them to generate an impulsion; after this bar has been intercepted in the passage through a stage placed a few inches lower, the progress of both boxes upwards and downwards will be found scarcely to differ from a perfectly uniform motion, especially if the velocity incited be not too feeble. This motion is only vertical indeed, but the same experiment could easily be performed in any given direction, with equal rollers running along an oblique plane.

We are, therefore, entitled to infer, that all bodies are quite passive or indifferent to a state of rest or of motion, and would continue for ever at rest, or

persevere in the same rectilineal and uniform motion, unless disturbed by the operation of some extrinsic force. This absolute *inertia* of matter is the great law on which rests the whole Doctrine of Phoronomics. Notwithstanding its extreme simplicity, it lay hid for ages, and was first discovered by the sagacity of Galileo. The ancients attributed to matter a certain inaptitude, reluctance, or renitency to motion. Nor was such a conclusion at all singular, at a period when only random observations were made, and the mechanical arts had not yet attained any precision. Every motion experienced upon earth insensibly relaxes and becomes extinguished; and it required mature reflection to discriminate this effect as a mere casual result, and not an inherent property of matter.

On the birth of accurate science, the term inertness or *inertia*, which had been borrowed from the indolence or sluggishness of animal effort, totally changed its meaning. Kepler, conceiving the disposition of a body to maintain its motion as indicating an exertion of power, therefore prefixed the word *vis*; and this compound expression, *vis inertiæ*, has been long retained. But the single term *inertia*, signifying merely passive indifference, should now be preferred, as more correct and appropriate.

The *inertia* of matter is, in every incident of life, unceasingly recalled to our observation. Thus, a person sitting erect in a chaise is thrown backwards, the

moment the horses start, or rather he retains his resting place for an instant, while the vehicle begins to advance; but having once acquired the progressive motion, he is pitched forward if the horses suddenly stop, that is, he holds on his course for an instant after the chaise has come to rest. If two wine-glasses, placed on a table at the interval of about half a-foot, have a thin narrow ruler laid across them, with a piece of money above the mouth of each, on striking a smart blow against the end of the ruler, it will shoot forwards, while the coins, not having time to acquire this impulsion, will merely drop into the glasses. Let two or three different kinds of smooth worn coins, or of the same kind only distinguished by marks, be piled on the bottom of an inverted egg-cup, which is then pushed suddenly along the table, it will be found that the coins will be thrown forward always exactly in the order in which they lay, the lowermost, receiving the greatest share of the impulsion, being projected farthest, and the uppermost, being the least affected, scarcely advancing at all. In like manner, while a ship glides swiftly through smooth water, a ball dropped from the topmast will strike the same spot of the deck, as if it had fallen when the vessel was at anchor. On this principle, the balancer easily tosses up oranges and catches them again, as he ambles in the ring; for these fruits acquire the motion of his palfrey. Hence the rotation of our globe about its axis does

not prevent a stone, falling from the top of a high tower, from alighting at the bottom. The minute deviation lately detected in this perpendicular descent, arising from the excess of the circular description of the top above that of the bottom, as we shall soon find, affords the most direct proof of the revolution of the Earth.

Every motion, then, is *Rectilineal* or *Curvilineal*. Rectilineal motion, again, is either *uniform*, or subject to *acceleration* or *retardation*.

In whatever way motion be generated, it will, if left free and undisturbed, persevere uniformly in the same straight line. In this simplest kind of motion, three things are to be considered: The velocity with which the body moves, the portion of time elapsed, and the corresponding space described. These may be denoted by the letters v, t, and s. It is evident that $s = vt$, and, consequently, that $v = \frac{s}{t}$ and $t = \frac{s}{v}$. Hence the space of description is compounded of the time and velocity, and this velocity is directly as the space, and inversely as the time; while the time itself is directly as the space, and inversely as the velocity.

In the case of accelerated or retarded motions, the inciting or obstructing force is especially to be distinguished: It may be generally denoted by the letter f. *Force* and *Velocity*, *Time* and

Space, are quantities in a state of continual flux, and passing through a series of proximate stages. These contiguous steps may be marked by repeated accents, and their successive intervals will form the minute differences or differentials of the varying quantity which are expressed by the prefix ∂. According to this notation, $\partial f = f' - f$, $\partial v = v' - v$, $\partial t = t' - t$, and $\partial s = s' - s$. Now, the velocity of a body being the result of the accumulated impression of the accelerating force, every minute accession which it receives must evidently be proportional to that intensity and to the duration of the instant in which it is exerted. Hence, $\partial v = f \partial t$, and $f = \frac{\partial v}{\partial t}$. Again, though the velocity be continually varying, it may be viewed as constant at least during an instant of time; the element of the space described will therefore be proportional to the velocity, and to the duration of its momentary flight. Whence $\partial s = v \partial t$, and $v = \frac{\partial s}{\partial t}$. Combine this expression with the former, and $f \partial s = v \partial t . \frac{\partial v}{\partial t} = v \partial v$. Thus, *the accelerating force multiplied into the variation of the space, is equal to the velocity multiplied into its corresponding variation.* But this expression admits of a simpler form. Since $(v' + v)(v' - v) = v'^2 - v^2$,

and v' may be considered as approximating indefinitely to v, it follows that $2\, v \partial v = \partial . v^2$ and $v \partial v = \frac{1}{2} \partial . v^2$, or the product of the velocity into its variation is equal to half the variation of the square of the velocity. Wherefore, $f \partial s = \frac{1}{2} \partial . v^2$, or *the generating force multiplied by each successive element of the space described, will give an amount equal to half the variation of the square of the corresponding velocity.*

Suppose this generating force to be constant, and the sum of all the $f \partial s$ must evidently be the product of f into the aggregate elements of s, whence in the case of a constant acceleration, $fs = \frac{1}{2} v^2$. But it was shown generally, that $f \partial t = \partial v$, and consequently $f\, t = v$; by substitution, therefore, $f\, s = \frac{1}{2} f^2 t^2$, or $s = \frac{1}{2} f\, t^2$. Now, abstracting from the resistance of the air, bodies near the surface of the earth are observed to fall about sixteen feet during a second. In round numbers, f, denoting the power of gravitation, is hence expressed by 32. Substitute this number, and the several formulæ will become definite. Wherefore, 1, $32\, t = v$, or the velocity acquired in falling, is equal to thirty-two feet multiplied by the number of seconds elapsed; 2, $s = 16 t^2 = (4t)^2$, or the space described, is equal to sixteen multiplied into the square of the time, or to the square of quadruple the time; 3, $s = \frac{1}{64} v^2 = \left(\frac{v}{8}\right)^2$, or the space of descent, is equal to the square of the

eighth part of the acquired velocity; 4, $t = \frac{1}{4}\sqrt{s}$, or the number of seconds in falling, is the fourth part of the square root of the height; 5, $v = 8\sqrt{s}$, or the velocity acquired, is equal to eight times the square root of the height; and since $v = 8\sqrt{s}$, and $t = \frac{1}{4}\sqrt{s}$, therefore 6, $vt = 2s$, or the body with its acquired velocity, would, in the time of its descent, shoot through double the space.

The same formulæ apply in the case of bodies thrown directly upwards, gravitation acting then as a retarding force. But similar results are deduced, from the inspection of a simple geometrical diagram. Since quantities may be expressed by lines or surfaces, let (fig. 37.) the equal portions AB, BC, CD, DE, &c. denote the successive instants of time elapsed, and the perpendiculars BL, CM, DN, EO, &c. equal to the whole segments AB, AC, AD, AE, &c., will express the velocities acquired, from the accumulated action of the same inciting force. The spaces described, in those several instants will hence be represented by the elementary rectangles CL, DM, EN, FO, &c. Wherefore, the whole space of descent will be expressed by the compound or serrated triangle KAT. But the indented boundary will obviously pass into a straight line, if the intervals be indefinitely multiplied, and the impulses graduate into a continued force.

Since the circumscribing square AKTV is double

the area of the triangle, it follows, as already stated, that a falling body, if no longer accelerated, would, with its acquired velocity, describe double the space in the same time. Hence, also, in constantly accelerated motions, the space described is always proportional to the square of the time, or to the square of the velocity. The inciting force is here conceived to be denoted by unit; but if it undergo a change, the annexed triangle will be made to correspond, by increasing or diminishing its base KT into KT′, in the same ratio. Hence, during the same interval, the acquired velocity, and the space of description, will be as the inciting force.

These theorems respecting uniform motion and motion constantly accelerated or retarded, are illustrated and confirmed by Atwood's machine. Thus, each box being loaded with 15½ ounces, if a bar equal to one ounce be laid over the box A (fig. 36.), this excess of weight is the 32d part of the compound mass, which will therefore acquire that share of the entire celerity produced by the acceleration of gravity. The box A will hence, in a second, drop through the space of 6 inches, and then, leaving its surplus load at the stage, it will continue to descend uniformly, at the rate of a foot each second. Suppose the boxes A and B to be loaded respectively with 16½ and 15½ ounces, the motion will be constantly accelerated, and the inciting force

being the same as before, the descents will form a series of squares, 32 times less than those of gravity. The box A will fall through 6 inches in one second, through 24 in two seconds, and through 54 in three seconds. Again, let each box hold $15\frac{1}{2}$ ounces, while A has a bar laid across it; after A has, in three seconds, reached 54 inches, and dropt the bar, let B take up another bar, and it will perform an opposite retarded motion, rising again to the point from which it fell. This alternate ascent and descent may continue for some time.

Such examples are confined to uniformly accelerated or retarded motions. But the principles may be extended geometrically to all cases where the inciting force is some function of the distance. Let AB (fig. 38.) express the space described, and the ordinate BC of the curve denote the corresponding force which urges the body. The element BC*cb* $=f\partial s$, we have already seen, is proportional to the accessions of the square of the velocity; and consequently the square of the velocity acquired in passing through AB, is represented by the area ABC, while the square of the final velocity attained is measured by the whole curvilineal space ACEDA.

If a body be suddenly turned aside from its course, it will suffer some loss of impulsion. Thus, if a ball, rolling on a horizontal plane, be diverted from the line AB, (fig. 39.) into the direction BC, the impulsion, which was the result of a force expressed by

AB or BC, is resolved into a force BD in the extension of AB, and another force DC perpendicular to this. In turning from B into the line BC, the body has, therefore, its velocity reduced, in the ratio of BC to BD, or of radius to the cosine of the angle of deflection CBD. The loss of velocity sustained at B, or DE, is hence, if the radius be reckoned unit, as the versed sine of CBD, or in small angles, as half the square of the sine CD. It may be found very nearly, by dividing the square of the number of degrees contained in the angle by 6566.

Conceive the change of direction from AB (fig. 40.) to DE to be divided by a series of intermediate deflections; the loss of impulsion will then be as the aggregate of half the squares of the sines of those deflections. Let their number be doubled, and the square of each sine will be reduced to one-fourth, and their sum to one-half. Multiply the bendings indefinitely, and the loss of velocity will thus become insensible. The gradually deflected motion in a curve is hence absolutely unimpaired.

Suppose a body, by the action of gravity, to slide down an oblique plane AC (fig. 41.): Draw the horizontal line CB and the vertical AB. It has been already proved, that the weight of the body is to its effort to descend in the direction of the plane, as the length AC is to the altitude AB. Wherefore, from what has been shown, the square of the time of fall-

ing through AB is to the square of the time of sliding along AC, in the compound ratio of AB to AC, and again of AB to AC, or as the square of AB to the square of AC. Consequently the time of falling through the perpendicular AB is to the time of the oblique descent through AC, as AB to AC. Again, the velocity acquired in the direct fall is to the velocity gained during the oblique descent in the ratio compounded of AB to AC, the times of description, and of AC to AB, the accelerating forces, that is, in a ratio of equality.

Let a body descend by its weight through a succession of planes AB, BC, CD, &c. (fig. 42.) Draw the vertical AG, and the horizontal lines AI, BE, CF and DG, and produce CB and DC to meet AI in H and I. When the body arrives at B, it is turned into the direction BC; and when it has reached C, it is deflected along CD. The loss of force occasioned by these deflections may be overlooked when the angles ABC, CBD, &c. are very obtuse, since they are only as the versed sines of the supplements. The velocity which the body gains in sliding down AB is the same as it would have acquired in descending over HB, or by a perpendicular fall from A to E. In descending through ABC, therefore, it will acquire the same velocity as if it had fallen from A to F. The body hence enters on the plane CD with the velocity which it would have gained in sliding from I to C, and the velocity which it accumulates

at D will be the same as in a perpendicular fall from A to G. In a series of many planes, therefore, the velocity acquired is always the same as in an equal vertical descent. If we substitute curves, the conclusion must evidently still hold. Hence, balls rolling along different curves, from the horizontal line ACE (fig. 43.) to a lower level BDF, will acquire equal velocities at B, D and F. Hence also, if a ball suspended by a fine thread, after descending through the arc of a circle, meet, in its subsequent ascension, with some obstacle which gradually shortens the thread, it will still attain the level from which it fell.

The same conclusion is derived likewise from another principle, and rendered even more general. The vertical inciting force at the horizontal line GHIK, (fig. 43.) is to the accelerating force in the oblique direction H*h*, as H*h* to G*g*. Wherefore, *ghik* being an adjacent parallel, the increase of the square of the velocity in falling from G to *g* will be to the corresponding increase of the square of the velocity during the oblique descent through H*h* in the compound ratio of H*h* to G*g*, and of G*g* to H*h*, that is, in a ratio of equality. A body descending along the curve, AHB, must thus receive the augmentation of velocity at each successive parallel, that another body dropped from A will acquire in the course of its fall. To produce this effect, it is not even requisite that the inciting force should remain constant, the essential condition being merely that it

should have the same intensity at every point of any parallel.

Let AB (fig. 44.) be an oblique plane, and BC its altitude; draw AD perpendicular to AB, meeting the vertical BD in D. The time of descending through AB is to the time of falling from C to B as AB to BC, and therefore the square of the time of descent through AB is to the square of the time of the vertical fall CB, as the square of AB to the square of BC, that is (Geom. vi. 15,) as BD to BC. But this ratio is the same with that of the squares of the times of falling through DB and CB; and hence, the fall through DB is performed in the time of the oblique descent along AB. Now, BAD, being a right angle, is contained in a semicircle, and therefore generally *the times of descent through different chords AB and EB are equal.*

Let a body be urged by a force whose intensity is directly as its distance from a given point. Thus, if the motion be performed from B to A (fig. 45.) under the exertion of a force which varies with the distance from the point A; having erected the perpendicular BD equal to AB and joined AD, it is evident that any other perpendicular CE must express the intensity of the inciting force at C. Wherefore, the elementary rectangle CE*ec*, or the product of that force into the element of the space, must represent the increment of the square of the velocity;

and, consequently, the trapezoid BDEC will represent the square of the velocity acquired in passing from B to C. But this trapezoid is the excess of the triangle BAD above CAE, or half the excess of the square of AB above the square of AC. With the radius AB describe the quadrant BG, produce EC and *ec* to F and *f*, and join AF. The square CF is obviously equal to the excess of the square of AF or AB above the square of AC, and consequently in this species of accelerated motion, the perpendicular CF bounded by the circumference will denote the velocity acquired at C, the final velocity being expressed by AG or AB.

The same conclusion is derived in another way. Draw Fϕ parallel to AB and join AF; the elementary triangle Fϕf being evidently similar to ACF, it follows that AC : CF : : ϕf : Fϕ or C*c*, and AC.C*c*=CF.ϕf. But, applying the formula $f\delta s = v\delta v$, the accelerating force f being here denoted by the space AC or s, and C*c* by δs, the ordinate CF, and its increment ϕf are evidently expressed by the velocity v, and its increment δv.

Other leading properties are easily deduced from the same figure. For, CF : AF : : Fϕ or C*c* : F*f*, and hence $\frac{Cc}{CF} = \frac{Ff}{AF}$. But $\frac{Ff}{AF}$ evidently denotes the minute portion of time in which the elementary arc F*f* is described by the final velocity AF or AB,

while $\frac{Cc}{CF}$ denotes the time in which the elementary space Cc would be described by the velocity acquired at C. Wherefore, collecting those elements, the accelerating body passes over the space BC in the same time that another body, moving uniformly with the final velocity, would describe the arc BF.

This final velocity is the same as what would be acquired by a constant acceleration from the half of only the first intensity; for bisect BD in H, and describe the rectangle HBAI, which will be equal to the triangle ABD; a body urged by the invariable force BH will hence acquire at A the same square of velocity as another which is incited by a force varying from the initial intensity BD in the ratio of the distance from A. But the velocity acquired by an uniform acceleration would carry the body through double the space AB during the descent from B to A. Wherefore the time elapsed during the uniform acceleration through AB, is to the time required for its description by a force of a regularly diminishing intensity, as the diameter of a circle to the length of the quadrant, or as a square to the inscribed circle, that is, as 1 to .785398, or nearly in the ratio of 14 to 11.

It thus appears, that the celerities generated by an uniform acceleration from B to C and to A, are measured by the chords BM and BG in the circle, or the ordinates CM and AL of a parabola, which

has BA for half its parameter. The celerities, again, derived from the action of a force, decreasing as its distance from A, are denoted, at the same points, by CF and AG. In like manner, while the chords BM and BG express the times of falling to C and A by an uniform acceleration, the corresponding arcs AF and AFG of the circle will express the times of descent, when the body is urged by a force proportional to the distance from the centre A.

Let a body slide down a small circular arc FB (fig. 46.) The accelerating force at any point G, will be as the sine of the corresponding obliquity, or as the sine of the arc BG, which differs not sensibly from the length of this arc itself. Wherefore, the body will be urged in its descent by a force very nearly proportional to its distance from the lowest point B. The final velocity attained at B in the circular sweep, will evidently be the same as what would be acquired in sliding through the chord EBFB. The inciting force at F in the arc is likewise double the constant force exerted in the chord ; for the oblique angle FHI is double of declivity FBH. Wherefore, the time of descent in the small arc is to the time of descent in its chord, as a square to its inscribed circle. It is obvious that a ball, suspended by a thread OB from the centre O, and therefore called a *Pendulum*, would oscillate in arcs of the circle.

Since the time of oscillation has a constant ratio

to that of descent through the chords or the diameter, *its square must be proportional to the length of the pendulum.* On the supposition that the diameter BF of the circle is 16.0953 feet, the descent through any chord will be performed, on the parallel of London and at the level of the sea, in a second, and consequently each half vibration would occupy .785398″, or a whole vibration 1.570796″, the expression for the semicircumference of a circle of which the diameter is unit.

Such is the time in which a pendulum of 8.04765 feet length must perform its vibrations. Hence the measure of a pendulum oscillating single seconds may be easily found. For the square of the circumference of a circle is to quadruple its circumscribing square as 8.04765 to 3.2616 feet, or 39.1393 inches, the length of a seconds' pendulum for the latitude of London. In round numbers, the analogy 10 : 4 : : 8 : 3.2 affords a very tolerable approximation; in short, the fifth part of the fall in any place on the earth's surface during a second, is nearly the corresponding length of a pendulum.

Hence, a clock will be made to go faster or slower, by raising or depressing the bob of the pendulum. Let l and l' be two approximate lengths of its rod, and $3l + l' : 3l' + l : : 86400''$: to the number very nearly of beats in a day. The shortening of the rod of a seconds' pendulum, only by the tenth part of an inch, would therefore occasion an accele-

ration of 114″, in the space of 24 hours. A single turn of the nut-screw, of which the thread is one-thirtieth part of an inch, will correspond to 34″.

In very small arcs, the vibrations of the pendulum may be viewed as isochronous. But the time of describing a large arc, such as BE, (fig. 46.) is considerably longer; for the inciting force at any point E being proportional only to the sine EK, and not to the arc itself, is now much inferior to what would be necessary to produce a descent in the same time. A seconds' pendulum would require 1.18″ to describe the full semicircle, and 1.0736″ to sweep over an arc of 120°. In small arcs, the daily retardation in seconds will be expressed nearly by five-thirds of the square of the number of degrees through which the pendulum traverses on either side of the vertical. Thus, if the vibration should alternate the arc of 3°, a clock will lose in the space of 24 hours a quarter of a minute, or a whole minute if this arc were doubled. The correction for the going of a clock, however, may be simplified by drawing, across the case, an horizontal line divided into inches and tenths, 36¾ inches below the point of suspension. The square of the number of inches through which the rod of the pendulum oscillates, will express in seconds the alteration of the rate in the course of a day.

To the *Cycloid* belongs the remarkable property, *that the time of descent through every arc is abso-*

lutely the same. For let ADB (fig. 57.) be a cycloid, of which CGD is the generating circle. From the property of the curve, (see Geometry of Curve Lines), the tangent at any point E is parallel to the chord GD of the generating circle; and consequently the inclination of descent to the horizon is equal to the angle DGH or DCG. But the sine of this angle to a radius CD is DG, of which the cycloidal arc DE is double. The accelerating force at each point of the cycloid is therefore proportional to the arc intercepted from D, and consequently the time of descent must, in all cases, be the same. But a pendulum KPE, suspended from K, and vibrating between the inverted semicycloids AK and BK, would describe the cycloid ADB. The oscillations of such a constrained pendulum would hence be perfectly isochronous. This cycloidal pendulum will, near the lowest point D, evidently coincide in its motion with that of a circular pendulum having the same length CD, which is the radius of curvature. But, in the large aberrations from the vertical, while the warping pendulum maintains its isochronous oscillation, the inflexible pendulum will suffer a visible retardation.

If the cycloid ADB were extended into a straight line equal to twice DN, and, with this radius, a cemicircle described upon it, a perpendicular erected from any point, as now shown in reference to fig. 45., would denote the acquired velocity, and would

cut off an arc proportional to the time elapsed. But the section of a semicircle, constituted on the expansion of a portion of the cycloid, situate equally on both sides of D, will intimate the same proportional quantities.

The *Cycloid* is likewise distinguished as *the curve of swiftest descent*. Thus, a body will slide from B to E, (fig. 57.) along the cycloidal arc BDE in a shorter time than by any other track, whether straight or curvilineal. To demonstrate this beautiful property, it will be sufficient to show that a body descending in the curve, would sweep over each portion of it in the shortest interval. Suppose, in general, that a body falling from A to B, (fig. 95.) makes its quickest transit in a vertical plane from A to B, passing by some point C in a given intermediate horizontal line CD. Let the velocity of description between A and C be denoted by E, and that between C and B by F. If an adjacent point *c* be assumed, it is obvious, that in the position of a *minimum* transit, the increment of the time of passage from A to *c* must be just equal to the decrement of passing from *c* to B. Wherefore, describing the arcs, or letting fall the perpendiculars, C*a* and *cb*, the elemental space *ca* must be described with a velocity E, in the same instant that *cb* is traced with the velocity F. Hence *ca* is to C*b*, or the cosine of the angle ACD to the cosine BCD, as E to F. The condition of the curve of swiftest

descent, is therefore such, that the sine of the angle which each portion of it makes with a vertical is proportional to the celerity of descent. But this is a distinguishing property of the cycloid; for the celerity acquired by a body in descending from the extremity A to E (fig. 57.) is measured by the chord CG, which likewise expresses the sine of the angle CDG, made by the curve at E with a vertical line.

It hence follows, that the diameter CD, denoting the time of falling through this height, the semicircumference CND will express the time of descending through the semicycloidal arc from B to D, while the chord DB itself will measure the descent over this inclined plane. The time of arriving in the cycloid from B to I will likewise be expressed by CNDK or D, the semicircumference with the arc K intercepted by the parallel IKN. The time of sliding down the inclined plane BI is evidently measured by L, the fourth term of this analogy, CK : CD : : BI : L.

To find the track of swiftest descent from B to any point M, it is only required to produce BM till it meet a given cycloid in I, and make BI to BM as CD to the radius of a generating circle, which, rolling along BA, would trace another cycloidal arc passing this M.

On this property of the cycloid are founded some applications, very useful in practical mechanics. Thus, in Switzerland and several parts of Germany,

slides have been constructed along the sides of mountains, by which the timber felled near their summits is conducted with extreme rapidity to the distant valleys. In such cases the angle of descent is taken greatest at first, and afterwards gradually diminished. A similar incurvation should be traced in laying the slips for launching vessels from the stocks.

Oscillations in different arcs of the cycloid are always performed in the same time, because the inciting forces, as we have proved, are exactly proportional to the spaces described. But the same property must obtain, whether the tracks of motion be curved or rectilineal. The only condition required, is, that the body should be urged by a force directly as its actual distance from the middle point of the path traced. This principle has a most extensive application in the economy of Nature. It includes not only the isochronous vibrations of pendulums and springs, but comprehends all those internal tremors excited in any system of atoms upheld by the antagonism of attraction and repulsion. Hence the regular propagation of pulses through air and through water; and hence likewise the equable transmission of the tremulous impressions along beams of wood and bars of iron.

Weights attached at different distances from the point of suspension, constitute a *Compound Pendulum.* But since short pendulums vibrate faster

than long ones, the nearer weights must conspire to accelerate the motion of such as are remote; while these again retard the former. There is hence a certain neutral point entirely exempt from the opposite impressions. This point is called the *Centre of Oscillation*, and its distance from that of suspension is equal to the length of a simple pendulum, which vibrates in unison with the compound one. Suppose (fig. 47.) the balls B, C, and D were fixed to a very slender inflexible rod, their centre of oscillation being O. It is obvious, that in every position of the pendulum, the tangents to the arcs of description will, at the several points B, C, O and D, make equal angles with the horizon, and consequently those points are all urged by the same measure of force. To render the descents in the arcs BE, CF, ON and DH isochronous, it would have required, however, the inciting energies to be proportional to them, or to the respective distances AB, AC, AO and AD. If AO, therefore, express the accelerating power at O, BO and CO will denote the excess of acceleration at B and C, and DO the retarding influence at D. But, from the principle of the lever, the effect of these disturbing forces in moving the pendulum must be as their distances from the point of suspension. Wherefore the accelerating action at the centre O must be equal to that of retardation; or $B.BO.AB + C.CO.AC = D.DO.AD$. But $BO = AO - AB$, $CO = AO - AC$, and $DO = AD - AO$;

by substitution consequently $B\,(AO-AB)\,AB + C\,(AO-AC)\,AC = D\,(AD-AO)\,AD$, or $B.AO.AB + B.AB^2 + C.AO.AC - C.AC^2 = D.AD^2 - D.AO.AD$; whence $AO\,(B.AB + C.AC + D.AD) = B.AB^2 + C.AC^2 + D.AD^2$ and $AO = \dfrac{B.AB^2 + C.AC^2 + D.AD^2}{B.AB + C.AC + D.AD}$. Wherefore *the distance of the Centre of Oscillation from the point of suspension, is found by multiplying each weight into the square of its distance from that point, and dividing by the sum of the products of those weights into their distances simply.*

The same procedure will apply, should any of the weights occupy a position above the point of suspension; only, in this case, their products into the several distances must be taken as subtractive. The general formula will admit of being modified also, in relation to G the Centre of Gravity. For, from the property of the lever,

$B.AB + C.AC + D.AD = (B + C + D)\,AG$; and consequently $AO = \dfrac{B.AB^2 + C.AC^2 + D.AG^2}{(B + C + D)\,AG}$.

Again, since $AB = AG - BG$, $AC = AG - CG$, and $AD = AG + DG$, it follows, by involution and substitution, that $AO =$

$$\frac{B(AG^2 + BG^2) + C(AG^2 + CG^2) + D(AG^2 + DG^2)}{(B + C + D)\,AG},$$

and therefore $OG = \dfrac{B.BG^2 + C.CG^2 + D.DG^2}{(B + C + D)\,AG}$.

Whence *the interval of the centre of oscillation below ~~the point of suspension~~ will be found, by dividing the sum of the products of the weights into the squares of their several distances from the centre of gravity, by the product of the sum of all the weights into the distance of the centre of gravity from the point of suspension.*

By either of these theorems, the position of the centre of oscillation is computed. Hence, an even slender rod, viewed as a mathematical line, and suspended from the end, has its centre of oscillation situate at two-thirds of its length ; but, in an isosceles triangle vibrating flatwise, it is three-fourths below the vertex ; and, in a parabola, the position is intermediate, being at the five-seventh part of the axis. Again, the distance of the centre of oscillation of a cone, suspended from its apex, is $\frac{4a^2+r^2}{5a}$, where a denotes the altitude of the cone, and r the radius of its base ; and consequently this distance becomes equal to four-fifths of the altitude, when the cone is viewed as indefinitely slender. Of a hemisphere vibrating from its pole, the centre of oscillation divides the radius, in the ratio of 26 to 9 ; but, of a whole sphere, it occupies seven-tenths of the diameter.

The centre of oscillation being found for one point of suspension may be readily discovered for any other, since the product of the distances of both those points from the centre of gravity remains always the same.

Thus, if a slender cylinder be supported by a line, the depression of its centre of oscillation below the middle point will be equal to the square of the length of the cylinder divided by twelve times the distance of that middle from the point of suspension.

Again, let a sphere be suspended by a fine thread. Then the length of this thread, together with the radius, is to the radius, as two-fifths of this radius, to the depression of the centre of oscillation below the centre of the sphere. Suppose a ball, or a lenticular bob flattened in the same proportions and 6 inches in diameter, to hang by a fine wire or very slender rod of 36 inches in length. Then $36 + 3 : 3 :: \frac{6}{5} : \frac{6}{65}$. So that in this case the centre of oscillation would be $39\frac{6}{65}$ inches below the point of suspension.

In general, the distance of the centre of oscillation below that of gravity, is inversely as the distance of the point of suspension from the same centre; for the system of atoms being unaltered, the product of the distances of the centre of gravity from those opposite points is constant. Hence the centre of oscillation and the point of suspension are interchangeable. Thus, if the compound pendulum ABCD (fig. 47.) were suspended from O, the centre of oscillation would occur in the point A. This pendulum would therefore perform its vibrations in the same time, either about A or O.

Let two equal balls B and C (fig. 48.) be attached to the inflexible line AC, which swings

from the end A; since BC is now bisected by the centre of gravity, $GO = \frac{BG^2 + GC^2}{2AG} = \frac{BG^2}{AG}$, and GO : BG : : BG : AG. Whence, by mixing, BG—GO : BG + GO : : AG—BG : AG + BG, or OC : OB : : AB : AC; the interval BC between the balls is, therefore, divided internally and externally in the same ratio, by the centre of oscillation and the point of suspension. In (fig. 49.) the ball B is placed as much above the point of suspension as in fig. 48. it lies below it; the centre of oscillation O is now thrown to a considerable distance, dividing BC externally in the ratio of AB to AC. Hence, by proportioning the distances of the balls of a short pendulum on both sides of the point of suspension, it may be made isochronous with one of any given length. Slow vibrations are thus obtained within narrow bounds. On this principle, is constructed a small instrument called a *Metronome*, for marking conveniently the beats in music.

The position of the centre of oscillation will determine the character of *Rocking Stones*, for we may consider them as only rolling about their *metacentre*. Suppose a wooden model, as in fig. 26., consisting of a cylinder 9 inches high and 3 inches in diameter; if a thin convex segment with a radius AO of 4 inches were attached to the base, it would not stand upon a plane, but, placed in a cavity less than three feet in diameter, it would rock. Thus, if the

radius AP of the concavity were 6 inches, AM would be $\frac{6.4}{6-4}=12$ inches; or if AP were 24 inches, AM would be $\frac{24.4}{24-4}=4.8$. Now the centre of oscillation of the cylinder being at two-thirds of its height, is 6 inches from the top, and its distance from the point of suspension when AM = 12, is $\frac{6.1\frac{1}{2}}{12-4\frac{1}{2}}=1.2$; but when AM = 4.8, it is $\frac{9}{4.8-4.5}=30$. Wherefore, since $30 = 1.2 \times 25$ and $\sqrt{25} = 5$, the latter arrangement will make the vibrations five times slower than the former.

Suppose a body, urged by a force inversely as the square of its distance, to fall towards a given centre, its celerity and time of descent may be easily derived from the general principles of phoronomics. Let the path traced reach from A to O (fig. 43 *.), bisect AO in I, and describe a semicircle, draw the tangent AL, and taking an ordinate BC, extend the chord OC to L, and assuming an adjacent ordinate bc, prolong the chord Oc to meet the tangent in l. Make OB : OA : : OI : OM : : OM : ON; whence $OB^2 : OA^2$: : OI : ON, and OI being constant, ON must represent the force of acceleration at B. Now, from similar triangles, Cx or $Bb : Cc$: : BC : IC, or OI. Again, the elementary triangles COc and LOl

being similar, $Cc : Ll :: BC : AL :: OB : OA :: OI : OM$, and by composition of ratios, $BC : OM :: Bb : Ll$. But $BC : AL :: OB : OA :: OM : ON$, and alternately, $BC : OM :: AL : ON$. Whence $AL : ON :: Bb : Ll$ and $AL.Ll = ON.Bb$. In the formula, $f \partial s = v \partial v$; f and ∂s being denoted by ON and Bb, it follows that AL will represent the velocity acquired at B, the increment being Ll.

To find the time of fall due to this law of acceleration. Since $AL : BC :: OA : OB$, it follows, that $\frac{OA}{AL}. Bb = \frac{OB}{BC}.Bb$. But OA being constant, the expression, $\frac{OA}{AL}.Bb$ denotes the element of the time, which, by decomposing the fraction $\frac{OB}{BC}$, is therefore equal to $\frac{BI}{BC}.Cx + \frac{CI}{BC}.Cx$. Now, from the similar triangles CBI and Cxc, $BC : BI :: Cx : xc$, and $BC : CI :: Cx : Cc$; whence $\frac{BI}{BC}.Cx = xc$, $\frac{CI}{BC}.Cx = Cc$, and the element of the time is compounded of xc and Cc, or of the increment of the sine BC and of its arc AC. The time of falling from A to B is, therefore, denoted, by the sum of BC and AC; and that of the whole descent to the centre O, by the semicircumference. It is hence evident, that the ordinate BCD of a cycloid which has AO for its altitude, must likewise express the

time of passing from A to B. The properties of these three modes of acceleration are all exhibited collectively in fig. 43 *. The velocity which a body uniformly accelerated acquires in falling from A to B, is represented by the chord AC, while the velocity accumulated by an acceleration inversely as the square of OB is denoted by the tangent AL. But the former is to the latter as OC to OA, or in the subduplicate ratio of OB to OA, for by similar triangles OC : OA : : AC : AL. The ultimate celerity of the one being as OA, that of the other becomes infinite.

The time of initial descent through a minute space being obviously the same in all the modes of acceleration, while the arc, together with its sine, is at first double of the chord, it follows that the time of falling to B, under an uniform action, must be represented by twice AC; the ordinate of the cycloid denoting the corresponding time elapsed, when the body is urged by a force inversely as the square of OB. The whole times of descent are hence respectively as 2OA to OF or ACO; that is, as a square is to its inscribed circle.

In the case of a body urged by a force directly proportional to the distance from a given centre, the velocity acquired at B will be represented on the same scale, by diminishing the ordinate BH of the quadrant in the ratio of AG to AO. The time elapsed during the fall to B will be expressed by the arc AH. The final velocity at O is therefore

denoted by OP perpendicular to AG, and the whole time of descent by the quadrantal arc AHG, or the semicircumference ACO.

These properties furnish the solution of some curious problems. Thus, suppose the axis of the earth were perforated from pole to pole: a body falling through the perpendicular hole, being attracted on all sides, would be urged downwards only by a predominating force proportional to its distance from the centre. The velocity acquired at this centre, reckoning the length of the axis 7900 miles, would hence be $8\sqrt{\left(\frac{3950.5280}{2}\right)} = 25834$ feet each second. The time of descent is $\frac{1.5708}{4}\sqrt{\left(\frac{3950.5280}{2}\right)} = 1268''\frac{1}{8} =$ $21'\ 8''\frac{1}{8}$, and the whole time of passage to the opposite pole $42',\ 16''\frac{1}{4}$.

Conceive a body under the mere influence of terrestrial attraction, to fall from the orbit of the moon to the earth's surface. At the mean distance of 60 semidiameters, the initial force would be diminished 3600 times: with the same continued acceleration, therefore, it would consume the period represented by $\frac{60}{4}\sqrt{(59.3956.5280)} = 526578''$, or 6 days, 2 hours, 16 minutes, and 18 seconds, in performing the whole descent. The final velocity, on this supposition, would be $\frac{8}{60}\sqrt{(59.3956.5280)} = 4680{,}69$ feet each second.

It is easily computed, that a tangent at the earth's surface would intercept, from a semicircle described on the radius of the lunar orbit, the arc of $165° \, 9' \, 54''$. But as twice the chord of this arc, or 3.9854020, is to the sum of the arc and its sine, or 3.1386812, so is the time of descent under an uniform acceleration, to the time required with an acceleration inversely as the square of the distance from the centre; which is hence 414645″, or 4 days, 9 hours, 10 minutes, and 45 seconds. Again, the final velocity, being augmented in the subduplicate ratio of 1 to 60, is 36256,45 feet, or about seven miles each second.

In general, r denoting in feet the radius of the Earth, the velocity which a body would acquire in falling with an uniform acceleration from the height nr to the surface will be expressed by $\frac{8}{n}\sqrt{(n-1)r}$; and consequently the velocity acquired under an acceleration inversely as the square of the distance from the centre will be denoted by $8\sqrt{r}\sqrt{\frac{n-1}{n}}$. When n is indefinitely great, the final celerity is hence $8\sqrt{r}$, or 36562.43 feet, the mean radius being 3956 miles. Abstracting, then, from the resistance of the atmosphere, a body shot directly upwards with a velocity of 36256.45 feet each second, would mount to the orbit of the moon; but, with the addition of one 120th part more, or 305 feet each second, it would reach the sun, and the farther acceleration of less than one foot would have enabled it to continue its flight into the regions of boundless space.

The same results are obtained more easily. Thus, r denoting in feet the radius of the earth, the expression for a body falling from the height x is $64r^2.\frac{-\partial x}{x^2}=2\,v\partial v=\partial.v^2$, which may be written thus, $64r^2\left(\frac{x}{xx}-\frac{x'}{xx'}\right)=64r^2\left(\frac{1}{x'}-\frac{1}{x}\right)=\partial\,.\,v^2$; and hence, by the summation of the differences of the successive fractions, supposing a to be the initial height, $64r^2\left(\frac{1}{x}-\frac{1}{a}\right)=v^2$. When x becomes equal to r, and the body reaches the surface, $8r\sqrt{\left(\frac{1}{r}-\frac{1}{a}\right)}=v$, or $8\sqrt{\left(\frac{ar-r^2}{a}\right)}$. If a be assumed indefinitely great, the final velocity is simply $8\sqrt{r}$.

If a body describe a curvilineal path, it must be continually deflected from its impulsive motion, by the influence of some regular force. When this force is directed to a fixed point, it is called *Centripetal*, and the antagonist effort of the body to fly off in a tangent, is termed *Centrifugal.* To begin with the simplest case, let the body A (fig. 50.) revolve in the circumference of a circle, whose centre is C. If it were abandoned to its mere impulsion, it would shoot forwards in the tangential line AB; but, in the time of describing the small portion of the curve Ab, it is constrained to sink towards the centre from B to b. Extend BdcE across the circle,

and (Geometry, III. 26.) $AB^2 = EB.Bd$; wherefore, since EB may be considered as equal to the diameter, the measure of deflection $Bb = \frac{AB^2}{2AC}$. Let the radius AC in feet be denoted by r, the time of revolution in seconds by t, the velocity by v, the power of gravity by g, the centrifugal force by f, and the ratio of the circumference to the diameter of a circle by π. Then $AB = \frac{2\pi r}{t}$, and the measure of deflection $Bb = \frac{AB^2}{2r} = \frac{2\pi^2 r}{t^2}$. But this is the momentary descent which would be caused by the force which confines the motion to a circle; wherefore, $\frac{2\pi^2 r}{t^2} = \frac{1}{2}gf$, or $\frac{4\pi^2 r}{t^2} = gf$, and $f = \frac{4\pi^2 r}{gt^2}$. *Hence the centrifugal force is directly as the radius of the circle, and inversely as the square of the time of revolution.* Assuming, for the sake of round numbers, $\pi^2 = 10$; and $f = \frac{40r}{32t^2} = \frac{5r}{4t^2}$, a very simple expression. Suppose the centrifugal force were equal to the action of gravity, and $5r = 4t^2 = (2t)^2$; or $t = \frac{1}{2}\sqrt{5r}$.

These formulæ may be changed into terms of the velocity, for $vt = 2\pi r$, and consequently $v^2t^2 = 4\pi^2 r^2$, and $\frac{2\pi^2 r}{t^2} = \frac{v^2t^2}{2rt^2} = \frac{v^2}{2r}$. But this quantity, which marks the deflection, is equal to $16f$, and therefore

$f = \frac{v^2}{32r}$. When the centrifugal force is equal to the power of gravity, $v^2 = 32r$, or $v = 8\sqrt{\frac{1}{2}r}$; that is, *the velocity is the same as what the body would acquire in falling through half the radius.*

Hence the centrifugal effort on the tension of a pendulum at the lowest point, after descending through a quadrant, would be just double its weight; for, in this case, $v = 8\sqrt{r}$, and consequently $f = \frac{64r}{32r} = 2$. Let d denote generally the vertical descent, and $v = 8\sqrt{d}$; whence $f = \frac{64d}{32r} = \frac{2d}{r}$. In small arcs, the vertical tension may be found, by dividing the square of the number of degrees described in each oscillation, by 13153.

Hence, also, a sling two feet long, circling vertically, with a celerity of 8 feet each second, would just sustain its load; if it were accelerated so as to perform a revolution in one second, the tension of the string would exceed that weight $2\frac{1}{2}$ times. A tumbler full of water is therefore easily whirled about the head, without spilling a single drop.

From the same formulæ, it is likewise computed, that, at the equator, the diminution of gravity occasioned by the centrifugal force arising from the rotation of the earth, amounts to about the 289th part. But since this number is the square of 17; it follows, that, if our globe turned more than seven-

teen times faster about her axis, or performed the diurnal revolution within the space of 84 minutes, the centrifugal force would have predominated over the power of gravitation, and all the fluid and loose matters would, near the equinoxial boundary, have been projected from the surface. On such a supposition, the waters of the ocean must have been drained off, and an impassable zone of sterility interposed between the opposite hemispheres. By a similar calculation, combined with the decreasing force of gravity at great distances from the centre, it may be inferred, that the altitude of our atmosphere could never exceed 26,000 miles. Beyond this limit, the equatorial portions of air would have been shot into boundless space.

The action of centrifugal force is familiar to us in the trundling of a wet mop. It likewise supports the vertiginous motion, and prevents the fall, of a spinning-top. In the practice of the arts, centrifugal force is often advantageously employed. By a rapid whirl, the globules of air, which are apt to form in filling a spirit-of-wine thermometer, are easily detached; and, in like manner, different liquids intermixed, but not chemically combined, are separated into distinct masses. Hence the flour, as fast as it is ground, is thrown out from the rim of the revolving mill-stone. Several manipulations of the potter and the glass-blower depend on the same principle. Hence the clay, under a gentle pressure, swells out regularly during the rotation of the wheel. Hence also the

lengthening of a rod of glass by whirling, and the spreading of a sheet by the process called *flushing*. But this extension is produced uniformly. For let OH (fig. 51.) represent a rod, or some radiated portion of the soft material, which is whirled about O as a centre, and conceive it to be distinguished into equal spaces OA, AB, BC, CD, &c. The centrifugal forces exerted at A, B, C, D. &c. are as the radii OA, OB, OC, OD, &c., the distances of those points from O. Wherefore the force at B exceeds that at A by a force AB; the force at C exceeds that at B by a force BC; and so, through the whole of the chain, the successive differences being all equal. The part A is therefore drawn from O, the part B from A, the part C from B, and so forth, all by the same excess of force. Consequently the whole column is stretched uniformly, and extended to the same thickness.

If a ball suspended obliquely from a point by a thread or thin rod, describe a circle in the horizontal plane; it will form what is called a *Conical Pendulum*. Let AC (fig. 52.) circulate about the axis AB, it is evident that a strain CH in the direction of the pendulum must be compounded of CG the vertical power of gravity, and of CF the horizontal centrifugal force. Wherefore, the action of gravity will be to this centrifugal force, as CG to CF, that is, from similar triangles, as AB to BC. But when the time of revolution remains constant, the centri-

fugal force is proportional to the radius BC, and hence the perpendicular AB, representing the power of gravity, must continue likewise unvaried; consequently, (fig. 53.) if a number of balls B, C, D, attached by wires AB, AC and AD, of unequal lengths, be carried round in the same time, they will always range themselves in a horizontal plane.

Let AC (fig. 52.) be expressed by l, AB by h, BC by r, and GH by f, the power of gravity CG being represented by unit. Then $f = \frac{\pi^2}{8} \cdot \frac{r}{t^2}$; but, since AB : BC : : CG : GH, or $h : r :: 1 : f$, therefore, $f = \frac{r}{h}$, and by substitution $\frac{\pi^2}{8} \cdot \frac{r}{t^2} = \frac{r}{h}$, or $\frac{\pi^2}{8} \cdot h = t^2$. Now, $\frac{\pi^2}{8}$ is constant, and hence the axis of the conical pendulum is proportional to the square of the time of revolution. In round numbers, $\frac{5}{4} \cdot h = t^2$, and $t = \frac{19}{17}\sqrt{h}$. But if the correct result be preferred, $1{,}2337 . h = t^2$, or $t = 1{,}11072 \sqrt{h}$, and $h = .81057 . t^2$. It may be sufficiently near, however, to assume $t = \frac{10}{9} \sqrt{h}$, and $h = \frac{43}{53} . t^2$. Hence the altitude of a pendulum, which circulates in a second, is 9.72684 inches; the altitude, corresponding to half seconds, is 2.43171 inches, and that answering to a circumvolution in the third of a second, 1.08076 inches.

The conical pendulum is applied to regulate the movements of a delicate piece of mechanism, which exhibits the continuous flow of time, and measures intervals less than the tenth part of a second. It is likewise adapted, with complete effect, as a *Governor* in the steam-engine and other rotative machines. For when the circulation exceeds a certain rapidity, the balls B and C (fig. 54.) flying out, shorten their perpendicular distance below the centre A, and consequently pull down the opposite point F of the parallelogram DAEF, and thus open the principal valve, or otherwise regulate the general movement.

If a body be thrown directly upwards into the air, it will, abstracting the resistance of that medium, ascend with an uniformly retarding velocity, till its motion is overcome by the action of gravity, when it will again descend, and acquire in the fall a celerity equal and opposite to its previous impulsion. The height to which it will rise must therefore be equal to the square of the eighth part of the velocity of projection; and the whole time spent during this ascent, and the subsequent descent, is the sixteenth part of that velocity. But if the body be thrown obliquely, it will be incessantly bent from its rectilineal flight, and brought sooner to the ground. The deflected space being, from the action of gravity, as the square of the time elapsed, must consequently be proportional to the square of the tan-

ments of description. Wherefore (Geometry of Curve Lines, p. 235.) the *trajectory*, or the curve traced by the projectile, is a PARABOLA.

Let v denote the velocity with which the projectile is discharged, and t the entire duration of its flight. Since it would be carried by its oblique impulsion, through the straight line AB, (fig. 58.) in the time which it would require to drop from B to C, therefore $AB = vt$, and $BC = 16t^2$. But, from the property of the Parabola, the square of the tangent AB is equal to the rectangle under BC, and the parameter of the diameter at A. Wherefore, $v^2 t^2 = 16 t^2 \times$ parameter, and the parameter itself $= \frac{v^2}{16}$. Hence the fourth part of this, or the focal distance AF, is equal to AG the vertical descent; and, consequently, the projectiles discharged from A, at whatever elevation, but with the same velocity, must trace parabolic paths, which have all the same directrix GHI, and whose foci F or F′ occupy the circumference of a circle described with the radius AG.

Now, from the property of the parabola, the tangent AD bisects the angle FAG, and consequently AFE, or its alternate angle GAF, is double of GAD; wherefore the supplemental angle AFD is double of the complemental angle EAD, or the elevation. Hence, in the right-angled triangle AEF, radius is to the sine of AFE, which is the same as

the sine of AFD, or of twice EAD, as AG or AF is to AE, the half of AC the *horizontal range.* Resuming the former notation, and e expressing the elevation CAB, the radius being unit; then

$sin\ 2e\left(\frac{v}{8}\right)^2 = AE$, and $sin\ 2e.\frac{v^2}{32} = AC$. Again, $sin\ CAB.AB = BC$, or $sin\ e.vt = 16t^2$, and therefore $t = \frac{sin\ e.v}{16}$.

Hence the greatest horizontal range is attained by an elevation of half a right angle, for the sine of double of this elevation is equal to the radius itself. In this case, the focus of the parabola occurs at F. It likewise follows, that two elevations, such as CAB and CAB′, equally distant on either side of this middle angle, will have the same horizontal range, because their doubles would evidently be supplemental angles having equal sines. Here the point E bisects the base AC of both those parabolas.

Suppose a bomb were thrown with a velocity of only 240 feet each second, the height to which it would rise, if thrown perpendicularly upwards, would be $(30)^2$ or 900 feet; if projected at an angle of 45°, the horizontal range would be the double of 900 or 1800 feet, and the time of flight 10.6 seconds. But if the shell had been thrown at an elevation either of 75° or 15°, it would have reached only the horizontal distance of 900 feet, the time of describing the

higher path being 14.5 seconds, while the lower path is described in 3.9 seconds.

The elevation and horizontal range of a projectile being known, let it be required to find where the ball will strike a given oblique plane. Suppose AB (fig. 59.) to represent the line of impulsion, AC the horizontal range, and AD the oblique plane. Erect the perpendicular CD meeting AD in D, draw DE parallel to AB, and EG parallel to CD; and the parabola will cut AD in the point G. By the application of such successive parallels, the curve might also be traced. (See Geometry of Curve Lines, pp. 297 and 447).

In the practice of Gunnery, however, unless in the case of very small velocities, the Parabolic Theory is of little avail, the resistance of the air being so prodigious as completely to derange all the effects. Thus, a 24 pound shot, discharged at the elevation of 45°, and with a velocity of 2000 feet each second, would, *in vacuo*, reach the horizontal distance of 125,000 feet, but has its range through the atmosphere confined to 7300 feet. Accurate experiments on the resistance of the air will introduce the proper corrections, and lead to the determination of the *Ballistic Curve*.

Suppose a projectile to be now deflected from its tangential course, not in the same constant direction, but in lines tending to a given point or centre of action. Let the body A, (fig. 57 *.) which, abandoned to itself, would travel uniformly along the path ABC,

after the lapse of the first instant it has arrived at B, instead of advancing through an equal space to C, is drawn aside to D, by a force directed towards O. But the segments AB and BC, assumed as extremely small, in comparison of the distance of the centre, the line CD may be considered as parallel to BO, and hence the elemental triangle OBD is equal to OBC, which again is equal to AOB. The series of triangles formed about the centre O, during successive instants, are hence all equal, and consequently the sector comprising them is proportional to the whole time elapsed. But those instants being taken indefinitely small, the polygon ABD, &c. will merge into a continuous curve. In all motions, therefore, controuled by the action of a central force, *the space traced over by a radiant is proportional to the time of description.* Such is the primary law of our Solar System, first detected by Kepler, and demonstrated by Newton.

Conceive a body to revolve in the periphery of an ellipse, urged by a force directed to the centre of the curve. In moving through the elemental arc E*e* (fig. 69 *.), the area of the triangle OE*e*, measuring a constant portion of the time, the deflection *es* will mark the corresponding centripetal force. This arc E*e*, or the velocity at E, is therefore inversely as the perpendicular OI upon the tangent; or, from the property of the ellipse, it is directly as the semi-conjugate diameter OL. But the square of E*e* in the circle is equal to the rectangle under *es* and

the chord of the equicurve circle or twice EK. Wherefore the square of OL, or the rectangle under OE and EK, is proportional to the rectangle under *e*ε and the same EK, and consequently *e*ε, *the measure of the centripetal force at E, is directly as the distance OE.* It likewise follows, that, in the revolution about the centre of an ellipse, the velocity at any point E is always proportional to OL, the semiconjugate diameter.

Next, suppose a body to describe an ellipse, under the action of a force emanating from one of its *foci*. This force being measured by *e*ε in the direction of EF, is therefore inversely as EH half the chord of the equicurve circle, and directly as the square of the velocity E*e*. But E*e* being inversely as the altitude FG of the elemental triangle EF*e*, it follows that the constraining force must be inversely as EH and the square of FG. Now, from the property of the ellipse, the square of OL is equal to the rectangle under EH and the semitransverse axis OA, and therefore Eε is inversely as the combined squares of OL and FG; but OD : OL : : FG : FE, and OL.FG = OD.FE; whence, in this case, the centripetal force is inversely as the squares of OD and FE, or OD being constant, it is as the square of FE simply. Such is the law of Universal Gravitation, that *a body is attracted to another by a force reciprocally as the square of their distance*—a fundamental principle, which Newton deduced from the

second great discovery of Kepler, respecting the figure of the planetary orbits.

When the revolution is performed about the centre of the ellipse, (fig. 69 *.) the element EO*e* of the time being equal to half the rectangle under E*e* and the perpendicular OI, the celerity E*e* is consequently inversely as OI. But, from the property of the curve, the rectangle under OI and the semiconjugate diameter OL is equal to the constant rectangle under the semiaxes OA and OE, and therefore OL is the reciprocal of OI; whence the celerity of E must be directly as OL; and thus the celerity at A or B is as OC, and C or D as OA.

From the relation of the ellipse to the circle, the area IBX (fig. 59 *.) is to IBS as YX to YS or as IP to IQ, or as the area of the ellipse to that of the circumscribing circle. But in the same ratio are the initial velocities at the vertex B, and consequently the time which a body, drawn by a force directly as its distance from the centre I, takes to descend in the ellipse, or the circle to an ordinate YXS is measured by the arc AS, or generally by the angle BIS. Hence the time of revolution is the same in all the ellipses constructed on the diameter BW. Nay, this period will yet continue the same while the length of the diameter is augmented, since the initial velocity in a circle must be proportionally increased.

These properties will hold, even though the curves

should collapse into straight lines. Hence the descents from A to O are isochronous, as in the case of cycloidal pendulums; and hence, likewise, the time of performing a complete oscillation, and of returning to the same point, is measured by the circumference of a circle described with the radius OA. Again, from the property of the ellipse, OA^2+OC^2 (fig. 69 *.)$=OE^2+OL^2$; and supposing the curve to be compressed into its diameter AB, then would $OA^2 = OM^2 + OL^2$, and therefore OL^2, which represents the square of the velocity at M, becomes YS^2 (in fig. 53 *.) the time of falling to M being likewise indicated, as before, by the arc BS.

Let the attractive force be now directed to the focus F (fig. 69 *.) of the ellipse; the element EF*e* of the time is equal to half the rectangle under E*e* and the perpendicular FG drawn to the tangent, and consequently E*e* is in the inverse ratio of FG. But, having let fall the perpendicular *f*R from the other focus, the rectangle FG, *f*R is equal to the square of OC, and is therefore constant; whence the celerity at E is directly as this perpendicular *f*R. The celerities at the vertex A, at C or D, and at the opposite extremity B, are thus respectively proportional to A*f*, OC, and *f*B or AF.

Since the elliptical sector IBX (fig. 53 *.) is to the circular one IBS, as YX to YS, or as IP to IQ, and the triangle OIX is to OIS in the same ratio, it follows, that the areas OBX and OBS, which, in the ellipse or the circle, measured the time elapsed, are

likewise proportional to IP and IQ, or the velocities at the vertex, and consequently those spaces must indicate equal intervals. The body descends in the same time, therefore, to any perpendicular YS, whether by an elliptical or a circular arc. Hence, the periods of revolution are equal in all those ellipses, including the circle itself, which have the same transverse axis or diameter.

In general, let r denote the radius of any circle, v the velocity, and t the time of circumvolution. The centripetal force being now expressed by $\frac{1}{r^2}$, it is evident, that $v^2 = \frac{1}{r^2}.r = \frac{1}{r}$, and $v = \sqrt{\frac{1}{r}}$, or *the velocity is in the inverse subduplicate ratio of the diameter of the orbit.* Hence $t = \frac{r}{\sqrt{\frac{1}{r}}}$ and $t^2 = r^3$, or *the square of the period of revolution is as the cube of the principal axis*—the third great law of the planetary motions, brought to light by the ingenuity and indefatigable research of Kepler.

Whether the controuling force be directly proportional to the distance of the centre, or inversely as the square of that distance, the body might have acquired its velocity of projection by falling with an uniform acceleration through a determinate space. In the case of circular revolutions, l denoting the vertical lapse, corresponding to the initial sweep of the trajectory, $4l.$BZ (fig. 53 *.) will express the

square of the velocity gained in descending to Z under a constant force; which, from the property of the circle, is likewise equal to l.BW, and, consequently, the measure BZ is always half the radius. This point Z coincides, therefore, with f the farther focus. If the curve described pass into an ellipse, the index of the primary velocity must still be half the radius of osculation at its vertex. To generate a circular description, it is hence requisite that the body should have the precise celerity due to a fall through half that radius. But, projected with inferior celerity, it would, about the same axis, trace elliptical orbits more or less compressed; the foci mutually retiring towards the extremities, while the conjugate diameter contracts. When the primary impulsion becomes extinct, the elongated ellipse merges in a straight line. On the other hand, if the projectile motion should exceed the limit of circular velocity, a new series of curves will arise. But when the acceleration is directly as the distance, on passing the limit of celerity, the foci will first unite in the centre, and the axis suddenly turning at right angles, the foci will now gradually dispart in this transverse position. As the velocity of impulsion is successively augmented, the ellipse will assume all the degrees of oblateness, till it finally vanishes in two parallel lines.

If the centripetal force be inversely as the square of the distance, the moment the primary impulsion transcends the limit due to a circular trajectory, the

orbit would suddenly change into an equilateral hyperbola; the farther focus, which had come to coalesce with the attractive one, now flying to the opposite side beyond the diameter. When the celerity of projection receives a farther increase, the incurvation at the vertex will be proportionally diminished. With an extreme impulsion, the body would shoot off in a straight line perpendicular to the axis.

It thus appears, that a circular revolution, which the ancients so fondly contemplated as the perfection of the celestial movements, is incompatible with the stability of the universe. The most absolute precision of impulse would have been necessary, and the very slightest subsequent addition of celerity, from the incidental influence of those disturbing forces which are incessantly in operation, would at once have transformed the circle into an hyperbola, and have carried the planet away for ever into the boundless expanse of heaven. In viewing the grand phenomena of nature, our admiration is drawn to those conservatory principles, which, in shorter or longer periods, correct every occasional deviation from the general balance of the system.

It is curious, however, to remark how very nearly the planetary orbits approximate to circles. In that of our earth, the two axes differ only by the 7086th part. In the trajectory of Mars, this difference amounts to the 231st part. It was accordingly the greater eccentricity of that orbit which led Kepler to

detect its elliptical form. The group of small kindred planets lately discovered revolve in curves still more elongated, the diameters of those of Juno and Pallas being nearly in the ratio of 30 to 29. These singular bodies might seem to rank between the ordinary planets and the comets, which wander in ellipses of extreme elongation, scarcely distinguishable from parabolas, during a great part of their visible track. In another circumstance, too, the analogy obtains; for while the larger planets deviate not more than 2 or 3 degrees from the plane of the ecliptic, Pallas crosses it at an angle of 35°, and the paths of the comets have every possible inclination. It remains to be discovered, whether diversified bodies, travelling in the celestial spaces, may not fill up more completely that prolonged gradation of existence, which appears so conspicuous in other parts of nature. Supposing the projection of the planets to be the result of some more general law, those which had an excess of impulsion would totally escape our range of observation, being swept away in hyperbolas into boundless space, there perhaps to form other stellar systems.

Abating the resistance of the air, a projectile discharged horizontally, with the velocity of 25853½ feet in a second, would circulate around our earth. For half the radius being 1978 miles, $8\surd(1978.5280)=25853.6$, and hence the time of re-

volution, on this hypothesis, would have been 5109.8 seconds, or $85\frac{1}{8}$ minutes. The deflective force being inversely as the square of the distance from the centre of gravitation, the smallest increase of celerity beyond the limit assigned would have led the body away in a boundless hyperbolic orbit. But, with a smaller velocity than $25853\frac{1}{2}$, the trajectory would contract into an ellipse, of which the projectile, whether discharged in a horizontal or an oblique direction, must describe a portion, till it strikes the earth's surface. The theory of Ballistics is thus comprised in the general system of centripetal forces. When the celerity of impulsion is moderate, as in the case of all artificial projectiles, the portion of their elliptical orbit, traced within our atmosphere, will not sensibly deviate from a parabola. Thus, reckoning the utmost rapidity of a ball, shot from the mouth of a cannon, to be 2000 feet each second, the focus would be only $\left(\frac{2000}{8}\right)^2 = 62500$ feet, or about $11\frac{4}{5}$ miles below the vertex of the curve. The semiconjugate diameter of the ellipse would have extended no farther than $305\frac{4}{5}$ miles, had the intervention of the body of the earth not opposed the complete description of the curve.

Assuming, therefore, the extreme portion of the trajectory as really parabolic, the horizontal velocity at the vertex V (fig. 58.) must be to the velocity of projection, as FV to the perpendicular FL; but,

from similar triangles, FV : FL : : FL : AG, and consequently FV : AG or AF : : FV^2 : FL^2. Now, this is the ratio of the spaces through which a body must fall, to acquire the velocities at V and A, and hence the projectile would have been carried directly upwards to G, the position of the directrix. Again, the time of descent, under an uniform acceleration, is the same as that of the passage from A to V, and therefore a body would have fallen through BC, during the whole sweep from A to V, and thence to C. The parabolic theory of ballistics is thus derived, by a slight modification, from the properties of an elliptical trajectory.

It will be more accurate, in some cases, however, to take into account the convergency of the radiant lines of attraction. Every body, for instance, is at the equator, carried round by the rotation of the earth, with a celerity of 1525¾ feet each second. A ball therefore dropped from a high tower, instead of falling through a mere vertical line, must really describe a portion of an ellipse, of which the focus lies 6.8888, or very nearly 7 miles below its vertex. Let BG (fig. 53 *.) represent the tower, and O, being the centre of the earth, the ball dropped from B, having the celerity of rotation due to the radius OB, will describe the small portion BC of the ellipse, and strike the surface at C. Divide the arc BK into equal parts, and draw from O the elementary triangles. It is evident that the sector BOC will de-

note the time of descent from B to C, and OBK the time which the tower takes to gain the position KC. But the curved space GBC, considered as parabolic, being two-thirds of GBKC, is equal to a segment G*gk*C, bounded by a concentric arc at two-thirds of the altitude CK. At the instant when the ball alights at C, the top of the tower has only travelled through an arc equal to *gk*, and, consequently, the bottom G is retarded by an interval equal to two-thirds of the excess of BK above GC. Thus, suppose at the equator the tower to be 576 feet high, the ball would fall to the ground in 6″; but the summit would, during this lapse, describe an arc exceeding that passed over the base by $\frac{576 \times 6.2832}{14360} = 3.024$ inches, and two-thirds of this, or 2.016 inches, indicates the deviation of the falling body to the east of the tower. Experiments of this kind, which have been successfully made at Pisa, and at Hamburg, prove indisputably the rotation of the earth about her axis.

Again, suppose a ball were fired at A from a cannon directly upwards; though it reached the same absolute height, yet spending a longer time in the atmosphere, it would fall considerably behind the point of discharge. The vertical impulsion, combined with the transverse rotation of the earth, would really give the projectile an oblique direction along the tangent AD. While it therefore describes the portion

of an ellipse ABC, the point A travels through G and beyond C to x; the circular sector AOCG marking the time of rotation from A to C, and the elliptical sector AOCB the time of the flight in the curve till its descent at C. Whence $\frac{1}{2}$OG is to $\frac{2}{3}$ GB, or 3OG to 4GB, as AC to Cx, the space through which the point CA is carried beyond C during the description of the trajectory. Thus, suppose as before, that the ball were shot vertically at the equator, with a velocity of 2000 feet in a second; the altitude to which it would mount is 62,500 feet; and the whole time of ascent, and of the subsequent descent, if the earth were at rest, would have been 125″. Wherefore $3 \times 3962 \times 5280 : 4 \times 62500 :: AC : Cx$, that is, 125″ to .4977″, the interval during which the projectile is detained in the air, after the point A has come into the position C. It amounts to the 251,16th part of the whole time of the flight; and being multiplied by 1525$\frac{3}{4}$, the velocity at the equator, gives 759,36 feet for the distance xC, where the ball must drop to the westward of the mouth of the cannon, which in the meanwhile has travelled over 36$\frac{1}{4}$ miles. It would be easy to show that this retardation of the ball is proportional to the cube of its velocity. With an impulsion, therefore, of 1000 feet each second, the western deviation in falling would be only 95 feet.

Though rising apparently in a vertical line, these projectiles really take an oblique course; for, in the

first supposition, AC : CD : : 1525¾ : 2000, and the resulting velocity in the direction of the tangent AD is 2515½ feet, the angle of elevation DAG being 52° 39′ 40″. To make the ball strike exactly at the same spot, it would require the cannon to be pointed eastwards at the very small declination of 3′ 27″.

The ascent and descent of bodies subject to the law of gravitation, may be viewed as only an extreme case of the motions performed in an elliptical trajectory, the curve being compressed into its axis, of which the foci come to occupy the opposite ends. The time of falling from A to B, or of describing the circular arc AC, will now be measured by the area OAC, composed of the sector AIC, and of the triangle ICO, both having the altitude of the radius IC, but the arc AC and its sine BC for their bases. The time of descent through AB is hence proportional to the sum of AC and BC, or to the ordinate BD in the cycloid, as was already shown. Again, the velocity acquired at B must be as a perpendicular from A, the farther focus, to the ultimate position of the tangent CT. Now, from the property of the tangent to the circle or ellipse, IB : IA : : IA : IT, and by division, IB : IA : : IA–IB, or AB : IT–IA or AT, and thence OB : IA : : BT : AT : : BC : AR; again OB : OA : : BC : AL, and, therefore, by equality, IA : OA : : BC : AL, and IA being the half of OA, the velocity at B indicated by AR is

likewise the half of AL, or is always proportional to this line, as was formerly investigated.

It was found that, the attractive force being inversely as the square of the distance from the centre of motion, the velocity in a circle follows the inverse subduplicate ratio of the same radiant. The initial velocity at A being denoted by $\sqrt{\frac{1}{OA}}$, the velocity acquired at S will be proportional to $\sqrt{\frac{SQ}{OS}}.\sqrt{\frac{1}{OA}}$ or $\sqrt{\frac{SA}{OS}}.\sqrt{\frac{1}{OA}}$. If OA be assumed indefinitely great, and the point S taken near the centre O, the ratio of SA to OA will be that of equality, and consequently the velocity at S is expressed simply by $\sqrt{\frac{1}{OS}}$.

It may be calculated, that near the surface of the sun, a body would fall through $455\frac{1}{3}$ feet in a second. Hence, likewise, the celerity acquired in this short interval of time, by descending from the remotest regions of space, would be $390\frac{1}{4}$ miles. The same rapidity would have enabled a projectile to escape for ever from the attractive force of the sun. Hence had Light darted with less than the 500th part of its actual velocity, it would have been recalled in its journey, by the predominant power of that great luminary.

An atom or physical point, moving uniformly in a straight line, will equally advance to any plane, or recede from it. Let AB (fig. 55.) be the direction of the motion, and conceive a perpendicular AP to be let fall upon a plane passing through B; in a certain portion of time, the point A arrives at the position C, when its distance from the plane will be reduced from AP to CQ. In the plane of the triangle ABP, draw CD parallel to BP, and AD will mark the corresponding advance of the point. But the right-angled triangle APB is evidently given in species; and its hypotenuse AC being given, the base AD must likewise be given. At each interval of time, therefore, the point A will make a constant approach to the plane BP. Should this plane lie on the opposite side, the point A will, for the same reason, uniformly recede from it; or if occupying a position parallel to AB, the point will neither approach nor retire from it. The recession, indeed, may be viewed as merely a modified case of advance.

But the converse of this proposition likewise holds—or a point which advances equally to any plane has an uniform and rectilineal motion. To simplify the demonstration, we may suppose the point to glide along one plane, and to make regular advances towards two other planes which are right angles to this. Let AP (fig. 56.) be perpendicular to the plane BP, and BP another plane at right angles to

the plane of the triangle ABP, in which the perpendicular AP is let fall. The path of the point A lying in the plane of ABP, or ABP′, the advances in the position C to the planes BP and BP′ will be indicated by AD and AD′, as limited by the parallels CD and CD′. Join DD′ and PP′; since the segments AD and AD′ correspond to the same interval of time, they are obviously proportional to AP and AP′, and consequently DD′ is parallel to PP′. Wherefore the triangle DCD′ is equiangular to PBP′, and DD′ : DC : : PP′ : BP, or alternately DD′ : PP′ : : DC : BP; but, from the property of diverging and parallel lines, AD : AP : : DD′ : PP′, and hence AD : AP : : DC : BP, or alternately AD : DC : : AP : BP. The right-angled triangle ADC is therefore similar to APB, and the angle CAD equal to the given angle BAP; whence the locus of C is a given straight line, or the point A must describe a rectilineal course; but its motion is likewise uniform, since AC has a given ratio to AD.

Now, suppose any system of atoms or physical points to move each of them in a straight line, and with an uniform celerity, their several advances to any given plane will be equable, and consequently the sum of those divided by the number of particles, or the measure of the approach of the common centre of gravity must likewise be proportional to the time. But the same property belongs to every plane,

and therefore the centre of the system travels uniformly in a rectilineal path.

From this proposition, it follows, that if any group of atoms impressed with uniform rectilineal motions, advance as much collectively on one part to a given plane, as they recede from it on the other, their centre of gravity must remain at rest, and the system will maintain its equilibrium. Suppose these atoms, when thrown into motion, to preserve their mutual arrangement, they must each of them, in the momentary effort, describe minute portions of arcs of circles, about the common centre of gravity. But such elementary arcs may be viewed as coincident with their tangents, which evidently measure the velocities impressed. In the case of an equilibrium, therefore, the sum of these velocities, which are called *virtual*, estimated in one direction, must be equal to their aggregate in the opposite direction; or if the weight of each distinct group, or the number of atoms which it contains, were multiplied into its velocity of aberration reduced to a given direction, the different products collected together would extinguish each other. Such is the general principle in Dynamics relative to *virtual velocities*, which determine the condition of the equilibrium of a system from the minute alterations which would follow the disturbing of it.

The quiescence of the centre of gravity is essential

to the equilibrium of any system of atoms. But this quiescence may be viewed as either absolute rest, or as only the stationary limits of extreme evagation, or *when the centre of gravity occupies the highest or lowest position.* This maximum likewise forms another condition of the balance of a body, which is *tottering*, however, if the centre of gravity be above the point of support, and *stable* only when below it.

Suppose the motion of a system of atoms to be gradually stopped by the influence of a certain obstructing force, the same force which restrained the farther advance of the system, repeating its operation again through the same space, must generate an equal square of velocity in the opposite direction; for the area of the curve ACDE, (fig. 37.) which represents the square of the velocity extinguished in the approach from A to D, will also express the square of the velocity accumulated during the subsequent retreat from DA. The like effect must evidently take place, however short the space may be in which this change of impulsion is produced. In all the graduated mutations of any system of atoms, therefore, the sum of the squares of their velocities, estimated in any direction, continues still the same. This principle, which has a very extensive application, is usually termed *the Conservation of Living Forces.* This figurative expression had

better been avoided; but it implies merely the permanence of the amount of the products of the squares of the velocity of each cluster of physical points into their number or weight.

Impulsion is never instantaneous. It is created, but in a shorter or a longer interval of time, only from the accumulation of some inciting energy. This accumulation, which carries the body forward, has been denominated *Moving Force*. It will hence, in each accession, be the combined result of the duration and intensity of action. The moving force of a body is therefore measured, by multiplying its mass or number of atoms into its acquired velocity. This product is concisely termed the *momentum*. In any system of matter, the *momentum* is composed of the advances of all the different portions of atoms during a certain portion of time to a given plane, and is consequently the same as the *momentum* of the mass, if the whole had been condensed in the centre of gravity. Through all the changes, therefore, which ensue in the motion of bodies, from congress or mutual action, their *momenta*, estimated in the same direction, remain still unaltered.

The communication of motion, or the transfer of impulse, likewise requires a finite portion of time, so small, however, as often to elude the most attentive observation. Whether I strike or push the end

of a rod in the direction of its length, the remote extremity will not advance simultaneously. View it as a series of connected atoms: The first is impelled towards the second, till the shock is extinguished by the accumulating powers of repulsion; but, in this constrained position of proximity to the second atom, the first repels and causes it to make a similar approach towards the third. By successive partial oscillations, the original impulse will thus be transferred along the whole chain of atoms. This internal process may be rendered more familiar to the imagination, by examining the mode in which any stroke is propagated through a spiral or helical spring. If I give a twitch, near the end of a very long cord stretched tight, the jerk, forming a slight sinuosity, will visibly dart along the whole line. In the ordinary cases of impact, motion becomes generated or transferred in a portion of time that is scarcely at all discernible. The very same velocity is produced, if the pressure be augmented just in proportion as the duration of action is diminished. But, even in the most extreme case, the influence of time must never be overlooked.

Bodies, with regard to their collision, are commonly divided into *elastic* and *non-elastic*. This distinction, however, is not well founded; for though some of them, approaching by their softness to the nature of fluids, are nearly indifferent to the change

of figure, yet they all recover from any compression of volume with unabated vigour. But if such bodies be struck with immense rapidity, they will even resist a change of figure. Thus, a stone thrown obliquely and with great velocity, will be made to rebound from the surface of water, just as if it had impinged against a sheet of ice; because the shock is then confined to a narrow spot, where the repulsive force, occasioned by the compression of the proximate particles of the fluid, heaves back the stone, before they had time to retire and compel the particles below them, by a diffusive motion, to give room for the entrance of that missile. Under such circumstances, therefore, every substance may display the properties of perfect elasticity.

It is hence more philosophical to distinguish impinging bodies into *Coalescent* and *Resilient*. These we shall treat separately.

I. Collision of Coalescent Bodies.—Let the ball A (fig. 60.) advance with the velocity AO in the same direction as B, which moves with only the velocity BO; it will evidently overtake the ball B at O, and, having *coalesced* or united with this, they will both travel forward in the same rectilineal path. Divide AB in G, so that AG be to BG as the weight of B is to the weight of A, and the point G will be the position of the centre of gravity, when they began to move. This centre must consequently have advanced from G to O at the moment of their collision;

but its uniform motion is not affected by the mutual action of the balls. Wherefore their combined mass will, after impact, proceed with the common velocity GO.

Since, by construction, A : B : : BG : AG, it is obvious that A.AO : B.BO : : BG.AO : AG.BO; whence the *momenta* of the balls A and B will be expressed by BG.AO, and AG.BO. But BG.AO= BG (GO+AG), and AG.BO=AG (GO—BG); these two *momenta* are therefore equal to (BG+AG) GO, the *momentum* of the compound after the collision has taken place; and thus an essential principle is maintained. Next, suppose the balls are impelled in opposite directions. Make (fig. 61.) AO to BO as the velocity of A is to the velocity of B, and they will evidently meet in the point O. Let G be their centre of gravity, which must lie either between A and O, as in fig. 61., or between B and O, as in fig. 62. After collision, therefore, the two balls, coalescing in a single mass, will proceed in the direction and with the velocity GO.

In this case, likewise, the collective *momenta* of the balls will continue the same after the shock. For since A : B : : BG : AG, the products A.AO and B.BO may be expressed by BG.AO and AG.BO; wherefore the *momenta*, estimated in the same direction, will be BG.AO—AG.BO = BG (AG±GO)—AG (BG±GO) = (AG+BG) GO = AB.GO, or the *momentum* of the joint mass

after collision. Its motion also will be progressive or retrograde, according as the centre of gravity lies between A and O or B and O, that is, according as the *momentum* of the ball A or that of the ball B predominates.

II. Resilient Bodies.—Let (fig. 63.) a glass or ivory ball A, advancing with the velocity AO, strike a similar ball B, which moves in the same direction, but with the velocity BO. The collision will evidently take place at the point O. The first effect of this shock is to produce a momentary union of the two balls, the impinging surfaces in both being partially flattened. The next act is to recover their globular figure by an elastic or *resilient* effort, which, if exerted, in an equal instant of time, would exactly redouble the change impressed; for, as in fig. 87., whatever velocity is gained or lost by either ball during their congress, must, by the reaction of the same repulsive forces, be repeated again. If the energy of recoil were exactly equal to the power of compression, the elasticity would be perfect. Let it be assumed as such in the collision now under review.

At the moment, therefore, when the ivory balls come into the closest union, they take the common velocity GO, (fig. 63.) and consequently A loses the velocity AG, while B gains the velocity GB. But, in the subsequent act of recovering their figure, by a mutual exertion of elastic force, the loss of A's

velocity and the gain of B's velocity, are each doubled. Hence, after their separation has been effected, the velocity of A is AO—2AG, and the velocity of B is BO+2GB. Make GP = GO; and, it will follow, that PA = PG—AG = GO—AG = AO—2AG, and PB = PG+GB = GO+GB = BO + 2GB; whence PA and PB will express the resulting velocities of A and B. It also appears that PG expresses the velocity of the centre of gravity, being equal to GO, the velocity which it had before collision.

Next, let the balls A and B, whose centre of gravity is G, meet in opposite directions at O (fig. 64.); GP being taken equal to GO; the velocity of A will, in the act of approach, have its velocity diminished from AO to AG, and by the subsequent recoil reduced to AP, while the velocity of B is augmented to BC and next to BP. In fig. 65., a similar, though modified, result takes place.

The case of perfectly resilient bodies admits the application of the principle of the *Conservation of Living Forces*. Thus, (in fig. 63, 64, and 65.) the weights of the balls A and B, multiplied into the squares of their velocities before and after collision, will form the same amount. In other words, $AO^2.BG + BO^2.AG = PA^2.BG + PB^2.AG$. To prove this, it will be sufficient to show that $AO^2.BG$ exceeds $PA^2.BG$, as much as $BO^2.AG$ is less than $PB^2.AG$. Now (Geom. II. 17.) $AO^2—PA^2 = (AO+PA)(AO—PA) = PO.2AG$; and, for the

same reason, it follows that $PB^2-BO^2=(PB+BO)(PB-BO)=PO.2BG$. Wherefore $(AO^2-PA^2)BG=PO.2AG.BG=(PB^2-BO^2)AG$.

If the balls A and B be of equal weights, they will interchange their velocities; for G the centre of gravity occupying now the middle point, it is evident that PA, the velocity of A after collision, is equal to BO, the velocity which B had previously; and likewise that PB, the velocity acquired by B, is equal to AO, the velocity which A brought into action. This exchange of mutual condition takes place equally, whether the balls move in the same or in opposite directions.

Let the ivory ball A (fig. 66.) strike another B at rest. The point O must evidently coincide with B, and G being the centre of gravity, make $GP=BG$. The velocity of A after the shock will be denoted by PA, and that of B by PB or 2CB. If the balls have equal weights, (fig. 67.) the point P will obviously fall on A, and consequently the ball A will stop and transfer all its motion to B. Hence, having placed any number of ivory balls in mutual contact along a straight line or horizontal groove, if the first be struck in the same direction, the last one only will fly off, leaving all the rest apparently unmoved. The impulse here is conveyed along the whole chain, neutralizing, in succession, each impinging ball, till it seizes and transports the extreme one.

Suppose B to be a firm obstacle (fig. 68.), or a

mass of indefinite extent at rest, all the points O, B, G, and P will then coincide, and therefore the ball A will be made to rebound in the opposite direction with a velocity BA equal to that with which it impinged. This property has a very general influence in the operations of nature.

The interposition of a third ball C between two unequal balls A and B may augment the velocity communicated. Thus, let the ball A of nine ounces weight, and velocity of one foot a second, strike directly a ball B of an ounce. The velocity which B will receive is $\frac{2.9}{1+9}=\frac{18}{10}=1\frac{4}{5}$. But suppose a ball of four ounces were interposed; the velocity which B would then acquire by a double transfer is $\frac{2.9}{4+9}\cdot\frac{2.4}{1+4}=\frac{18}{13}\cdot\frac{8}{5}=2\frac{14}{65}$. Let the intervening ball be now only two ounces, and B will, through this medium, obtain a velocity equal to $\frac{2.9}{2+9}\cdot\frac{2.2}{1+2}=\frac{18}{11}\cdot\frac{4}{3}=2\frac{6}{33}$.

The intervention of the third ball thus augments very considerably the impulse delivered to B. The effects, too, are evidently not quite the same in both. Assume, therefore, an intermediate ball of three ounces. On this supposition, the velocity acquired by B would be $\frac{2.9}{3+9}\cdot\frac{2.3}{1+3}=\frac{18}{12}\cdot\frac{6}{4}=4\frac{1}{4}$, which is evi-

dently somewhat greater than either of the former results. The intervening ball C, being now three ounces, is evidently a mean proportional between nine and one, the weights of the balls A and B. It will be found in general, that *a maximum velocity is communicated, by interposing a ball which is a mean proportional to both extremes.*

To investigate this curious property, let (fig. 69.) AB and BC represent the weights of the perfectly elastic balls A and B, and the perpendicular BD, the weight of the inserted ball C. The velocity communicated to B will be expressed by $\frac{2AB}{AB+BD} \cdot \frac{2BD}{BD+BC}$, and its reciprocal is consequently proportional to $\frac{(AB+BD)(BD+BC)}{AB.BD}$; but this expression may be expanded into $\frac{AB.BC+AC.BD+BD^2}{AB.BD}$. About the triangle ADC describe a circle, and produce DB to meet the circumference in E, and the velocity of the ball B becomes $\frac{BE.BD+AC.BD+BD^2}{AB.BD}=\frac{AC+DE}{AB}$, which must therefore be a *minimum.* But AC and AB are both of them constant quantities, and hence the chord DE must be a *minimum.* Bisect AC in O, and the distance BO from the centre of the circle is consequently given. The circle itself must therefore be the least possible: Now, of all the circles

which can pass through the points A and C, the smallest is obviously that which has AC for its diameter. Hence ADC, being contained in a semicircle, is a right angle, and the perpendicular BD, or the weight of the intermediate ball, is (Geom. VI. 15.) a mean proportional between AB and BC, the weights of the balls A and B.

The velocity communicated to the extreme ball is likewise increased, by multiplying the interposed balls. Thus, a ball of 64 ounces with a velocity of one foot each second, striking another ball of only 1 ounce, will impress the velocity $1\frac{63}{65}$. If a ball of 8 ounces be interposed, the velocity will become $3\frac{13}{81}$; if two balls of 16 and 4 ounces be inserted, the velocity will amount to $4\frac{12}{125}$; but, if four intermediate balls be placed in the continued proportion of 32, 16, 8, 4, 2 ounces, this velocity will rise to $5\frac{451}{729}$.

Suppose a ball of 32 ounces, and with a velocity of one foot each second, to strike against a series of balls of 16, 8, 4, 2, and 1 ounces; the velocity communicated to the last one will be $4\frac{52}{243}$. But if the order of these balls were reversed, and the smallest impinged with the same *momentum*, or a velocity of 32 feet; the velocity transferred to the largest ball will be $\frac{32}{243}.32 = 4\frac{52}{243}$, the same as before. With the same original impulsion, therefore, a ball is, by this arrangement, made to move as fast as another, only the thirty-second part of its weight.

This curious result might appear at first to contradict the principle, that, in every communication of impulse, the same *momentum* is maintained. But when the collision commences with the smallest ball, the rest of the range, except the last one, are thrown partially backwards, and consequently their several *momenta* are to be deducted from the final *momentum* of the projected ball. On the other hand, again, when the collision is propagated through a descending series of balls, they all advance with certain velocities after the shock has been transmitted through them. The *momenta* thus acquired and retained should consequently be joined to the *momentum* of the final ball, in estimating the full aggregate effect. Computed in this way, the results will be found exactly to correspond.

In regard, however, to the impulsion communicated through the ascending series to the extreme ball, an obvious and important advantage is gained. The final *momentum*, in the example which we have chosen, is, without any sensible expense of time, more than quadrupled. Were this idea pursued, it might, in various instances, suggest the means of saving labour, and improving the performance of machinery.

Suppose a ball of perfect elasticity to impinge obliquely against another at rest. If the ball A, (fig. 70.) moving in the direction CA, strike B at the

point D, the plane of collision will evidently be perpendicular to the line BDA passing through both centres. Parallel to that plane, draw CE meeting the extension of BA in E. The oblique velocity CA may be decomposed into CE, parallel to the tangent at D, and EA perpendicular to it. But CE has no concern in the collision, while EA is directly exerted against the ball B. Divide EA in G, so that EG shall be to AG, as the weight of the ball A is to the weight of the ball B, and make GF equal to EG. Since AG is the velocity lost by the ball A in the first act of approach, 2AG—AE=AF must be the velocity of its recoil. But the velocity CE remaining unaltered, draw the perpendicular FH equal to it, and the oblique line AH will exhibit the velocity and direction of the resilient ball A. The larger ball B which is struck will move with the force and direction EF.

When the two balls are equal, the point G will bisect EA, and the point F coincide with A. The velocity EA will therefore be transferred from the ball A to the ball B, which will move in the perpendicular direction DB, while A, retaining the velocity CE, will proceed in the parallel direction AI.

If the ball B be considered as indefinitely large, the points G and F will coincide with E; this ball will continue at rest, while the ball A will be reflected in the line AK, making an angle EAK equal to

the incident angle EAC. Such is the effect of any firm obstacle extended in the plane of collision.

The propositions respecting the impact of Coalescent and Resilient balls are easily verified by experiment. If they be suspended at the same height by parallel threads, they will acquire velocities very nearly proportional to the arcs which they describe, and, being dropped at the same instant, they must always impinge at the lowest point. But were they made to oscillate in the arcs of a cycloid, their motions would be absolutely isochronous, and their acquired velocities exactly as the spaces passed over.

In treating of the impact of elastic bodies, I have confined myself to the case of *perfect* resiliency, where the partial change of figure occasioned by the shock is recovered with the same energy and rapidity. But such accurate and complete development of force never occurs, except in the reflection of light and the expansion of air. In other instances, a deficiency of reaction lessens the velocity of recoil. Thus, a marble ball dropped from the height of 16 inches upon a marble floor, will perhaps rebound only 9 inches. On this supposition, the resilient energy being in the subduplicate ratio of the space, is only three-fourths of the force of impact. Were the proportion constant in the same bodies, those modified effects could easily be computed. But the reaction is rendered more complete, by the great ra-

pidity of the shock. Had the same ball fallen from the height of 16 feet, it might perhaps have rebounded 12 feet, instead of 9. Such perfection of resiliency is hence the limit to which all substances tend, as their motions approach to extreme celerity.

If a system of atoms, turning about a fixed axis, impinge at a given point against a firm obstacle, the shock will either be completely extinguished in that point, or partly spent upon the axis of motion. When this impulsion is only partially checked by the obstacle, a jarring stroke will be communicated to the axis; but when the whole of it is absorbed in reaction, the axis will suffer no tremor. The point of impact in which this balance of moving forces occurs, is therefore called the *Centre of Percussion.*

To simplify the investigation, let it be restricted to the case of a compound pendulum or inflexible rod, with the balls B, C and D attached to it, and swinging from the point A (fig. 71.). If P, the centre of percussion, be stopped, those balls must expend their *momenta* against the lever AD, to which P may be considered as a *fulcrum.* The velocities of impulsion at B, C and D are evidently as the several distances BA, CH and DA from the point of suspension; but these velocities must all be extinguished by the obstacle in the same moment of time; and consequently during the short lapse preceding the extinction, they will act in the same proportion,

as inciting forces or pressures. The ball B will hence press against the lever for a moment with a force B.AB, the ball C will press with a force C.AC, and the ball D, during the same instant, will press with the force D.AD. Now, from the property of the lever, the forces at B and C will cause the strains $B.AB.\frac{BP}{AP}$, and $C.AC.\frac{CP}{AP}$ at the point of suspension A, while the force at P, on the other side of the fulcrum, will exert an opposite strain $D.AD.\frac{DP}{AP}$. Wherefore, since an equilibrium of impression must obtain, $B.AB.\frac{BP}{AP} + C.AC.\frac{CP}{AP} = D.DA.\frac{DP}{AP}$, and thence $B.AB.BP + C.AC.CP = D.AD.DP$. But such was likewise the condition that determined the Centre of Oscillation, which in this case, therefore, must coincide with the Centre of Percussion. Both these centres are indeed the same, unless the dimensions of the compound body should extend considerably in width in proportion to its height. In all the ordinary cases, they may be assumed as identical.

If the compound pendulum ABCD, (fig. 71.) in the state of rest, were struck perpendicularly to AB at the centre of percussion P, it is evident that the whole would be made to turn freely about A, the point of suspension. If BCD were conceived to be

merely a rigid system of clustered atoms, the direct impact at P would cause it to revolve about A, without jarring against that point. The first effect of the shock, therefore, would be the same, whether the system were fixed at A or not. The several balls B, C and D would begin by describing the minute arcs *Bb*, *Cc*, *Dd*, and then shoot forwards with velocities proportional to those spaces. The point A, about which the system thus begins its motion, is called the *Centre of Conversion* or *Spontaneous Rotation*. Let G*g* be the initial arc described by the centre of gravity, and draw A*'b'c'gd'* parallel to AD; the velocity B*b* is composed of the direct velocity B*b'* or G*g* and a retrograde velocity *b'b*; the velocity C*c* likewise consists of the direct velocity C*c'* or G*g* and the retrograde velocity *c'c*; but the velocity D*d* is compounded of the direct velocity D*d'* or G*g* and of the direct velocity *d'd*. While the centre of gravity or the whole system is carried forwards with the common velocity G*g*, the balls B and C retreat with the velocities *b'b* and *c'c*, and the ball D advances with the velocity *d'd*. These several velocities are thus proportional to the distances from the point G or *g*, and consequently proportional to the centrifugal forces occasioned by the balls continuing after impact to revolve in the same time about that centre. But, since B.BG + C.CG = D.DG, these divellent forces are exerted equally on both sides, and can have no influence whatever in

distraining the rectilineal advance of the centre of gravity, which must therefore pursue an undeviating and rectilineal course. Every other point of the system will describe a species of cycloid, either *curtate* or *prolate*, (See Geometry of Curve Lines, p. 357.)

From the equilibrium of the opposite *momenta* of the balls, or B.BG+C.CG=D(A).DG, it likewise follows, that the progressive motion of the whole system, or of the centre of gravity, is neither augmented nor diminished at all by their rotation. The rectilineal and uniform advance of this centre, and the constant circulation maintained around it, are therefore two distinct and independent movements.

Hence, *while a rigid system of atoms performs a single revolution, its centre of gravity will travel over a space equal to the circumference of a circle described about the centre of spontaneous rotation.* Thus, if the narrow cylinder AB (fig. 72.) were struck perpendicularly at the point P, situate at two-thirds of the whole length from the end A, the cylinder would begin to move about that extremity, and consequently the centre of gravity, or the middle point G, must advanee in the direction of the stroke over a line that is equal to 3.1416 times the length AB, during the time in which the cylinder performs a complete revolution. But if the point of impact be taken nearer G, the corresponding centre of

rotation will be thrown beyond A. Suppose the cylinder to be struck at p, the distance Ap, being three-fifths of AB; then $GV = \frac{AB^2}{12Gp} = \frac{AB}{1.2} = \frac{5AB}{6}$, or V, the centre of rotation, lies at one-third of the length beyond A, and consequently G will advance $\frac{5}{6} \times 6.2832$ or 5.236 times AB, while the cylinder performs a revolution. But if the stroke were aimed exactly at the centre of gravity, the point of conversion V would be thrown to an indefinite distance, and the cylinder would hence be carried directly forward, without any sort of rotation, merely in a parallel position.

Again, suppose two equal balls A and B (fig. 73.) to be connected by an inflexible straight line. If this pendulum were suspended from A, the centre of oscillation would evidently occur in B, and therefore the rectangle under the distances of the point of the suspension and of the centre of oscillation from the middle point G must, in every case, be equal to the square of GB. Hence, if the line AB were struck at P, which bisects GB, the centre of rotation would be transferred to V, at a distance AV equal to AG. The centre of gravity G would, therefore, at each revolution, describe a rectilineal path, equal to the circumference of a circle whose diameter is AB. Let AP be only two-thirds of AB, and the point of conversion V will be thrown to a distance Gv, equal to the triple of AG; the cen-

tre of gravity would hence travel over 4.7124 times AB, during each circumvolution of the connected balls.

If, conformably to that simplicity which pervades all the works of Nature, the planets derived their motions both of rotation and revolution at once from the impression of the same original force, our earth must have received the initial impact at the distance of the 165,6th part of the equatorial radius from the axis, or very nearly 24 miles. The centre of conversion would be distant 66¼ radii, a little beyond the orbit of the moon. In like manner, it may be computed, that Jupiter had received the stroke which impressed his very disproportionate diurnal and annual revolutions, at the .351th part of his radius, or 14040 miles from the centre.

If a rigid system of atoms, impelled by a progressive and revolving motion, be stopped at the centre of gravity, its rotation about that point will evidently continue the same as before. The shock being extinguished by the resistance of the obstacle, the circulation of the system is maintained, without any effort exerted against the axis, which may be henceforth either fixed or loose. But if the centre of rotation remain fixed, the circumvolution of the system can also be arrested at any point in the radius of description. This absolute cessation of motion must consequently be transfused through the whole mass,

during the same very small portion of time. Let the radius ABCD (fig. 74.) advance into the proximate position *Abcd*, while the revolving impulsion is expended. The atoms at B, C, and D therefore pass over the minute spaces B*b*, C*c*, and D*d*, with velocities proportional to these, or to the distances AB, AC, and AD, from the centre of rotation. Consequently, the retarding forces that extinguish the *momenta*, and which, multiplied into the spaces, are as the squares of the velocities, must be proportional to AB, AC, and AD. The pressures exerted during impact by the groups or balls at B, C, and D, are hence as B.AB, C.AC, and D.AD; but, from the property of the lever, the powers exerted by those pressures, being augmented in the ratio of the distances AB, AC, and AD, are proportional to $B.AB^2$, $C.AC^2$, and $D.AD^2$. There is consequently a certain point R in the radius, at which, if all the balls B, C, and D were conceived to be collected, the shock of circumvolution or angular motion would be the same. This point has been therefore called the *Centre of Gyration*. From its condition, $(B+C+D)AR^2=B.AB^2+C.AC^2+D.AD^2$, and therefore $AR=\sqrt{\left(\frac{B.AB^2+C.AC^2+D.AD^2}{B+C+D}\right)}$.

The centre of gyration, however, is not strictly confined to a mere point, but includes the whole circumference of a circle described at the assigned distance about the centre of rotation. It has been al-

ready shown, that G being the centre of gravity, the distance of the centre of oscillation or percussion is $\frac{B.AB^2+C.AC^2+D.AD^2}{(B+C+D).AG}$, and thus $AR^2=AG.AO$, or $AR=\surd(AG.AO.)$ Consequently, *the distance of the centre of gyration from the point of suspension, is a mean proportional between the distances of the centre of gravity and of oscillation.*

Hence, the distance of the centre of gyration of a straight line revolving about one extremity, is a mean proportional between the whole length and its third part; and hence also the distance of the centre of gyration of a circle, or of a circular sector, is a mean proportional between the radius and its half. If two equal balls B and C (fig. 48. and 49.) be attached to an inflexible line, either on the same or opposite sides of A the centre of rotation, the distance of the centre of gyration will in either case be $\surd\left(\frac{AB^2+.AC^2}{2}\right)=\surd(AG.AO)$. In a sphere, the distance of the centre of gyration from that of motion is a mean proportional between the radius and two-fifths of it; but, in a hollow sphere, this distance is a mean proportional between the radius and two-thirds of the radius.

The power of turning any rigid system of atoms is thus proportional to the square of its distances from the axis of motion. This power may therefore be termed the *Momentum of Rotation*, and its total

amount in any revolving body is expressed, by the mass multiplied into the square of the distance of the centre of gyration.

The descent through a declivity is performed either by sliding or rolling. In the former case, the body maintains the same position, and is immediately urged forwards by that part of its weight which comes into action. But when the body rolls down any path, the accelerating force has not only to incite the progressive advance of the centre of gravity, but to create a corresponding rotation in the whole mass. By this distribution of force, the action exerted on the centre of gravity becomes diminished, and its velocity, generated in a given space, is hence reduced in a subduplicate ratio. Thus, if a thin hollow cylinder were set to roll along an inclined plane, the matter being all collected to the circumference, the rotatory impulsion must be equal to the progressive *momentum* of the centre; wherefore, it will descend in a given time only through half the space over which it would slide lengthwise. On the other hand, if the matter of the cylinder had been concentrated in the axis which is fixed to two very thin circular ends, no sensible portion of the inciting force would be lost in producing rotation, and a body of such a spindle shape would in equal times travel twice as far as the former drum.

These different effects are rendered very conspi-

cuous, by having two cylinders of light wood, but loaded equally with lead, the one near the centre, and the other about the circumference, each of them being terminated by narrow protuberant rings or beads. The cylinder that has its mass approximated to the axis of motion will be found to roll down any inclined plane almost as fast as if it had merely slid, and to acquire, in the same time, nearly double the velocity which is attained by the cylinder that had its weight thrown to the circumference.

In a homogeneous cylinder, the distance of the centre of gyration being a mean proportional between the radius and its half, the inciting force will be divided into three shares, two of these employed in generating the progressive motion, and the other consumed in producing that of revolution. It therefore descends with only two-thirds of the power of gravity. A hollow cylinder has its centre of gyration situate in the circumference, and consequently one half of the inciting force urges the progressive motion, while the other half merely supports the rotation. In a solid sphere, the force of acceleration being distinguished into seven shares, five of these are exerted in propelling the centre, while the remaining two are spent in maintaining the conjoined rotation; its descent is consequently urged by five-sevenths of the power of gravity. But in a hollow sphere or very thin spherical shell, the inciting force is allotted in the ratio of three to five, so that

the propelling energy is only three-fifths of gravitation.

Since the square of the velocity acquired, in descending through the same path, is proportional to the accelerating force, the time of descent must evidently follow its inverse subduplicate ratio. Hence the time of a semi-vibration of a pendulum in a cycloidal and circular arc is to the time in which a solid cylinder would descend through the same arc by rolling, as $\sqrt{2}$ to $\sqrt{3}$, or very nearly, as 9 to 11; but this same vibration would be to the time spent in rolling of a hollow cylinder, as 1 to $\sqrt{2}$, or nearly as 5 to 7. Again, the semi-vibration of a pendulum is to the time of rolling over the same arc by a solid sphere as $\sqrt{5}$ to $\sqrt{7}$, or nearly, as 11 to 13; but to the time in which a hollow sphere would roll through it, as $\sqrt{3}$ to $\sqrt{5}$, or as 7 to 9.

These proportions are reversed when the rolling bodies are by their acquired celerities again carried up an inclined plane. The hollow ball or cylinder, which in their descent had been retarded by the large share of their inciting force consumed in generating the rotation, are now, by this accumulated *momentum*, borne with greater rapidity in rising over a similar plane. If they roll in a cycloid or in a small arc of a circle, their motions will be reciprocated in the same periods as those of sliding bodies, but very unequally divided. The hollow spheres and cylinders will in their descent always consume

a longer interval of time than is required for their subsequent ascent. The contrary would take place if the matter were collected near the centre of these bodies.

If a body be struck in a direction which passes through its centre of gravity, it will advance in that line with an uniform celerity, maintaining invariably the same parallel position. But if the scope of impulsion should incline on either side of the centre of gravity, the body will, besides its progressive and rectilineal motion, acquire about that point a coexistent and independent rotation. To illustrate farther this important property, let the body ACBD, (fig. 75.) having the perpendicular planes AB and CD crossing in the centre of gravity G, be struck at L with a force expressed by LO, the double of LN, and at right angles to AGB. Join LG, and produce till LG=GP, and let fall the perpendiculars LM, PR and PQ. The force LN, which is half of the original impulse, may be decomposed into the forces LG and GN oi DL. But the force LG, viewed as acting upon any point P in that direction, is equivalent to GP, which again may be decomposed into the forces QP and GQ or RP. But QP is evidently equal to LN or NO, while GQ is equal to DL or GN. The equal forces QP and NO, thus exerted at equal distances on both sides of the centre of gravity, must produce an equilibrium, and urge forward that point by their joint action. Wherefore the centre of gravity is borne along in

the same or parallel direction DC, with the entire force of impulsion LO. But there still remain the forces DL and RP, which may be conceived to act at the equidistant points M and R at right angles to the diameter MR, and hence generate a revolution about the centre G, with a force represented by 2ML or QN. This rotatory motion, since the efforts on opposite sides are equal, cannot impair, or in any way disturb the progressive momentum.

It is evident that, every part of the plane performing its revolution in the same time, the centrifugal force exerted by the point P will have its intensity and direction denoted by GP. But this oblique force may be resolved into the forces PQ and PR, at right angles to the cross diameters. Now, from the property of the centre of gravity, the perpendicular distances PQ, of every point on both sides of the line AGB, balance or extinguish each other; and, for the same reason, the perpendicular distances PR on both sides of CGD are exactly balanced. Consequently the centrifugal forces GP of all the points about the centre of gravity, completely extinguish each other at the axis, and leave it free and undisturbed.

This axis would evidently pass through the centres of gravity of every similar parallel plane which has the same position. In each of these, an equilibrium of centrifugal action must likewise obtain. Wherefore, the axis of a symmetrical body, which

may be conceived to be composed of such planes, will continue permanent.

An irregular body may commence a rotation about any axis which passes through its centre of gravity, because the aggregate of all the centrifugal forces, estimated in the direction of perpendiculars to a plane touching the axis, must produce a mutual extinction. But though this collective action would, if exerted at a single point, maintain a perfect balance; yet being unequally distributed over the axis, it may, from the principle of the lever, predominate upon one side of the primary more than upon another, and therefore continue to bend the axis with greater or less effect in some particular direction. Thus, let the body be cut by a plane through any point H (fig. 76*) above or below the common centre of gravity. Suppose g to be the centre of the particular section, and through H, where the axis pierces it, draw the perpendicular diameters $a\mathrm{H}gb$ and $c\mathrm{H}d$. The centrifugal force of any point P exerted against the axis at H, being expressed by Hp, is decomposed into the forces pq and pr, at right angles to Hb and Hc; but from the property of the centre of gravity, the sum of all the pq's on both sides of $a\mathrm{H}gb$ is nothing, and so is likewise the sum of all the pi's perpendicular to the cross diameter gi. Wherefore, there is an excess ir of the lateral force pr, which for the whole plane will amount to the num-

ber of points multiplied into ir or gH. This preponderance will consequently be exerted at H, in bending the axis in a direction parallel to aHgb, and with an efficacy proportional to the distance Hg from the centre of gravity. Let the parallel sections, therefore, be multiplied, corresponding to different points along the axis; the several excesses of centrifugal action being thus variously combined, and their effects estimated by the application of the principle of the lever, will give a certain resulting impression, which tends to push the centre or axis about the fulcrum, (fig. 77.) with the force and in the direction OZ.

Conceive such parallel sections at equal intervals, multiplied indefinitely in various directions, and at the distances 1, 2, 3, 4, 5, &c. from the centre O; and let their corresponding efforts to derange the axis, when all reduced to the same vertical plane, be denoted by a, b, c, d, e, *&c.* It is evident, that the aggregate power exerted to bend the axis in this plane, will be expressed by $a + 2b + 3c + 4d + 5e +$ &c. From the property of the centre of gravity, all the different efforts about the axis must counterbalance each other, or $a+b+c+d+e$, &c. $=o$. But, that the axis should remain undisturbed, the action of those powers must produce a mutual balance, or $a+2b=3c+4d+5e+$ &c. $=o$. These are hence the only two equations for determining the innumerable quantities a, b, c, d, e, &c., which will therefore ad-

mit of values indefinitely varied, and subject only to such very general conditions. Suppose those values to be represented by a countless multitude of curves applied to the axis; their mutual intersections will form an infinite variety of dispersed points. Among such immense diversity, therefore, it would be possible to trace a succession of them in a straight line. This marks the direction of a permanent axis, which unites the conditions implied in the two fundamental equations.

It may therefore be inferred, that every rigid body, however irregular in its form, has at least one axis about which the cumulative centrifugal action would be exactly balanced. This axis is called the *Principal Axis*; but, depending on it, there are always two other axes which have the same denomination. For let OB and OC (fig. 78.) be drawn through the centre of gravity perpendicular to the principal axis OA and to each other. Let the plane AOB cut the section, which passes through H in HL; from any point P in that section draw PL at right angles to HL, and LK parallel to AO, and join PK. The centrifugal force of the point P about the axis OB being proportional to its distance PK, this force is reduced to KL acting in the direction of OA, and therefore bending OB with an energy $KL \times OK$. The combined efforts to change the position of this new axis will consequently be expressed by $\int, OK \times KL$, or $\int, HL \times OH$; but the latter expres-

sion was shown to produce accumulatively a mutual balance about the axis OA, and hence the disturbing impressions exerted on OB will extinguish each other, and leave it perfectly free in its rotation. The same reasoning will apply to the third axis OC, which is at right angles both to OA and OB. Every body, however irregular, has consequently those three Principal Axes. In the case of a symmetrical body generated by the revolution of a plane, the primary axis evidently is an individual line, but the other two principal axes are indeterminate, being any perpendicular diameters of the circle of the equator.

The momentum of rotation about the primary axis is always a *minimum*. For this axis, in a symmetrical body, must pass through the centre of gravity of each vertical plane, and consequently, by Prop. 21. Book III. of Geometrical Analysis, the sum of the squares of all the lines drawn from that centre to every particle of each plane, which constitutes the *momentum of rotation*, is less than the sum of the squares of their distances from any other point. In the case of irregular bodies, the primary axis will evidently cut the successive planes as near as possible to their centres of gravity, and therefore the aggregate squares of the distances of all the physical points will yet be a *minimum*. It can likewise be proved, that about one of the remaining principal axes the momentum of rotation is a *maximum*.

V. MECHANICS,

which consists in the application of the Principles of Dynamics to the construction and composition of Machinery.

The various wants of society are supplied by the operations of human industry on the surface of the globe. It was necessary to divide and re-unite bodies, and to transport portions of matter from one place to another. But those labours are abridged, and rendered greatly more productive, by the proper exercise of skill. The incessant efforts to augment our powers by the aid of tools, called forth the earliest germs of ingenuity and invention. Without the simpler implements of art, mankind could never have emerged from the savage state; and to the prodigious improvement and extension of machinery in modern times, we are indebted for all the comforts, enjoyments, and delicacies of highly civilized life.

The elements of Machines may be ranked under two distinct classes—those of a general, and those of a particular nature. To the former belong what I should call the *Concentrator of Force*, and the *Engine of Oblique Action*, which, when composed of connected cords, has been named the *Funicular*

Machine; the latter include the five ordinary mechanical powers—the *Lever*, the *Wheel* and *Axle*, the *Inclined Plane*, the *Screw*, the *Wedge*, and the *Pulley*. We shall consider these several instruments in their order.

1. *The Concentrator of Force.*—This engine exhibits, in a very striking manner, the accumulation and transfer of impulsion among bodies, and may therefore be regarded as, next to Atwood's ingenious machine, a most important addition to our stock of illustrative philosophical apparatus. It not only sheds a clear light on some abstruse parts of mechanical theory; but may with advantage be directed, in a variety of important cases, to the practice of the arts.

This Concentrator consists of a ponderous wheel, composed of a thin circle of iron, loaded at the circumference with a broad swelling ring of lead, and fixed to a strong steel axle, to which is likewise attached three or more barrels or short cylinders, of different diameters, the smallest formed of brass, and divided in two parts that are capable of locking together at pleasure. The axle, placed in a horizontal position, runs upon gudgeons on the top of a high and solid frame; and the machine may be set in motion, either by turning a winch, or more commonly by the descent of a small weight fastened to a silk line, which passes over a pulley, and is lapped

round one of the barrels. Fig. 79. represents the only model which I have yet used, the frame being about 5 feet high, the wheel has 18 pounds weight, and is 17 inches in diameter, while the diameters of the successive barrels are only 6, 4, and 2 inches. The principal application of this engine is to raise from its platform any great weight. If 1 pound, for instance, in descending through 30 feet, gradually communicate its impression to the wheel, and the instant it reaches the ground, the detached part of the brass barrel should lock and catch hold of the loop of a cord holding a half-hundred weight or 56 pounds, this mass will be almost immediately lifted up near 6 inches, and there suspended. But what is remarkable, and appears at first sight paradoxical, the effect is precisely the same, about whatever barrel the line be wound. The result, however, is quite altered, when different descending weights are used, the elevation produced being always proportional to them. Thus, the descent of 2 and of 4 pounds through 30 feet, will respectively raise 56 pounds to nearly 1 and 2 feet.

It is not difficult to explain generally these effects. Let the descending power be very small compared with the weight of the ponderous wheel, and suppose its action at first to be exerted at the rim. This rim, in which is condensed the entire mass, will now describe a space equal to the measure of descent, and the whole power may be considered as inciting its

revolution. Consequently the square of the velocity acquired by the wheel must be proportional to the descending power multiplied into the space through which it falls. When the accelerating force acts on a barrel smaller than the wheel, the energy exerted at the rim is proportionally diminished, but the space described by it is augmented in the same ratio, and hence the square of the velocity resulting from those combined causes will continue unaltered. This square of the velocity, or measure of impulsion, is extinguished by the efforts expended in raising the weight. Wherefore the power multiplied into the quantity of descent is equal to the weight multiplied into its corresponding ascent. The power and the weight are thus inversely proportional to the spaces which they severally describe.

This general proposition must be viewed, however, as only a very near approximation to the truth, since the small quantities which would affect the result are, for the sake of simplicity, rejected. Thus, a minute portion of the inciting force is wasted in generating the slow descent of the falling body, while the final impulsion of the wheel incurs a slight loss merely in sustaining the momentary gravitation of the weight to be raised. To investigate the rigorous *formulæ*, let p denote the falling body, and s the space which it describes; m the mass of the wheel, g the distance of its centre of gyration, and a, the radius of the

barrel round which the thread is lapped. Since the circumference of this barrel must move just as fast as the descending weight, the effect would be the same if, instead of the wheel, a mass expressed by $\frac{mg^2}{a^2}$ had been collected at the distance a from the axis. The whole inciting force p is hence shared proportionally between p and $\frac{mg^2}{a^2}$, and therefore the part of it exerted on the barrel is $\frac{pmg^2}{mg^2+a^2p}$; which, being divided by $\frac{mg^2}{a^2}$, gives $\frac{pa^2}{mg^2+a^2p}$ for the accelerating energy. The velocity communicated to the circumference of the barrel, by the descent of the weight p through the space s, is hence $= 8\sqrt{\left(\frac{pa^2s}{mg^2+a^2p}\right)} = 8a\sqrt{\left(\frac{ps}{mg^2+a^2p}\right)}$. The velocity communicated to the centre of gyration is therefore $8g\sqrt{\left(\frac{ps}{mg^2+a^2p}\right)}$. But this final velocity may be again extinguished, by the operation of an opposite retarding force. Let w denote the height to be raised, h the height of its elevation, and b the radius of the barrel which lifts it up; then $8g\sqrt{\left(\frac{ps}{mg^2+a^2p}\right)} = 8g\sqrt{\left(\frac{wh}{mg^2+b^2w}\right)}$, and consequently $\frac{ps}{mg^2+a^2p} = \frac{wh}{mg^2+b^2w}$. Hence,

the other quantities remaining, the value of h is easily determined from that of w.

As an exemplification of this *formula*, let the measures of the model be substituted. Here $p = 1$ lb., $s = 30$ feet, $m = 18$ lb., $w = 56$ lb., $g = \frac{2}{3}$, and a and b the smallest barrels $= \frac{1}{12}$; whence $\frac{30}{\frac{4}{3}.18+\frac{1}{144}} = \frac{56.h}{\frac{4}{3}.18+\frac{56}{144}}$, and, by reduction, we have $\frac{4320}{1153} = \frac{8064.h}{1208}$, and $h = .5613$ feet or 6.735 inches. If the barrels of 2 and 3 inches radii were employed, h would be respectively 6.717 and 6.688 inches; but, if the accelerating weight had been 4 pounds, the ascents corresponding to the smallest, the middle, and the largest barrel, would be '26,868, 26,589, and 26,124 inches.

In general, the quantities a^2p and b^2w are very inconsiderable in comparison of mg^2, and may be therefore safely omitted. Whence $ps = wh$, and $h = \frac{p}{w}.s$, or $w = p.\frac{s}{h}$. There is no limit, then, but the strength of the axle, to the weight which may be raised, by the action of the Concentrator of Force; the only indispensable condition being, that the height is always inversely as the load. The fall of 2 lb. through 30 feet would lift five hundred weight or 560 lb., over one inch and a third part. Such a small primary force might hence, when accumulated

in this way, prove sufficient to *start* the greatest load, or overcome the most powerful obstacle.

Instead of raising great weights, the Concentrator might be adapted to tear asunder thick wires or metallic rods. The power exerted will then be inversely as the spaces through which those rods stretch, before they suffer fracture. The effects will consequently be augmented, by shortening the lengths of the rods. To the bottom of the engine, screw a strong bar above four feet in height, and to this fasten rods from six inches to a foot in length. If the limit of extension, which precedes the final disruption, were only half an inch, the power exerted, though produced by the descent of a single pound, would amount to about 1500 lb. A rigid and unyielding body is hence the most easily torn or broken.

But the impetus accumulated by the Concentrator may be wholly consumed, in merely stretching a very elastic substance which has a sufficient length. If a light contorted spring, for instance, be opposed to the rotation of the mass, it will, by its large though languid extension, gradually destroy the motive energy. A thick woollen cord, loosely plaited, and tied to a ring at the bottom of the machine, will produce a similar effect. A slender hempen string, though possessing little of a stretching quality, may still serve the same purpose, if it be taken of an adequate length. It is only required that

half the final strain multiplied into the corresponding extension should be equal to the product of the falling weight by its quantity of descent. A tension of 60 lb., acting through a height of one foot, would be sufficient to muffle and extinguish the momentum of rotation generated by the descent of one pound through 30 feet. To produce this effect, therefore, it is only wanted to select such a length of cord as will extend one foot, by the application of a strain of 120 pounds. A more slender substance, if proportionally more stretching, would have the same effect. In either case, a weight exceeding the absolute tension, and attached to the end of the string, would not, during the moderated consumption of the shock, be stirred in the slightest degree from its place.

The strength of a cord depends on its thickness, but the power to resist impulsion is determined by its elasticity and its length. This principle, which has been much overlooked, enters largely into the consideration of practical mechanics. Hence the practice of stemming a ship's way into a harbour by the friction of a long rope, the momentum being thus gradually spent. A short rope, firmly fastened to the pier head, so far from staying the vessel, would instantly snap. For the same reason, a ship riding at anchor is obliged to lengthen her cables. When these are composed of chains, the tension resulting from a diminution of curvature is precisely the same

as if a contractile force had been exerted. It is perhaps a general error in civil architecture to aim at mere solidity. Lightness combined with elasticity will often resist the shocks of ages, while stiff and ponderous materials are crumbled into ruins.

It may be curious to mark the time of accumulation of *momentum* in the Concentrator, and that of its subsequent expenditure. The force accelerating the descent of p is $\frac{a^2p}{mg^2+a^2p}$, and consequently the time of this descent in seconds is $\frac{1}{4}\sqrt{\left(\frac{mg^2s+a^2ps}{a^2p}\right)} = \frac{1}{4a}\sqrt{\left((mg^2 + a^2p)\frac{s}{p}\right)}$. The time required to lift the weight will hence be always $\frac{1}{4b}\sqrt{\left((mg^2+b^2w)\frac{h}{w}\right)}$.

But these expressions may be abbreviated, by omitting the quantities a^2p and b^2w. Wherefore, the time of descent is nearly $\frac{g}{4a}\sqrt{\frac{ms}{p}}$, and the time of the subsequent ascension of the weight is $\frac{g}{4b}\sqrt{\frac{mh}{w}}$, or $\frac{g}{4b}\sqrt{\frac{mps}{w^2}}$, because $ps=wh$. Hence, g and m remaining the same, the time of generating the impulsion is compounded of the inverse ratio of the radius of the barrel, and the inverse subduplicate ratio of the falling body, with the direct subduplicate ratio of the space of descent; while the time of expending this impulsion is in-

versely as the weight to be raised, and directly in the subduplicate ratio of the falling body and of its descent. Thus, one pound, having a line coiled about the smallest barrel, will descend through 30 feet in $\frac{8}{4}\sqrt{(18.30)}$ seconds or $46\frac{1}{2}''$; connected with the middle barrel, it would descend in $28\frac{1}{4}''$; but applied to the largest barrel, it would require only $15\frac{1}{2}''$. The succeeding act of lifting an half-hundred weight would be performed in $\frac{8}{4.56}\sqrt{(18.30)}$ seconds, or five-sixths of a second. If the descending power had been four pounds, the times spent by the successive barrels in generating the momentum of rotation would be $11\frac{5}{8}''$, $5\frac{13}{16}''$, and $2\frac{7}{8}''$; but the time of hoisting the weight of 56 pounds would be $1\frac{2}{3}''$.

Let $a=b$, and the time of descent will be to that of the subsequent ascension as $\sqrt{\frac{ms}{p^2}}$ to $\sqrt{\frac{mps}{w^2}}$, or as $\sqrt{\frac{mps}{p^2}}$ to $\sqrt{\frac{mps}{w^2}}$, that is, inversely as p to w. Hence the greater is the weight to be raised, the more rapid is the act of its ascension; and it is the same thing, whether an obstacle be overcome, or a disruption effected. The impulsion which required $46\frac{1}{2}$ seconds to accumulate, now exerting a strain equal to 1500 pounds, would tear a rod of metal asunder in less than the thirtieth part of a second.

The slowness with which this impulsive energy collects, is remarkably contrasted with the rapidity of its subsequent discharge. The greatest effects may be concentrated within such a portion of time as eludes the observation of the senses, and appears really instantaneous.

The Concentrator of Force thus finely elucidates the acquisition and the transfer of impulsive energy. The same accumulated momentum produces diversified effects, according to the way in which it is disposed. It will raise a ponderous mass, tear asunder a solid body, or will expend all its action in merely stretching a substance of a very distensible quality. These different purposes are attained in the operations of the mechanical arts; but sound theory is yet required to guide and improve the practice.

This engine, constructed on a large scale, might hence, with obvious advantage, be adopted as a most powerful auxiliary in various operations of art. Many situations occur which require an immense effort to be made on a sudden, and within a very limited space. This can be accomplished only, by storing up, as it were, a magazine of force, which may be opened and discharged at some precise moment. Even moderate animal exertion, if applied during any considerable time, will communicate to the Concentrator an impulsion sufficient to burst the firmest obstacles, and to lift, through a short space, the most enormous loads. The only thing wanted

is, by the application of geometrical principles, to graduate at pleasure the transfer of impulsion.

This engine involves likewise the theory of the *Fly*, which is annexed to various machines, not to augment their power, but merely to equalise their motion. The variable inciting forces are thus, by the intervention of a heavy wheel, blended together in creating one great momentum, which afterwards maintains a nearly uniform action. The use of the Fly in mechanics hence resembles that of a reservoir, which collects the intermitting currents, and sends forth a regular stream.

2. *Machine of Oblique Action.*—This depends on the theory of the composition of forces. A force exerted in the proper direction will balance any two forces; but if one of these be sustained by some fixed point, the first force may be considered as acting only against the other. Let the force OB (fig. 80.) be supported at B, produce BO till OD be equal to it, and complete the parallelogram AOCD; the force OA will counteract a force OC, or produce an equal force in the opposite direction. If the angle BOC be very oblique, OC will be much greater than OA, for OA is to OC as the sine of the angle COD to the sine of AOD. Suppose AO, BO, and CO to be cords tied at the point O; the force OA would occasion at C a strain OC. If, therefore, while the cord OB is fastened at B, an extension of the cord

OC were passed over a pulley at C, the force OA would cause the partial elevation of a weight represented by OC. This arrangement has been called *The Funicular System*. It is, to a certain extent, familiar to the seaman, who has often recourse to that mode in bracing the sails. He pulls at P laterally the rope AP′C (fig. 81.) into the oblique position AOC, and his power being at first indefinitely augmented, enables him to *start* the yard, and move it through a minute space; and by fastening the slackened rope again at B, he can renew the process.

If, instead of cords, there be substituted steel bars, all jointed at O, while OB turns about the fixed point B, a most powerful thrust will be produced in the direction OC. This thrust would even be increased in a slight degree, if exerted in the direction BC, by making the end of the bar OC to slide along the line BC; for OA will be to CP, as the pressure at O to the thrust at C. But the angle BOC being very obtuse, the ratio of the pressure to the thrust may be considered as the same as that of the sine of its supplement COD to the radius. If this angle COD were five degrees, the thrust would hence be $11\frac{1}{2}$ times the pressure; but if the angle were only the tenth part of a degree, the thrust would exceed the pressure 573 times. At the moment, therefore, of the collapse of the bars BO and

CO into a rectilineal position, the power is infinitely augmented.

The end C of the bar OC may be easily made to move in the direction BC, by combining (fig. 82.) two opposite bars BN and NC similar to BO and OC. The thrust will, in the case of great obliquity, be inversely as the angle NCO. Such a combination of bars, variously modified, has lately been adopted, with great effect, in several operations of art—for opening the steam-valve of Watt's engine—for the construction of the printing-press—and for extracting the steel core from the hollow brass cylinder now used as a roller in the printing of cotton. In all these instances, a vast momentary effort only is wanted.

The common mechanical powers are the chief elements of all machinery. They amount to five, but may be ranged in three divisions: 1. The *Lever*, and the *Wheel and Axle*, which is only an extension of it; 2. The *Inclined Plane*, and its modifications, The *Screw* and The *Wedge*; and, 3. The *Pulley*, which admits of multiplied combinations. Of these instruments, the theory has been already given, in as far as regards equilibrium; and to produce motion, it is only required to apply additional force. But it will be satisfactory to arrive at the same conclusions, by employing the principle of vir-

tual velocities. We shall consider these elementary engines in their order of succession.

1. *The Lever*, which consists of an inflexible bar ACB, (fig. 83.) either straight or bent, resting on a point C called the *fulcrum*, the *power* being applied at the end A of the arm AC, to raise a *weight* at the end B of the other arm CB. Throwing out of view the weight of the lever itself, and supposing it to be at first horizontal; let it shift into the proximate position A′CB′. The minute arcs AA′ and BB′ thus described may be regarded as tangents, and, consequently, while the power P descends vertically through a space AA′, the weight W rises through a space BB′; wherefore the opposite *momenta* being equal, $P \times AA' = W \times BB'$, and $P : W :: BB' : AA' :: BC : AC$, or the power and weight are inversely as their distances from the fulcrum.

Let ACB (fig. 84.) represent a bent lever, and from its extremities A and B, let fall the perpendiculars AH and BI, upon the vertical CH passing through the fulcrum C. Conceive the lever to move into the proximate position A′CB′, and parallel to CH draw A′L meeting AH in L and BM meeting B′M a parallel to AH in M. From similar triangles, $A'L : AA' :: AH : AC$, $AA' : BB' :: AC : BC$, and $BB' : BM :: BC : BI$; whence, by composition of ratios, $A'L : BM :: AH : BI$. But AL is the minute descent of the power, and BM the

corresponding ascent of the weight; consequently $P \times A'L = W \times BM$, and $P : W :: BM : A'L$, or, by equality of ratios, $P : W :: BI : AH$. The power and weight are, therefore, inversely as the perpendiculars let fall upon the vertical CH.

The same result is deduced from the property which belongs to the centre of gravity, of occupying the lowest place possible, whenever an equilibrium is attained. Join AB, and $AH : BI :: AG : BG$; whence $P \times BG = W \times BG$, and the power and weight being supposed to be attached at A and B, the point G would be the centre of gravity of the loaded lever. This centre would, therefore, while the lever turns about its fulcrum, describe an arc of a circle, and hence falls into the lowest position in crossing the vertical CH.

Levers are usually distinguished into three kinds, according to the relative position of the power, the weight, and the fulcrum. 1. When the fulcrum (fig. 85.) lies between the power and the weight. This kind includes the crow and handspike, pincers, and scissars. The toothed hammer is only a bent lever of this kind. Its invention was in mythology ascribed to Neptune, his *trident* being only a three-pronged crow. The arm PC is commonly longer that WC, and consequently the weight exceeds the power. The number of times which the weight contains the power, is always called the *mechanical advantage* or *purchase*. 2. When the weight lies between the

fulcrum and the power (fig. 86.). This kind includes the crow in its more general application, the baker's and druggist's knife, the common door, the wheel-barrow, nut-crackers, and oars. 3. When the power is applied (fig. 87.) between the fulcrum and the weight. To this kind belong the sheep-shears. It has a mechanical disadvantage, but admits of a proportionally wider motion. The bones of animals are therefore levers generally of this sort, pulled by the moderate contraction of muscles inserted near their joints.

Conceive the forces represented by the power and the weight to be exerted upwards, (fig. 88.) in a lever of the first kind, their joint efforts will be sustained by an opposite fulcrum at C. Instead of the fulcrum, substitute a load suspended from the same point, and this will be shared at the ends P and W inversely as the distances PC and WC. Such is the distribution of pressure in the case of a pole bearing an intermediate weight. If it hang from the middle, the carriers will always share the burden alike; but if the load be only placed upon the pole, as in fig. 89. a vertical from the centre of gravity will, in going up hill, divide the space unequally at C, and the lower person must therefore suffer a greater strain.

Two horses of unequal strength may yet be yoked to draw equally, by a proportionate division of the bar. This is partly effected in the ordinary way, by attaching the perch to a short projection from the

middle of the bar. To range a number of men along the arms of a pole, for the purpose of transporting heavy loads, is very unskilful, though frequently done in this country. Those who are nearest must evidently take the greatest share of the burthen, while the remote bearers have not the means of exerting their strength. The method practised in the East is much preferable, the strain being successively subdivided by a system of levers crossing each other. In China the same object is obtained still more simply, by placing the load on the middle of long bamboos, which cross at different angles, each end of them being borne by a labourer.

When a load is laid upon a plane whose weight may be neglected, the pressures sustained by three points have been already assigned. But they may be derived more easily perhaps from the property of the lever. For suppose the triangle ABC, (fig. 90.) bearing the weight P, to rest upon the angles A, B and C. Through P draw BPD, CPE, and APF. Conceive the plane to be lifted by the point B about AC as a fulcrum; BPD being then a lever, the load P would be to the force exerted, as BD to PD, or as the triangle ABC to APC. In the same manner, were the triangle lifted by the point A, the load P would be to that strain, as the triangle ABC to BPC. Wherefore, collectively the forces required to raise the triangle from the several points A, B and C, or the pressures which these exert are pro-

portional to the opposite triangles BPC, APC, and APB. When B coincides with the centre of gravity, the several internal triangles are equal, and consequently the strain is equally distributed.

The action of the simple lever is evidently confined within a very narrow space. But, by means of a small addition, it can be rendered capable of repeating its operation to an unlimited extent. This is effected by annexing to the end of the short arm two claws, which work alternately in the teeth of a ratchet wheel. Such a machine is called the *Universal Lever*. It is used occasionally for raising weights, but more commonly for dragging logs of timber to the saw-mill.

The mechanical advantage may be repeatedly multiplied, by a combination of levers of the first kind. Thus, the power being applied at (fig. 91.) A, let AB act upon the end C of another lever, and this upon E; the termination F will lift a very great weight. Suppose the *purchase* of the lever AB to be 4, that of CD, 3, and that of EF, 2; then one pound applied at A will support $4 \times 3 \times 2$, or pounds at F, and a slight additional force will set it in motion. The space of ascent, however, must evidently be very confined. But the wheel and pinion being only an extension of the lever, the same system of combination may have its action continued or incessantly renewed in a train of toothed wheels,

One of the most ordinary but useful applications of the lever is to weigh substances, or rather to compare their weights by some standard. This can be accomplished, by employing either a single weight or a set of subdivided weights. When the former mode is adopted, the long arm of the lever is divided into portions equal to the short one, which has a counterpoise appended to it. The standard consisted of a weight resembling in shape the pomegranate, and hence called in the East, *Romman;* which was fixed to a ring that could slide along the successive divisions, and mark the relative weight of the substance examined. In allusion to this circumstance, the *Steel Yard*, as it is now termed, was formerly, through misconception, named the *Roman Statera.* If the fulcrum or point of suspension be taken still nearer the end, the scale of weights will be proportionally augmented. The small balance of the Chinese consists of a tapering rod of ivory, like a quill, perforated in four points, to be suspended by different threads, the subdivisions being extended along each corresponding side.

The common balance, though less expeditious, is capable of greater accuracy than the steel yard. As it has equal arms, it requires a series of intermediate weights. For philosophical purposes, the easiest way is to reckon always by grains. The geometrical progression 1, 2, 4, 8, 16, 32, 64, &c. forms the simplest arrangement; but it will be found more

convenient to follow the decimal division, and the successive sets of 1, 2, 3, and 4; 10, 20, 30, and 40; 100, 200, 300, and 400; 1000, 2000, 3000, and 4000, &c. would save much trouble in adding up the weights.

A False Balance has one arm somewhat longer than the other. But the fraud is easily detected, by interchanging the places of the weight and the substance to be weighed; for the weight assigned will then be diminished, in the same proportion as it was before augmented. It perhaps deserves remark, that the true weight is rather less than half the sum of those opposite indications. Suppose the perpendicular BD (fig. 92.) to represent the true weight, and let BD be to BA in the ratio of the arms of the fraudulent balance; having completed the semicircle, it is evident that AB will indicate the weight of the substance when placed in the scale of the longer arm, and BC its weight when suspended from the shorter arm. But BD is less than the radius, or the arithmetical mean between AB and BC.

Since, from the property of the semicircle, BD is a mean proportional or a geometrical mean between AB and BC, the true weight of any body might be discovered by means even of a false balance. For let it be weighed first in one scale, and then in the other; and, the results being multiplied together, the square root of their product will give the accurate value.

Every correct balance must thus have arms of precisely equal lengths, or its fulcrum placed equally distant from the extreme points at which the scales are suspended. But the delicacy of the instrument is derived from the proximity of its fulcrum to the straight line joining those two points. To preserve a stable equilibrium, however, the fulcrum must occupy a position somewhat above that line. The beam should be strong and light, the preferable form consisting of two hollow cones: it should turn with a fine knife-edge upon a plate of agate, polished crystal, or hard steel; and the scales should likewise be hooked from sharp edges. The sensibility is farther augmented, and the risk of injury obviated, by various other contrivances. To such perfection have the arts been carried in this country, that Ramsden's famous balance would turn with only the seven-millionth part of its load.

In treating of the properties of the common balance, it will be requisite to consider the beam as a bent lever. We shall assume it as devoid of weight, and then apply the small correction that such an omission renders necessary. Let AC and CB (fig. 93.) be the equal arms, C the fulcrum, and A and B the points of suspension. Suppose the mass in the scale attached at B to exceed somewhat the weight applied at A; the beam will move into an oblique position, so that the vertical line CE will divide AB into segments AE and BE, which

are inversely proportional to the forces acting at A and B. Let W denote the weight with its scale, W′ the opposite scale and mass to be weighed, while a expresses the length of the arm CA or CB, d the perpendicular CD, and I the angle of declination DCE. Since W : W′ : : BE : AE, by composition W′+W : W′—W : : AB : 2DE : : AD : DE, and consequently $\frac{W'-W}{W'+W}=\frac{DE}{AD}$. But DE=CD . *tan* I, and W′ + W may be viewed as merely equal to 2W; wherefore, by substitution, we have W′—W= $2W\left(\frac{d.tan\,I}{a}\right)$. The difference of weight is thus, in the same beam, proportional to the tangent of declination. The value of d in relation to a may be easily determined by experiment; and, for the convenience of calculation, a line of tangents, divided centesimally, might be annexed to the index of the balance, instead of a graduated arc.

Let the weight of the beam itself be now taken into account. If G (fig. 94.) be the place of the centre of gravity, from which a vertical cuts AB in F; the effort of the beam to recover its horizontal position would, from the property of the lever, be the same as if a part of its weight, diminished in the ratio of BE to EF, were applied at B. Let m denote the weight of the beam, and g the depression CG of its centre of gravity; then EF=g.*tan* I, and consequently the additional pressure at B, indicated

by such declination, must be $\frac{mg}{a}.tan.I.$ The values of a and m being once known, that of g can easily be discovered experimentally, from observing the angle of deviation occasioned by the appending of a single grain at B. But the position of the centre of gravity, and consequently the value of g, is altered at pleasure, by merely screwing a nut or bob higher or lower on the index. If this bob were so adjusted, that the whole weight of the beam is to one grain, as the length of an arm to the depression of the centre of gravity below the fulcrum, the tangential line of the index would mark the quantity of correction in centesimal parts of a grain.

The value of d and g may be readily and accurately found, by observing the times of the oscillations of the loaded beams. Let fig. 98. represent the oblique lever, in its deviations on either side of its position of equilibrium. The tendency to redress itself is measured by DE, which is proportional to the tangent of the angle DCE, or to the small arc AA′. The oscillations of the beam are hence, under the same circumstances, all isochronous.

But the time of those oscillations may be deduced from theory. Since the accelerating force is to the weight appended at A, as DE to CA; it is hence to the action of gravity, in the oscillation of a pendulum CA from A to A′, as DE to AA′ or CD, or as CD to CA. Wherefore, CD is to CA, as CA to

the length of a pendulum which would vibrate in concert with the loaded beam. The length of this isochronous pendulum is consequently the diameter of a circle (fig. 95.) described through the fulcrum C and the points of suspension A and B. Conversely, the quantity of depression CD may be found, by dividing the square of CA by CF, the length of a pendulum corresponding to the observed time of vibration. Thus, if the arm CA were 8 inches long, and oscillated in 12 seconds; CD would be $\frac{64}{144.30,126}$, or the 80th part of an inch.

In this investigation, I have thrown out of view the weight of the beam, which can have little influence when charged with its full load in the scales. But this slight discrepancy might easily be corrected, for let the vibrations of the naked and loaded beam correspond to those of the pendulums CK and CL, and $\frac{\frac{1}{2}m.CK^2+W.CF^2}{\frac{1}{2}m.CK+W.CF}=CL$, or very nearly $CF=\frac{2W.CL^2-m.CK^2}{2W.CL-m.CK}$.

The delicacy of a balance may hence be inferred, from the slowness of its oscillations. When the arms extend almost in a straight line, and the centre of gravity is brought near to the fulcrum, the beam will turn with the smallest additional weight. This extreme sensibility, however, proves inconve-

nient in practice, by requiring the protection of a glass case, and rendering the process of accurate weighing very tedious. It were perhaps better generally to give the beam a greater flexure that it may vibrate more quickly, and to apply a certain correction, which is easily computed from the tangent of declination.

II. *The Wheel and Axle* is reckoned the next mechanical power. It consists of a wheel fixed to a smaller cylinder moving about the same centre; the power is applied to the circumference of the wheel, and the weight to be raised is attached by a cord lapped about the cylinder. This instrument may be regarded as a continued lever. The power is, therefore, to the weight, as the radius of the axle to that of the wheel; or, if the principle of virtual velocities be preferred, the power and weight are inversely as the circumferences of the wheel and of its axle. If spokes or arms be applied to the wheel, the circumference described by these must be considered as the path of action.

As varieties of the same instrument, we may name the *Capstan* or *Windlass*, and the *Common Gin*. In the latter, the rope which draws up the weight is wound about a drum, with a long projecting arm, which a horse puts in motion by treading round a circular path. Of a similar nature is the

crane, driven by men or cattle, walking within its circumference; but here the purchase is only in the ratio of the distance of the point of impulsion from a vertical through the centre of the wheel, to the radius of the axle.

The wheel and axle may turn also on different centres, and have their circumferences connected in mutual action, either by means of a belt or strap, or by the indentation of a system of cogs or teeth. This latter arrangement is usually called *Wheel* and *Pinion*.

The wheel has sometimes its axle of a tapered or conical shape, which gives it a varying purchase. This construction is adopted in the fusee of a watch, and is employed likewise advantageously in raising minerals, by an uniform pull from very deep pits, the rope at its greatest length being coiled about the narrow end of the axle, and advancing towards the wide end as it gradually shortens.

The *Double Capstan* is a very ingenious contrivance, originally brought from China, for augmenting in any degree the efficacy of the Wheel and Axle, without reducing its strength. It consists of two conjoined cylinders of nearly equal diameters, turning about the same axis, the weight being supported by the loop of a very long cord, of which one end uncoils from the smaller cylinder, while the other end laps constantly about the larger cylinder. (See fig. 96.) The elevation of the weight at each re-

volution is therefore equal to half the difference between the two circumferences, and the effect of the arrangement is the same as if the cord sustaining the weight had been wound about a cylinder, which has a circumference merely equal to that quantity. The mechanical advantage of the instrument, combined with its pulley, is hence in the ratio of the diameter of the larger cylinder to half its excess above that of the smaller one. Nearly the same effect is procured, by employing a single conical axle; one part of the train of cord unrolling itself from the smaller end, while the other part is coiled up towards the larger end.

III. The inclined Plane is accounted the third Mechanical Power. It has been already shown that an equilibrium would be produced, if the force exerted were to the weight to be raised, as the height is to the length of the plane. But the same conclusion is derived, from the consideration of Virtual Velocities. For, suppose a weight were drawn up the plane AC (fig. 33.) by a cord passing over a pulley at C. While this weight, in moving from A to C, acquires a vertical elevation BC, the power exerted will descend in the opposite direction through a space equal to AC. The power and weight must hence be inversely, as AC to BC.

The Inclined Plane is often combined with a circular motion. Suppose ABE (fig. 97.) to be a

spiral fixed to a perpendicular axis at P. As it turns round, it will push in the direction PB; and the proximate radius PD being drawn, and the small arc BC described, the thrust exerted at D must evidently be to the rotatory power applied at B, as CB to CD. If the curve be the Equable Spiral, it will maintain an uniform pressure, and the mechanical advantage will then be as the circumference of description to the quantity of protrusion PC, during a single revolution. But a power may be thus evolved, with an intensity varying as circumstances shall require. If a circle, for instance, be made to turn about an eccentric point P (fig. 98.), it will produce an effort that increases continually, from the intermediate position C, till it becomes infinite at the remote extremity B of the diameter. The heart-shape (fig. 99.) is employed in the composition of many of the most useful machines. A similar arrangement will conveniently change a revolving into an incessant reciprocating motion.

The Inclined Plane is employed chiefly in facilitating excavations, and raising the materials for the construction of edifices. On the same principle, the draught on roads over a hilly country is diminished, by conducting them with a circuitous but gentle ascent.

IV. The Screw is a most efficient Mechanical Power. It consists of a ridge or groove winding about a cylinder, and cutting at the same angle every

line on the surface drawn parallel to the axis. The Screw is called *exterior* or *interior*, according as it is formed on the outside or inside of the cylinder. The acting cylinder has always a handle annexed to it, and the screw is therefore really a compound of the Lever and Inclined Plane. As a machine, it is commonly employed for producing compression, but it may be likewise used in lifting heavy weights. The power applied to the screw is to the pressure exerted or the load sustained, as the interval between the adjacent threads, to the circumference described by the point of impulsion.

The action of the screw may be rendered far more intense, by applying to it the principle of the *Double Capstan*. Suppose a hollow screw adapted within a firm nut, should work upon an exterior screw with a finer thread, the remote end of this would evidently be pushed forward at each revolution, and by the difference only between the intervals of the threads. Such an instrument was proposed by the late Mr Hunter to be applied as a *Jack*, for the moderate elevation of great weights, and also to serve as a delicate micrometer.

An endless screw, working in the teeth of a wheel, has its power multiplied by their number. But it may act at once on two rachet wheels divided almost alike, the difference in breadth between the parallel teeth being the space of exertion. By such a contrivance, which also reckons the revolutions,

the mechanical purchase might be enlarged to any extent.

The coining engine consists of a screw carrying ponderous arms. The impulsion accumulated by the swing, produces a stroke similar to the Concentrator of Force; but the violence of the blow is softened, and the shock partly consumed, by the prolonged friction of the slanting grooves of the screw, by which the stamper advances to the die.

V. The Wedge, is sometimes employed in raising bodies, but more commonly in dividing and cleaving them. As an elevator, it resembles exactly the inclined plane, for the action is obviously the very same, whether the wedge be pushed under the load, or the load be drawn over the wedge. But when the wedge is driven forward, the percussive tremor excited in the block destroys for an instant the adhesion or friction at its sides, and augments prodigiously the penetrating effect. From this principle chiefly is derived the power of the wedge in rending wood and other substances. It then acts besides as a lever, insinuating itself into the cleft as fast as the parts are opened by the vibrating concussion. To bring the action of the wedge, therefore, under a strict calculation, would be extremely difficult, if not impossible. Its peculiar operations must be discovered by experience. All the various kinds of cutting tools, such as axes, adzes, knives, chisels,

saws, planes, and files, are only different modifications of the wedge.

VI. The PULLEY is a very useful auxiliary in the composition of the generality of machines. In its simplest form it has already been considered under the head of Statics. But whatever addition is made to the power of equilibrium must generate motion.

The *Purchase*, however, procured by the various combination of pulleys, may be computed from other principles. We may either estimate the subdivision of primary strain, or compare the celerity of the weight to that of the power. Pulleys occur sometimes singly, but oftener conjoined in blocks, and then they turn on the same or different centres, and have their diameters either equal or unequal.

The usual combination is represented in fig. 100. Suppose n to be the number of pulleys in each block, the weight will be supported by $2n$ cords, each bearing an equal share. The last moveable cord must therefore sustain the $2n^{th}$ part of the weight. Such is the power required to maintain the equilibrium, and consequently the purchase obtained by the blocks is $2n$, or the number of extended folds of cord.

The principle of Virtual Velocities gives the same result. For, suppose the weight to rise one inch; each of the cords would slacken an inch; and the

last one to which the power is applied, would be lengthened $2n$ inches. The power is consequently to the weight, as 1 to $2n$.

If a weight be attached to a pulley which is supported by a cord having one end fastened to a fixed obstacle, the loose end will evidently sustain only half of the weight, or, for every inch which the weight rises, the power which draws it upwards will move over two. But instead of this power, we may apply another pulley mounted in the same way. (See fig. 101.) The sustaining power will now be only a quarter of the weight. The similar application of a third pulley, would reduce the load borne up, to one-eighth part. The action is thus doubled, by each additional pulley. The purchase finally obtained in this system of pulleys is therefore denoted by 2^n. The line of traction is here directed upwards; but it may be reversed and adapted to perpendicular pressure, by passing the loose cord over a fixed pulley.

Another combination of pullies may be noticed, which, though less convenient, has still greater efficacy. In this system, each cord is fastened to the weight, and passing over a pulley, has its end tied to another pulley. (See fig. 102.) The first pulley, fixed to a firm obstacle, merely changes the direction of the force; but the other pullies in succession greatly augment its action. Thus, while the weight rises one inch, the moveable pulley will sink an inch, and each of the folds of the cord bent over

it, will shorten as much, permitting, therefore, the second moveable pulley to sink three inches. This pulley, now, by its descent, lets the folded cord on each to slacken six inches, which, joined to the inch which the weight mounts, gives 7 inches for the shifting of the third moveable pulley. This, again, giving 14 inches of extension to its adapted cord, and the inch derived from the ascent of the weight, leaves the next pulley to drop 15 inches. Hence, one moveable pulley will, in this way, enable one pound to support 3 pounds; two moveable pullies will enable it to support 7 pounds; three such pullies will augment its action to 15 pounds; and a fourth pulley would make it uphold 31 pounds. In general, if n be the number of moveable pullies combined in this manner, the *purchase* obtained by the system will be denoted by $2^{n+1} - 1$.

In the composition of machines, it is often required to augment or diminish, in a given ratio, the celerity first impressed. This object is effected by means of a train of wheels, put in motion by the application of a strap, or by the action of engrained teeth. It is essential, however, that the force should be exerted perpendicular to the impelling surfaces. Let A (fig. 103.) be the centre of the pinion or smaller wheel, which drives another wheel about the centre B. Suppose, first, that this pinion acts upon

the teeth of the larger wheel, merely by implanted pins or cylinders of insensible diameters, either perpendicular to its plane or radiating from its centre. The point E, which represents one of those pins, must therefore meet the curved surface of the tooth DE at right angles, and consequently press against it in the direction of the normal CE. But the right angle CEA is contained in a semicircle, and hence the curve DE is only a portion of the exterior epicycloid, described by the revolution of the circle AC, which has half the diameter of the pinion, about the circumference of the primary circle. Produce EC to meet BF drawn parallel to AE. The force with which the pinion presses against the tooth at E has the same effect, in turning the wheel about the centre B, as if it acted upon the point F; and, being proportional to the radius AC, it urges the circulation of that wheel, by an energy proportional to the distance BF. But AE is to BF, as the radius AC to the radius BC; and consequently the equable pressure exerted by the pinions will maintain the uniform circumvolution of the wheel.

The pinion will communicate its entire impulsion to the wheel, evidently, whether it acts on the concave or the convex surface of each tooth. In the former case, it must turn the wheel to the left; but in the latter, to the right. Its action is in either way exerted uniformly.

Suppose now the pinion, instead of carrying linear pins, should be armed with thick cylinders or spindles, a form of construction which gives it the name of *Lanthorn*, *Trundle*, or *Wallower*. Divide the distance AB (fig. 104.) between the centres, into segments proportional to the number of the spindles and of the teeth, and AC and BC will be the primitive radii or *pitch-lines*, whose circles mark by their contact the path of action. Since the spindle must press perpendicularly against the surface of the tooth, it is evident that the line OC drawn through the centre O, and the point E of impulsion, must be a normal to the curve. But the centre O of the spindle describes the arc GO of an exterior epicycloid, and consequently the tooth is only a parallel curve, shifted backwards by an interval equal to the radius OE or GD.

Let the pinion be furnished with teeth which act upon those of the wheel. The line of pressure EC (fig. 105.) must be necessarily perpendicular to both curves at their point of contact E. Wherefore, while the tooth DE of the wheel is traced by the rolling of a circle on the outside of the circumference of the wheel, the tooth GE of the pinion is described by the revolution of the same circle within the circumference of the pinion. The corresponding teeth are thus only portions of an exterior and an interior epicycloid generated by the same primitives. The diameter, however, of the revolving circle is left indeterminate. If it be taken equal to half the pinion,

treated in the same way, will describe the forms which are capable of accurate contact, and fitted therefore to communicate the full energy of impulsion.

To form a Templet or Pattern Tooth for wheels, the easiest mode of proceeding is by means of a mechanical construction. Let a small segment FCG (fig. 106.) of a circle denoted by the pitch lines, including the space of at least two teeth, be described on a smooth board, which is then accurately rounded; and to this apply a corresponding segment DCE of the generating circle, bearing a pencil or tracer at D, both circumferences being rubbed with rosin or chalk to prevent sliding. The point D being made to apply at F, let the other points in DCE be successively brought in contact with FCG; the tracer, as it recedes from D, will mark out the required exterior epicycloid FD. The interior epicycloid of the pinion is described in a similar manner, only the generating circle must evidently be less than the circle which contains it. For the sake of accuracy, the circular segments may consist of steel, and have the surfaces divided by fine flutings.

If the rolling arc DCE belong to a large circle, the portion FD of its epicycloid will not differ sensibly from the evolute of the circle of which FCG is a segment. This discrepancy is less perceptible when the teeth are small. In ordinary cases, there-

fore, it will be sufficiently accurate, to assume the initial part of ane volute of the primary circle FCG for the form of the teeth. The curve is traced by the end of a thread, as it unfolds from the circumference between F and G, or more correctly by a fine straight saw, applied successively along the arc FCG.

But a more accurate mode of construction may be derived from the property, that an epicycloid is the evolute of a similar curve. (See Geometry of Curve Lines, p. 372.) For let R and r be the radii of the fundamental and generating circles; and it follows, from Prop. V, that $\frac{R(R+r)}{(R+2r)^2}.4r$ will denote the radius of curvature at the beginning of the epicycloid. Instead of R and r, the number of teeth of the wheel and half of those of the pinion may be adopted, as having the same ratio. Find, by calculation, therefore, CE (fig. 107.) the value of this expression, and from E describe an arc CF, which may be viewed as coincident with a portion of the involute of the exterior epicycloid. Unfold a thread or fine steel spring from CF, and its extremity will describe the tooth CG.

When an internal epicycloid is wanted, the radius of curvature of its opening involute will become $-\frac{R(R-r)}{(R-2r)^2}.4r$. The point E will hence lie without the fundamental circle, and the evolution of

the arc CF (fig. 108.) will now produce an internal tooth CG.

If the generating, had half the diameter of the fundamental, circle, the radius of the involute for the exterior epicycloid would be $\frac{3}{2}r$, but infinite for the interior. In the latter case, the first part of the involute becomes a straight line, the evolution of which traces a perpendicular, the teeth here being only the extremities of the several diameters. Let $R = 3r$, and the radii of the inner and outer involutes will be $\frac{48}{25}r$ and $24r$; and they will become $\frac{20}{9}r$ and $12r$, when $R = 4r$.

It would be easy to give an approximative geometrical construction. Bisect CB (fig. 109.) in H, and make AH : CB : : AB : CI, and CB : AC : : CI : CE, from E describe the small arc CF, and in it assume any points K, L, &c. either equidistant or brought closer together as they approach to F. From these points, describe the successive arcs CM, MN, NO, &c., which will nearly coalesce into the form of a tooth, and, if necessary, their junctions may be somewhat rounded.

If the fundamental circle be indefinitely large, its circumference will become a straight line, and the epicycloid will pass into the common cycloid. Hence the mode of tracing the initial portion of this curve,

which suits the form of cams. Let CP (fig. 110.) be the path of rotation, and A the centre of the generating circle; make CE equal to twice its diameter, from E describe an arc CF, in which assume any points, and from these describe the minute arcs CL, LM, MN, &c. terminated by the successive tangents IL, KM, FN, &c. The compound arc CP will, for a considerable space, scarcely differ from a cycloid.

In crown wheels and bevelled geer, the faces of the teeth must be all tapered, as if they proceeded from a common centre. The pinion may be considered as a portion of a cone, which, rolling along the horizontal rim of the wheel, traces the contour of the teeth, by describing a modified epicycloid. The same general principles will indicate all the variations.

It is of practical consequence that the teeth of wheels should be all equally worn. In their mutual congress, therefore, they ought to produce a series of the most diversified contacts. To effect this object, the numbers of teeth in the pinion and the wheel must be as discordant as possible. If the one were any aliquot part of the other, the same incessant coincidence would soon recur, occasioning a partial and disproportionate attrition. Prime numbers should be preferred; and the larger they are, the smoother will the wheels work. The odd tooth in

the pinion is hence by our mill-wrights very appositely called the *hunting cog*.

The effects produced by machines are extremely diversified. To extend action to a distance, or to divert its direction; to reduce or multiply the celerity impressed; to modify an uniform progression into an accelerating or retarding one; to maintain a parallel motion; to change a rectilineal into a circular motion, and the reverse; to convert a reciprocating play into a constant circulation or equable rectilineal flow;—these are a few of the objects generally aimed at. Pressure is conveyed to any distance, by a system of parallel levers, supported at certain intervals, the extremities of their arms being connected by a train of beams. The direction of any power is easily changed, by means of a crank or bent lever. The same contrivance, aided by the action of a fly, converts a reciprocating into a circular motion. This effect is likewise produced by a rack, working alternately on the opposite sides of a toothed wheel. The celerity is modified at pleasure, by affixing to the axle solid blocks, sometimes called *heart wheels*, (see fig. 99.) and fashioned like spiral or eccentric curves. These lines may be traced with such accuracy as to evolve the precise succession of impulse required. But all the transitions should, if possible, be gradual, any sudden change occasioning a concussion which wastes the force, and tends to disjoint and shatter the parts of the machine.

The most elegant mode of transmitting force in any direction, is by a contrivance termed the *Universal Joint*. (See fig. 111.) Instead of the cross, a ring or a ball is often preferred. But the angular motion communicated in this way is not quite equable, and becomes even irregular in the case of great obliquity. By doubling, however, the combination of axes, as in fig. 112, the impression may be turned aside almost into an opposite direction, and may be rendered uniform, by a compensation of irregularities.

An alternate motion is beautifully converted into a revolving one, by a vertical rod which carries affixed to it a wheel indented to another wheel of the same size. The central wheel, describing at each reciprocation a double circumference, must turn twice round. This mechanism is, from a vague analogy, called the *Sun and Planet Wheel*.

A reciprocating beam can be made to raise and depress a rod very nearly in a vertical line, by means of a regulated parallelogram. Thus, C (fig. 113.) being the centre of the beam, let an arm BE turn about the firm joint B, and guide the end E of the parallelogram ADEF; if CD be taken a mean proportional to AD and BE, the other end F, carrying the rod of a piston, will travel in a path which is very nearly rectilineal and perpendicular.—These two fine contrivances were happily combined in Watt's Steam Engine.

The efficacy of any complex machine is generally computed from the statical relation of the power to weight to be raised. This, however, is merely the proportion which would maintain a state of quiescence; and to produce actual motion, it requires the application of more force. Nor does the velocity thus created correspond to the simple ratio of the additional power, but follows a modified and much slower law of increase. The performance relatively becomes greatest, when the force exerted has attained a certain limit of intensity. It is of the utmost consequence, therefore, in the economy of machines, to approximate at least to this measure of advantageous exertion. But the problem involves so many and such intricate considerations, that theory can seldom furnish a direct solution, and requires all the aid of experience. I shall select only two instances illustrative of the general principle; the first relating to the Wheel and Axle, and the next to the Inclined Plane.

I. Let it be required (fig. 114.) to find the radius OB of a circle, from whose circumference the descent of an appended weight shall raise, with the greatest possible celerity, another equal weight attached to a given circle AC fixed on the same axis. These weights being each of them regarded as unit, the force at B which would maintain the equilibrium, is denoted by $\frac{OC}{OB}$, and

consequently the inciting force by $\frac{CB}{OB}$. But, both circles being supposed devoid of weight and having the same angular motion, this motive power will be shared as the velocity of OC^2 to OB^2, and therefore the acceleration of the weight from B is $\frac{CB}{OB}\left(\frac{OB^2}{OB^2+OC^2}\right)=\frac{CB.OB}{OB.^2+OC^2}$. Such is the velocity generated at that point, and consequently the velocity at A must be $\frac{CB.OC}{OB^2+OC^2}$. This expression is hence a *maximum*, or its reciprocal $\frac{OB^2+OC^2}{CB.OC}$ a *minimum*. But for OB^2, substituting $(CB+OC)^2$, it becomes $\frac{CB}{OC}+2+\frac{2OC}{CB}$. Omitting, therefore, the constant number 2, and multiplying the rest by OC, which is likewise a constant quantity, and the result $CB+\frac{2OC^2}{CB}$ must be a *minimum*. Let OC (fig. 115.) and 2OC together, form the diameter of a circle, and draw the chord BCF; it is evident that the rectangles $BC.CF = OC.2OC = 2OC^2$, and $CF = \frac{2OC^2}{CB}$. Wherefore, BCF is the shortest chord passing through C; it consequently approaches nearest to the centre G, and must be perpendicular to GC

or to the diameter OE. Hence $BC=CF$, and $BC^2=2OC^2$, or $BC=OC\sqrt{2}$.

In approximative numbers, therefore, OC being reckoned 5, CB will be 7, and OB 12. The accelerating force exerted at B is $\frac{7}{12}\cdot\frac{144}{169}=\frac{84}{169}$, or very nearly half the power of gravity; but this action would cause the weight from A to mount only $\frac{5}{12}\cdot 8$ feet, or 40 inches in a second.

2. Suppose a weight, acting over a pulley at A (fig. 116.), to be just sufficient to support a load on the inclined plane AC; it is required to substitute another more sloping plane AD, along which this load would be drawn or made to rise from the level DB to the elevation A, by the same power and in the shortest time possible. The urging weight being reckoned unit, the load which it sustains on the plane AC will be denoted by $\frac{AC}{AB}$; but this load, transferred to the plane AD, would be held at rest by a weight or vertical force expressed by $\frac{AB}{AD}\cdot\frac{AC}{AB}$, or $\frac{AC}{AD}$. The excess of weight engaged in producing the compound motion, is therefore $\frac{AD-AC}{AD}$, or $\frac{DE}{AD}$, AE being cut off equal to AC. This force, communicating equal velocities to the load and the primary weight, must be shared be-

tween them according to their respective masses. But these remaining constant, the accelerating power in both is proportional to $\frac{DE}{AD}$. Now, since the space described is as this force multiplied into the square of the time, the load will be carried from D to A in the shortest time, when AD divided by $\frac{DE}{AD}$, or $\frac{AD^2}{DE}$, is a *minimum*. Suppose AF were made equal to EA, and DG found a third proportional to DE and DA; while EA is given, the point D, and hence G must in its extension be assigned, so that DG shall have the least possible extent. By division, DE : DA : : EA : AG, and DE : EA : : EA : FG; consequently, the rectangle DE, FG, being equivalent to the square of EA, is constant. But EF, or the double of EA, is likewise constant; and therefore, the sum of those extreme segments DE and FG must be the least possible. As they contain a given space within the shortest perimeter, they must consequently form a square, and thus DE, EA, AF, and FG are all equal. Wherefore AD, the length of the plane required, is double of AE or AC.

Suppose BA were 3 feet and AC 5 feet, one pound suspended vertically would support $1\frac{2}{3}$ pounds leaning upon AC; but this load, when transferred to AD, which is 10 feet, would be upheld by half a pound, and, consequently, the remaining half pound must supply the accelerating force. The energy ex-

erted is therefore $\frac{1}{2}\cdot\frac{3}{8}=\frac{3}{16}$, and the load would be drawn from D to A, or lifted vertically in $\sqrt{\frac{10}{3}}$ seconds, or 1,825″. The direct action of gravity would have produced the same effect in .433″.

But such investigations are unfortunately rendered in a great measure superfluous, by those unavoidable imperfections which are incident to all machines. A very large portion of the force applied is in most cases consumed by the various obstructions which it has to encounter during the course of its transmission. The resistance of the air causes a certain retardation; but the chief impediment arises from the rubbing or attrition of the surfaces which come incessantly into contact. Most solid bodies, when brought close together, are disposed to cohere mutually, and with various degrees of tenacity. This peculiar force, being exerted perpendicular to the surface of contact, can evidently have no influence whatever in impeding a lateral traction. But all substances appear to possess likewise a certain adhesive property, which opposes any change of mutual contact, and retards even the horizontal passage of one plane along another. This latent obstructing power constitutes *Friction*, which has such extensive influence in diminishing the performance of all machinery. The effect, however, is much diversified, according to cir-

cumstances, and can be discovered in any particular case, only by experiments conducted in a scale of sufficient magnitude.

There are two general modes of exploring the nature and operation of Friction. The first ascertains the weight required to draw a body under the pressure of a given load along the horizontal surface of another. The second method is still simpler, and consists merely in raising the end of the upper plane till it acquires the declination at which the succumbent load begins to slide. This extreme declination is hence called the *Angle of Equilibrium, Quiescence, or Repose.* Let CA (fig. 33.) represent the position of the plane at the moment the load slips from its place. If the vertical FD represent the weight, FE will express the perpendicular pressure sustained by the plane, and OED must denote the corresponding friction, which is just balanced here by the tendency to descend. Wherefore, the Pressure is to the Friction, as FE to ED or as AB to BC, that is, as radius to the tangent of the angle of repose.

These two modes of experimenting, however, will seldom give precisely the same results. Most bodies require a greater force to pull them from their contact, than what is afterwards sufficient to maintain their adhesive progression. But the obliquity of the plane of descent evidently marks only the initial obstruction, and not the subsequent enfeebled, yet unceasing, action of Friction.

Thus, if the plane CA were a pine board, upon which is laid a block of oak with a smooth surface, it may be elevated to an acclivity of 30 degrees before it disengages its load. But should the angle of declination exceed only 10 degrees, on striking the side of the plane with a smart blow, the oaken block will start from its seat, and then glide downwards. While the plane retains this lower inclination, the load will not rest, on being again replaced, unless it be a few seconds held in contact.

It was inferred from the earlier observations, that Friction is a constant retarding force, proportioned nearly to the pressure, but commonly varying from the third to the sixth part of this quantity. In polished freestone, it exceeds the half of the pressure, and the rough surface of bricks augments it to three-fourths. This great obstruction contributes most essentially to the stability of arches and vaults.

A body subjected to no obstruction will descend, it was demonstrated, in the same time, through any chord to the lowest point of a vertical circle. But even when retarded by the influence of friction, it will yet slide down equally through the chords of a certain determinate arc. For the whole weight being represented by CF (fig. 95.), the pressure on the oblique plane AF is denoted by AC, and consequently the measure of the friction by AM, a definite portion AC. The remaining force of acceleration is hence expressed by MF, which will be

described under the attrition, in the same time that AF is traced by a free descent. But the sides AC and AM of the right angled triangle CAM having a given ratio, that triangle is given in species; consequently the angle AMC is given, and likewise the exterior angle CMF, which must therefore be contained in a given segment of a circle. The body would hence slide down the planes MF or PF, in the time that it would have fallen directly from C to F. It likewise follows that the tangent NF to the arc AMF will mark the position of the plane of *repose*.

The angle of repose often determines the contour of natural objects. Thus, fine sand slides more easily than ordinary mould, and hence sand-hills have generally a softer ascent than the grassy flanks of mountains. The latter, without being broken into precipices, may rise at an angle of 40 degrees; but the former will seldom support an acclivity above 25 degrees. Again, the angle of repose of iron pressing upon iron being 16 degrees, if the threads or spirals of a vice wind closer than this inclination, the screw must hold at any place to which it is carried.

By removing the visible asperities from the surfaces of bodies, their mutual attrition is diminished. But any higher polish than what merely prevents the grinding and abrasion of the protuberant particles, has no material effect in reducing the measure of Friction. Other peculiar circumstances appear to have more extensive influence.

The experiments on Friction which Coulomb made between the years 1779 and 1781, being devised with ingenuity, performed with great care, and executed on a large scale, are the most original, and by far the most important that have been made. His general method was to draw a sort of sledge, mounted on different sliders and variously loaded, along a large horizontal bench, fitted likewise with corresponding slips of wood or metal. This sledge was either allowed to rest a certain time, or it was instantly put into motion. Some idea of the diversified results may be conceived, by referring them to a common standard of comparison. Assuming the pressure as equal to one hundred parts; the friction of oak against fir was 66 in the direction of the fibres, but amounted only to 16 when moved with the velocity of a foot each second; the friction of oak against oak, in the direction of the fibres, was 45, and across them only 27, the effect being still reduced by motion to 10; the friction of fir against fir, in the direction of the fibres, was 56, which sunk to 17 during motion; the friction of elm against elm, in the direction of the fibres, was 46, which motion reduced to 10; the friction of copper upon oak was lengthwise 18, and only 8 when kept in motion; the friction of iron upon oak was 28, and diminished during motion to 11. But the mutual friction of metals appeared in general to be less affected by motion. Thus, iron against iron had its friction, in passing from rest to

motion, diminished only from 28 to 25; again, the friction of brass upon iron, which reached 25, was contracted by motion to 17.

The application of grease, soap, or other soft liniments, to the rubbing surfaces, generally diminishes the friction, though in very different degrees. Oak, fresh greased, has its friction upon oak, only depressed from 45 to 43; but this was reduced to 26, after the tallow had been thoroughly imbibed into the wood and left a glossy smoothness. The greasing of the surfaces of copper and iron reduces their friction to 10.

When a coat of tallow is applied to metals, the friction attains almost instantaneously its limit. But the different sorts of wood, treated in the same way, suffer a slow change in the condition of their surfaces, which continues for a considerable time to augment the intensity of friction. A piece of greased oak, the moment it is laid upon another, is drawn, with a force of 4, but would have this friction augmented in three seconds to 10, in one minute to 16, in two hours to 28, and in the space of five or six days to 44. The friction of wood against metal increases likewise during a sensible time. A closer approximation of surfaces must evidently be produced by some gradual process, which is enfeebled by the softness of the substance impressed. Hence the friction of wood is greatly diminished by rapid motion, while that of metals continues with very little alter-

ation, whether they be drawn along slowly or extremely fast.

This general view of the phenomena of Friction affords a glimpse of its origin. Matter, however passive to external impressions, is not strictly inert. Its attractive and repulsive energies, modified only by the mutual distance of the particles, are incessantly in action, and produce varied effects and in different times. Nor are such operations confined to the substances commonly termed chemical; they occur to a certain extent, though more concealed from observation, in other bodies which manifest no peculiar signs of activity. "Nature speedily extinguishes every motion upon earth, and seems to diffuse a principle of silence and repose; which made the ancients ascribe to matter a sluggish inactivity, or rather an innate reluctance and inaptitude to any change of place. We shall perhaps find, that this prejudice, like many others, has some semblance of truth; and that even dead or inorganic substances must, in their recondite arrangements, exert such varying energies, so like sensation itself, as could not fail, if completely unveiled in our sight, to strike us with wonder and surprise.

"When one solid is drawn along another, if the opposing surfaces be rough and uneven, there is a necessary waste of force, occasioned by the grinding and abrasion of their prominences. But Friction subsists after the contiguous surfaces are

worked down as regular and smooth as possible. In fact, the most elaborate polish can operate no other change than to diminish the size of the natural asperities. The surface of a body, being moulded by its internal structure, must evidently be furrowed, or toothed, or serrated; and such is the information derived from close inspection with the Microscope. Friction is, therefore, commonly explained on the principle of the inclined plane, from the effort required to make the incumbent weight mount over a succession of eminences. But this explication, however currently repeated, is quite insufficient. The mass which is drawn along is not continually ascending; it must alternately rise and fall, for each superficial prominence has a corresponding cavity; and since the horizontal boundary of contact is supposed to be horizontal, the total elevation will be equal to their collateral depressions. Consequently, though the actuating force might suffer a perpetual diminution in lifting up the weight, it would, the next moment, receive an equal increase by letting it down again; and those opposite effects, destroying each other, would have no influence whatever on the general motion.

"Adhesion appears still less capable directly of explaining the source of Friction. A perpendicular force acting on a solid, can evidently have no effect to impede its advance; and though this lateral force, owing to the unavoidable inequalities of

contact, must be subject to a certain irregular obliquity, the balance of chances must on the whole have the same tendency to accelerate as to retard the motion. If the conterminous surfaces were hence to remain absolutely passive, no Friction could ever arise. Its existence betrays an unceasing mutual change of figure, the opposite planes, during the passage, continually seeking to accommodate themselves to all the minute and accidental varieties of contact. The one surface, being pressed against the other, becomes, as it were, compactly indented, by protruding some points and retracting others. This adaptation is not accomplished instantaneously, but requires very different periods to attain its *maximum*, according to the nature and relation of the substances concerned. In some cases, a few seconds are sufficient; in others, the full effect is not produced till after the lapse of several days. While the incumbent mass is drawn along, at every stage of its progress, it changes its external configuration, and approaches more or less to a strict contiguity with the under surface. Hence the effort required to put it first in motion, and hence too the decreased measure of Friction, which, if not deranged by adventitious causes, attends generally an augmented rapidity.

" Friction, then, consists in the force expended in raising continually the surface of pressure by an oblique action. The upper surface travels over a

perpetual system of inclined planes; but the system is ever changing, with alternate inversion. In this act, the incumbent weight makes incessant, yet unavailing efforts to ascend; for the moment it has gained the summits of the superficial prominences, these sink down beneath it, and the adjoining cavities start up into elevations presenting a new series of obstacles which are again to be surmounted; and thus the labours of Sisyphus are realized in the phenomena of Friction.

"The measure of friction must depend merely upon the angles of the natural protuberance, which are determined by the elementary structure or the mutual relation of the two approximate substances. The effect of polishing is only to diminish those asperities and augment their number, without altering in any respect their curvature or inflexions. The constant or successive acclivity produced by the ever-varying adaptation of the contiguous surfaces, remains therefore the same, and consequently the expense of force will still amount to the same share of the pressure. The intervention of a coat of oil, soap, or tallow, by readily accommodating itself to the variations of contact, must tend to equalize it, and therefore must lessen the angles, or soften the contour of the successively emerging prominences, and thus diminish likewise the friction which hence results *."

* *Experimental Inquiry into the Nature and Propagation of Heat*, pp. 298-303.

The Theory now quoted is the result of strict induction, and seems quite accordant with all the observations hitherto made on the subject of Friction. This obstructing power must, in the attrition of the same surfaces, be proportional to the pressure, since a certain share of the incumbent weight is required to surmount the prominences or natural acclivities which rise in perpetual succession along the range of contact. But the friction becomes sensibly diminished, if the surface in action be reduced to the smallest dimensions. Thus, while the friction of a ruler of brass against a similar one of iron, is expressed by 26; it was found to be only 17, after the sledge had been mounted upon four round-headed brass nails. The reason of this diminution appears to be, that the contact is not so close, when the nails are drawn along the iron surface, the great convexity towards the edges being less affected by the projecting eminences. The same consideration will explain the diminished friction of an axle against its box. Thus, an iron-axle turning in a brass socket has its friction reduced from 26 to 16, or lessened by more than one-third part. The curvature of the axle produces nearly the same effect as the softening of the natural acclivities of its box. Hence this peculiarity of loose contact is lost, if a coat of tallow be interposed. When fresh greased, the friction between the axle and socket is 9, the same as in the case of plane surfaces. So likewise axles of lignum

vitæ running in boxes of oak or elm, have only a friction from 3 to 4, but increased to 6 after the unguent is rubbed off.

In the mutual contact of metals, the friction attains, almost instantaneously, its *maximum*. But when metal rubs against wood, or one piece of wood against another, the friction always augments by resting, and reaches not its full measure till after the lapse of some portion of time. The duration of this progressive increase appears to depend on certain peculiar circumstances, but chiefly on the pliancy of the conterminous surfaces. Two pieces of wood acquire the utmost friction in the lapse of an hour or two; while iron laid upon oak will have its friction still augmenting for the space of five or six days. The application of a coat of tallow seems to protract the limit of friction. This limit is attained by the greased surfaces of iron and copper in four minutes; while pieces of wood, treated in the same way, will have their friction gradually augmented during nine or ten days.

If two planed boards of oak be coated with tallow, and rubbed against each other till they become smooth and shining, the friction will at first increase most rapidly, and still continue progressive for a very long time. This friction, which at the instant of contact, is only the twelfth part of the pressure, becomes one-half more in the space of a single minute, is doubled in eleven minutes, and tripled in twenty-four

hours. Amounting now to the fourth part of the incumbent weight, it acquires not sensibly any farther augmentation. If t represent in minutes the whole time elapsed, the friction of such finely greased oak will be expressed nearly by $\frac{1}{12}+\frac{1}{25}\,log.\,(1+10\,t)$.

This gradual augmentation is still farther prolonged by applying a soft layer of tallow. If this were spread to the thickness of the twentieth part of an inch over polished boards of oak, it would penetrate into the wood about the eighth part of an inch, and the adaptation of the impregnated surfaces would be effected with extreme slowness. The friction would be *doubled* in very little more than a second, *tripled* in $6\frac{1}{2}''$, *quadrupled* in $36''$, *quintupled* in $3'.9''$, and would continue its progress nearly nine days, beyond which no farther increase would be perceived, the friction being then augmented tenfold, and equal to two-fifths of the whole pressure. If t denote the time in minutes, the friction will be expressed with tolerable accuracy by this formula: $\frac{1}{25}+\frac{1}{18}\,log.\,(1+240\,t)$.

Where friction is augmented by the duration of contact, it must evidently be greater in slow than in rapid motions. Hence, it is not affected at all by the celerity of the attrition of metals. In all other cases, however, the influence of friction decreases in some proportion to the quickness of traction. This diminished obstruction, resulting from

a swift congress, is visible in the mutual attrition of pieces of wood: it is still more prominent in the heterogeneous contact of metal with wood, but is most remarkable when a coat of tallow has been interposed. A ship is launched along sliders, which commonly slope only from 4° to $5\frac{1}{2}$°. The lowest friction is here exerted, all previous adhesion being destroyed by blows of the mallet, and shocks given in the act of withdrawing the wedges. The momentary friction being 4, leaves an accelerating force of 3, that hurries the vessel forwards, notwithstanding its immense pressure of perhaps thirty-five tons on every square foot of the slide. If any imperfection in the track should arrest the progress of the ship, it will soon gain such adhesive power as to render its removal extremely arduous. A tremulous agitation is the only expedient to urge forward the ponderous mass. Hence the reason of the sudden falling down of weak or decayed structures. They are upheld long beyond the term of equilibrium, by the rooted adhesion of their parts; but any accidental shock dissolves this union, and the whole pile is precipitated to the ground.

The adhesion arising from prolonged contact, explains likewise some striking facts in the practice of the mechanical arts. Hence an axe, struck into a heavy block of wood, will, notwithstanding its wedge form, take such a firm hold as to lift it up. For

the same reason, a round iron bolt, with a slight degree of taper, being driven into a hole bored in a stone, will safely raise the ponderous mass, and it is only disengaged by the smart blow of a hammer. The binding action of nails depends on the same principle. Their adhesion to wood is not exactly as the quantity of surface compressed, but seems to be nearly proportional to the square root of the cube of the depth of insertion. Thus, to extract an iron nail from a fir plank, requires a force denoted by $220\sqrt{d}$, in pounds averdupois; d marking in inches the depth of penetration. In dry elm, the tenacity of cohesion is about a fourth part greater.

The friction of pivots is peculiar. Being so near the axis, it is never affected in any degree by the swiftness of rotation. This friction is not exactly proportional to the pressure, but nearly a mean between the pressure and its square root. Thus, if one pivot weighs 10 and another 810 grains, the friction of the latter will be 28, and not 81 times greater. The probable reason of this discrepancy is, that the loaded pivot occasions a depression at the spot on which it turns, which incurvation of the plate diminishes the friction. The tapering of the pivot lessens this still more. It should not exceed an angle from 30 to 45 degrees, and the cone may be reduced even to 10 or 12 degrees, if the weight should not exceed an hundred grains. The retardation

which a steel pivot suffers, varies according to the nature of the substance in which it turns. On garnet it loses only 10 parts of its motion, on agate 12, on rock-crystal 13, on glass 18, and on steel 23.

If a hard body rub against a very soft substance, the friction will no longer continue uniform, but increase remarkably with the celerity. Thus, when the sliders of the bench are covered with lists of cloth, the loaded sledge being mounted as before upon rules of wood or metal, the motion incited by a predominant weight, so far from accelerating, is quickly retarded, and would soon become entirely extinguished. But the impediment now created partakes more of the nature of the resistance of fluids; it consists in the consumption of force occasioned by the continual depressing of the spongy and elastic substance of the cloth with the celerity of the passage of the sledge. The effect of such obstruction, being compounded of the quantity of matter displaced and of the velocity of its removal, must therefore be proportional to the square of that velocity. If a denote the incipient friction, and v the velocity, the corresponding friction will be expressed by $a+mv^2$, where m is a coefficient to be determined by experiment.

When a cylinder is made to *roll* upon a plane surface, it encounters a new sort of obstruction,

quite distinct in its character, and generally much inferior to that of Friction. These retarding forces are strikingly contrasted in the rolling and sliding of different cylinders of wood down inclined planes The cylinders will not begin to slide lengthwise, though disengaged by percussion, unless the slope of the plane exceeds 10 degrees; but they will roll easily at much smaller angles. The inclination of 4 degrees will be sufficient to enable a cylinder of elm, one inch in diameter, to roll down an oaken board, and an angle of 3 degrees will decide the rolling of a cylinder of lignum vitæ of the same dimensions. But a single degree of slope will make an elm cylinder of four inches diameter begin to roll, and three-fourths of that angle will occasion the rolling of a like cylinder of lignum vitæ. The loss of force in the act of rolling is hence inversely proportional to the diameter of the cylinder.

If adhesion were confined to the mere line of contact, it could have no effect whatever in hindering, on a horizontal plane, the revolving motion of a cylinder. But this power, subsisting an instant after contact has taken place, may be conceived as constantly drawing it down at a certain distance behind. Thus, if A (fig. 117.) be the contact of the cylinder and B the point where the adhesion is mainly exerted, its efficacy to restrain the rolling of the cylinder will evidently be as AB to AC. On the supposition that AB is constant, this retarding force is in-

versely as the radius AC. In the case of elm, the distance AB of vigorous cohesion, must be the 28th part of an inch; but in that of lignum vitæ it is only the 40th part.

Rollers, by removing friction, are extremely useful in practice of Mechanics. The only obstacle is the necessity of frequently replacing them under the load; but this inconvenience may, in a great measure, be obviated by substituting cylindrical axles fixed to large wheels.

Balls appear to roll still more easily than cylinders of the same diameter, though they have never been subjected to any nice experiments. On a large scale, however, they are often employed most advantageously in aiding the removal of enormous masses.

If the diameter of a cylinder in inches be denoted by d, that portion of the incumbent weight which is required to maintain the rolling of a cylinder of elm upon a plane of oak, will be expressed by $\frac{1}{14d}$, but the retardation of a cylinder of lignum vitæ is only $\frac{1}{20d}$. Balls of the same dimensions and materials would, in rolling upon such a plane, probably consume only two-thirds of those measures of force.

From some trials made by Tredgold with small models, it would follow, that the obstruction which cast-iron wheels encounter in rolling along an iron

railway, employing the same notation, amounts only to the $\frac{1}{23d}$ part of the load. By the application of conditional equations, we are enabled to infer, from three measures incidentally furnished by Wood, of the retardation along a cast-iron railway, encountered by an ordinary waggon carriage, with ponderous wheels of 35 inches diameter, that the obstruction from the rolling alone amounts to the 235th part of the whole incumbent pressure, which is more than three times greater than what might be expected from the action of small models. But when the rims of the wheels are case-hardened, this obstruction is reduced to about the 650th part of the load, and thus very nearly corresponds with the experiments in miniature. The difference of celerity probably modifies the effect of rolling.

In the composition of machines, we should avoid attrition as much as possible, and prefer rolling movements, wherever circumstances will admit. But friction itself may be diminished indefinitely, by bringing its action nearer to the centre of revolution, or by transferring its influence from the circumference, to the axle, of a wheel. Thus, the axle of the wheel AB (fig. 118.), sustaining a pressure, being made to roll on the summit of the circumference of CB, its friction against the gudgeon at A

will be transferred to the axle C. Supposing the axles to have the same diameters, the influence of this friction in retarding the machinery must, from the principle of the lever, be now diminished in the ratio of CB to CD. For the sake of convenience the axle of the incumbent wheel A is generally planted at the intersection of two equal wheels C and E. The weight, considered as pressing vertically, becomes hence shared between them, and the effect of friction is reduced, in the ratio of their diameter, to that of their axles. These axles may yet be made to roll each of them on the circumference of an equal pair of wheels, as in fig. 119, and the friction will be again diminished in the same ratio. The wheels thus introduced, for the purpose of approximating the operation of friction to the centre of motion, are called *Friction-Wheels*. If their diameters, or rather the vertical chords, were ten times greater than the diameters of their axles, two wheels would reduce the friction to one-tenth, four additional wheels to one-hundredth, and eight wheels more would contract the friction to the thousandth part. Scarcely any obstruction would then be left but that of rolling, which is comparatively inconsiderable.

Since the primary axle does not rest precisely on the summit of the friction-wheel, but leans against a point beyond this, and therefore divides its oblique pressure between two equal wheels, the friction is

really diminished in a ratio somewhat less than that of the diameters of the wheel and axle. Let A and B be the centres of those wheels, and C the centre of the incumbent axle: join ADC, and let fall the perpendicular DE upon AB. If AD express half of the load borne by the axle, DE will denote the pressure exerted at D, or by the secondary axle at F. Wherefore the friction will be diminished in the ratio of the perpendicular DE to CD. This perpendicular, in most cases, however, differs little from the radius AD itself.

The theory of friction wheels explains likewise the advantages derived from the construction of wheeled carriages. In these machines, the draught is facilitated by three distinct circumstances: 1. The excessive obstruction, which the rim of the wheel would encounter if dragged along the road, is changed to the very inferior friction of the axle against the bush of the nave. 2. This reduced friction has its influence in impeding the progress of the carriage, still farther diminished, in the ratio of the diameter of the wheel to that of the axle. And, 3. The dimensions of the wheel enable it to surmount easily any obstacle which may occur; the effect being the same as if it were drawn over an inclined plane, from its point of contact to the top of the prominence. The friction of the rim of a locked wheel, even on the smoothest road, might perhaps exceed the half of the whole load; but the friction of the axle in its

box, which is substituted for it, would amount only to the eighth part when iron is inserted in a copper bush, and scarcely the seventh part when oak turns in lignum vitæ, the grease in both cases being worn smooth. The power of this friction within the nave, in retarding the motion of the carriage, must be directly as the diameter of the axle, and inversely as the height of the wheel. A large wheel and a small axle are hence the most advantageous. For this reason, an iron axle, though it has twice as much friction as an oaken one of the same dimensions, may be preferred for its smallness. In carriages rightly fitted and carefully greased, the whole friction seldom exceeds the thirtieth, but need not amount to the hundredth part of the load.

To assign the force expended in overcoming an obstacle, let the wheel (fig. 120.) touch A the horizontal line of traction; if it meet a protuberance BD, it must be lifted over this with the progressive motion AB: the draught is therefore to the load, as AB to BD, or, from the property of the circle, as BE or AF to AB. But $AF^2 : AB^2 :: AF : BD$, and consequently AF is to AB in the subduplicate ratio of AF, the diameter of the wheel, to BD the height of the obstacle. Large wheels are hence best adapted, not only for diminishing the effect of friction, but for surmounting the inequalities of the road.

Cylindrical wheels will answer best on level roads; for their breadth, though it has no effect on the

quantity of friction, may yet lessen their sinking into the ground. In hilly and uneven roads, a slight declination of the spokes, called *dishing*, will, like an arched vault, give strength to the wheel, and prevent its accidental twisting.

The obstruction which a loaded carriage has to overcome, when drawn along a smooth level road, is always composed of two very distinct portions; first, the *attrition* of the axle against the box of the nave, and, secondly, the *adhesion* of the rim of the wheel as it rolls over the yielding surface of the ground. These elements of retardation, though quite different in their nature, have been often confounded under the general term *friction*. But it would evidently be rash to infer the properties of adhesion from experiments made on ordinary friction; nor is it ascertained whether this adhesion be constant or variable, and whether, by great celerity, it lessens, like attrition, or augments, as in the resistance of fluids.

The attrition of the nave is proportional to the weight of the carriage and its load, but the adhesion to the ground is proportional to the whole aggregate mass, including the wheels and axle. Both of these sources of retardation are diminished with the height of the wheel, but the attrition is likewise diminished by the smallness of the axle. In ordinary cases, we may reckon the effect of adhesion at least five times greater than that of attrition. Thus, a horse travelling at the rate of three miles

an hour, exerts a pull of 81 lbs. in drawing a loaded cart: of this force only 12 lbs. are spent on the attrition of the axle, but the remaining 69 lbs. are consumed on the adhesion of the rim of the wheel to the level road. Supposing the absolute friction of the nave to be one-eighteenth of the incumbent weight, and the diameter of the axle the fifteenth part of that of the wheel; then $18 \times 15 \times 12 = 3240$ lbs. or $28\frac{5}{4}$ cwts, leaving about 25 cwts. for the load. The attrition being thus equal to the 270th part of the pressure, the adhesion along the road may be taken at the 50th part of the entire charge; which gives $50 \times 69 = 3450$ lbs., being an excess of 210 lbs. for the additional weight of the wheels and axle.

It is matter of just regret, that the relation between the influence of attrition and of adhesion, in retarding the motion of wheel-carriages, has never been ascertained with any degree of precision; yet nothing could be more easily determined than the comparative expenditure of power, occasioned by the operation of those distinct causes: Let the retardation of a loaded carriage, with ordinary wheels, be found; and next that of a light carriage, mounted on very heavy wheels, perhaps of solid cast iron. This obstruction might be examined on different roads and railways, the rims of the wheels being shod with a thin ring of wood, or of iron, cast or hammered, or case-hardened. Since the attrition is proportional only to the weight of the carriage, and of

its load, while the adhesion is produced by the whole collective pressure of carriage and wheels, the several results would afford data for distinguishing the blended shares. These experiments, performed on a scale of sufficient magnitude, would throw a clear and steady light on the principles of the formation of roads, and the construction of carriages; and might therefore be deemed of national importance. Similar inquiries ought certainly to have been instituted by the proprietors of railways, before they ventured, with such limited and imperfect information, to embark in their expensive schemes.

Though ordinary friction is generally diminished by rapid motion, the attrition of the nave cannot be sensibly affected by the swiftness of rotation, since the rubbing surface being so near the centre, its celerity is proportionally reduced. It is different altogether from the adhesion of the rim of the wheel in rolling along the road, which bears an affinity to the resistance of soft and yielding substances, and therefore increases probably after some ratio of the velocity.

In the drawing of carriages, it is of the utmost consequence to blunt or avoid the shocks occasioned by the inequalities of the road. Such concussions waste the power of draught, in proportion to the celerity of their deranging influence. The application of springs to a carriage, by restraining and enfeebling those irregular movements, must therefore

be an unprofitable expenditure of power. The advantages thus procured are greater in rapid travelling. It has been found that a carriage mounted on springs may be drawn along a rough road at the rate of two miles an hour, with three-fourths of the force necessary to effect it without springs. At the rate of three miles and a half an hour, the corresponding force is only two-thirds; but, at the velocity of five miles, it becomes reduced to one half.

But the motion of machines, in which pulleys are concerned, suffers another obstruction, arising from the stiffness of the ropes, or the force consumed in continually bending them into a new direction. This impediment, in the case of a single strand, must depend on the rate of inflexion, and will therefore follow the inverse proportion of the diameter of the pulley or revolving cylinder. If those strands were laid parallel, their combined stiffness would be as their number, or as the square of the diameter of the rope which they form. In practice, however, this obstructing force appears adapted to a slower ratio. A certain portion of the stiffness of a rope is owing to its peculiar tenacity, but the greatest part of it proceeds from the tension occasioned by the appended weight. But this is liable to much diversity, which can be ascertained only by experiment.

Two methods have been adopted for exploring

the stiffness of ropes. The first was proposed by Amontons, which consists in a strong beam, having two pulleys fixed to it, and over them a long rope is passed; and both ends being lapped about a cylinder, are then fastened to a frame loaded with heavy weights. See fig. 121. A slender silken line is next wound about the cylinder, having an appended scale to hold small weights. These being added, till the cylinder begins to descend, will evidently measure half the stiffness of the rope, since they are exerted in turning it not about the centre, and at the distance of the radius, but at the whole length of the diameter.

This mode of experimenting was pursued on a large scale by Coulomb, and the results were confirmed by a simpler method of investigation which he afterwards employed. The cylinder was now placed upon two parallel and horizontal edges, having the rope coiled about it, and equally loaded at both ends; a weight was now added on the one side, just sufficient to put the cylinder in motion; and the small obstruction in rolling obtained by a previous experiment, being deducted from this, leaves an accurate measure of the stiffness of the rope. See fig. 122. An approximation, reduced to the English standard, may easily be deduced from the different facts ascertained by Coulomb. Let D express in inches the diameter of the pulley, *d* that of the rope, and P in pounds averdupois the,

weight applied to it; then will $d^{\frac{5}{3}}\left(\frac{140+3P}{10D}\right)$ denote very nearly the stiffness of a new hempen rope. In the case of old ropes, the fraction $\frac{7}{5}$ is more correctly the index of d. Small cords have their stiffness diminished by wetting, but large cords are thus rendered somewhat stiffer. The application of tar adds about one-sixth part more to the stiffness of ropes.

But we sometimes need to create friction, as well as to lessen or extinguish it. All machinery is gradually set in motion, and cannot be stopped suddenly without incurring imminent danger. As the celerity was progressively acquired, so to insure safety it must be slowly retarded. Friction is the simplest and most effectual means of arresting all motion. For this purpose, its action is augmented, by transferring it in complex machines from the centre to the circumference of revolution. But simpler modes of exerting the retarding force of friction are frequently adopted in practice. The attrition of a rope against a round post is the common way of stemming the motion of a ship.

This peculiar species of friction deserves a distinct investigation. Let a flexible cord ABC (fig. 123.), supporting a weight P, and drawn by a force Q, wind about the circumference of a fixed pulley or cylinder, whose centre is O. This cord must press against the cylinder at any point, in proportion to

its degree of inflexion. Conceive the arc ABC to be divided into elementary portions, and the tension of the cord in the direction of a tangent at B will be to the pressure of the element bB, as the radius OB to bB. If $1 : m$ denote the ratio of the pressure of bB to its friction, $\frac{m.bB}{OB}$ will express the proportional increase of tension from b to B. Suppose the successive tensions to be represented by P, P′, P″, &c. then $P' - P = P.\frac{m.bB}{OB}$, and $P'' - P' = P'.\frac{m.bB}{OB}$, &c. Wherefore, the hyperbolic logarithm of $\frac{Q}{P} = \frac{m.AC}{OB} = m$. angle AOC. Hence, assuming equal angles about the centre O, the corresponding tensions will form a geometrical progression. Thus, assuming the arc AC to be a quadrant, if a weight P of one pound balanced a traction of two pounds at C, it would uphold four pounds at the end of the semicircumference, eight pounds at three quarters of a circuit, and sixteen pounds at a complete circumvolution; but the same power would, at the end of two, three, and four turns, &c. sustain no less than 256 lb., 4096 lb., 65,536 lb. The progression thus augments with extreme rapidity; and after a few turns of the cord, a very small weight will be sufficient to support the most enormous load. This efficacy is nowise modified by the size of the cylinder, about which the line is coiled, but depends en-

tirely on the number of its circumvolutions. Hence the firmness procured by the lapping of cordage; and hence, likewise, the principle of various fire-escapes, by which the celerity of descent from a great height is retarded, and the shock against the ground rendered soft and easy.

In all mechanical structures, each member should not only be able to resist the strain under which it is constantly subjected, but should also be capable of withstanding those occasional shocks to which it is ever exposed. The stability of the fabric depends no doubt mainly on the strength of its several parts; but the same degree of strength will exert very different forces, according to its direction, and the elasticity that accompanies it. In combining the materials, therefore, the power brought into action should always be proportioned to the respective strain. The skilful disposition and arrangement of the component members, hence contribute often more essentially than their size or absolute strength to the security and duration of any structure.

In estimating the strength of materials, they are generally considered as having a prismatic or columnar form. But they may likewise be exposed either to a longitudinal or a transverse strain. Their strength is therefore exerted in four different ways: 1. In sustaining a longitudinal tension; 2. In withstanding a longitudinal compression; 3. In resisting

a transverse pressure; and, 4. In opposing the act of twisting or wrenching.

1. *Longitudinal Tension*.—The tension which a stone pillar, a bar of metal, a beam of wood, or even an hempen rope, can bear when pulled lengthwise, must evidently depend on the cohesion of any cross section. As the material stretches out, the longitudinal attraction of the particles becomes augmented. This increase, at first, is proportional to the dilatation, but it afterwards advances very slowly, and a small additional strain is then sufficient to produce that limit of extension which occasions total fracture or disruption of the column. Its length will nowise affect the utmost strain which it can bear, this being determined merely by the smallest cross section where the dislocation of the particles will take place.

Let ZA (fig. 124.) be a prism stretched in the direction of its length. The particles of the section B will be pulled from those of the section C, till an attraction be generated equal to the whole tension. But the particles of the section C must settle in equilibrium, and are consequently drawn back by an equal force. In this acquired position, therefore, they must attract the particles of the section D with the same force. The original tension is thus transferred successively to the extremity Z, where it is finally exerted, the effects of its action being neutralized in all the intermediate sections.

The cohesive power thus evolved, is hence the accumulative attraction of all the particles in any section. The corresponding longitudinal distension is at first proportional to it, but afterwards increases in a more rapid progression. Thus, a bar of soft iron will stretch uniformly, by continuing to append to it equal weights, till it be loaded with half as much as it can bear; beyond that limit, however, its extension will become doubled by each addition of the eighth part of the disruptive force. Supposing the bar to be an inch square, and 1000 inches in length, 36,000 lb. averdupois would draw it out 1 inch; but 45,000 lb. will stretch it 2 inches, 54,000 lb. 4 inches, 63,000 lb. 8 inches, and 72,000 lb. 16 inches, when it would finally break, the extension being now increased eight times beyond its ordinary rate.

Let AB (fig. 125.) be a prism or bar of any material, and suppose its prolongation BC to express the whole longitudinal force exerted, in causing the small extension A*a*. While the length of this bar BC continues the same, it is evident that *a*A must be proportional to B, the distraining weight BC. Make, therefore, *a*A : BC : : AB : CD; or, alternately, A*a* : AB : : BC : CD, and CD must be constant. Since BC now bears the same relation to CD as *a*A to AB, any portion of CD will, by its weight, produce a corresponding distension of AB. Thus, a column of the thousandth part of the length of CD would extend AB one thousand part, and

the same weight acting by compression, would occasion an equal contraction. The column CD thus found, is called the *Modulus of Elasticity*; it depends entirely on the nature of the cohesive substance, and may be determined by a single experiment.

The Modulus of Elasticity, though of great importance, has been ascertained but in a few instances. That of white marble is 2.150,000 feet, or equal to the weight of 2,520,000 pounds averdupois on the square inch; while that of Portland stone is only 1,570,000 feet, corresponding, on the square inch, to the weight of 1,530,000 lb.

White marble and Portland stone are found to exert, on every square inch of section, a cohesive power of 1811 lb. and 857 lb.; wherefore, suspended columns of these stones, with the altitudes of 1542 and 945 feet, or only the 1394th and 1789th parts of their respective measure of elasticity, would be torn asunder by their own weight.

Of the principal kinds of timber employed in building and carpentry, the annexed table will exhibit the respective Modulus of Elasticity, and the portion of it which limits their extreme longitudinal cohesion.

Teak,	6,040,000 feet.	168th.
Oak,	4,150,000 feet.	144th.
Sycamore,	3,860,000 feet.	108th.
Beech,	4,180,000 feet.	107th.

Ash,	4,617,000 feet.	109th.
Elm,	5,680,000 feet.	146th.
Memel Fir,	8,292,000 feet.	205th.
Christiana Deal,	8,118,000 feet.	146th.
Larch,	5,096,000 feet.	121th.

A Tabular View may be likewise given of their absolute cohesion, or the load which would rend a prism of an inch square, and the altitude of the prism which would be torn asunder by the action of its own weight.

Teak,	12,915 lb.	36,049 feet.
Oak,	11,880 lb.	32,900 feet.
Sycamore,	9,630 lb.	35,800 feet.
Beech,	12,225 lb.	38,940 feet.
Ash,	14,130 lb.	42,080 feet.
Elm,	9,720 lb.	39,050 feet.
Memel Fir,	9,540 lb.	40,500 feet.
Christiana Deal,	12,346 lb.	55,500 feet.
Larch,	12,240 lb.	42,160 feet.

It is singular, that woods of such diversified structure should yet differ so little on the whole in the measures of their elasticity and cohesion. Specimens of the same sort will occur, which are sometimes as much varied as the several kinds themselves.

The modulus of the elasticity of hempen fibres has not been determined, but may probably be rec-

koned about 5,000,000 feet. Their cohesion is, for every square inch of transverse section, 6,400 lbs. The usual mode of estimating the strength of a cable or rope of hemp, is to divide by five the square of its number of inches in girth, the quotient expressing in tons the utmost strain it could bear. But a simpler computation is to double the square of the diameter of the rope. This estimate, however, applies only to new ropes formed of the best materials, not much twisted, and having their strands laid even. Yarns of 180 yards long are usually worked up into a rope of only 120 yards, and lose one-fourth part of its strength, the exterior fibres alone resisting the strain. But the register cordage of the late Captain Huddard exerts nearly the whole force of its strands, since they suffer a contraction of only an eighth part in the process of intertwisting.

The metals differ more widely from each other in their elastic force and cohesive strength, than the several species of wood or vegetable fibres. Thus, the cohesion of fine steel is about 135,000 lb. for the square inch, while that of cast lead amounts only to about the hundred and thirtieth part, or 1800 lb.

According to the very accurate experiments of Mr George Rennie in 1817, the cohesive power of a rod an inch square of different metals, in pounds averdupois, with the corresponding length in feet, is as follows :

Cast steel,	134,256 lb.	39,455 feet.
Swedish Malleable Iron,	72,064 lb.	19,740 feet.
English ditto	55,872 lb.	16,938 feet.
Cast Iron,	19,096 lb.	6,110 feet.
Cast Copper,	19,072 lb.	5,003 feet.
Yellow Brass,	17,958 lb.	5,180 feet.
Cast Tin,	4,736 lb.	1,496 feet.
Cast Lead,	1,824 lb.	348 feet.

It thus appears, that a vertical rod of lead 348 feet long, would be rent asunder by its own weight. The best steel has nearly twice the strength of English soft iron, and this again is about three times stronger than cast iron. Copper and brass have almost the same cohesion as cast iron. This tenacity is sometimes considerably augmented by hammering or wire-drawing, that of copper being thus nearly doubled, and that of lead, according to Eytelwein, more than quadrupled. The consolidation is produced chiefly at the surface, and hence a slight notch with a file will materially weaken a hard metallic rod. Hence the advantage of case-hardening. English malleable iron has 7,550,000 feet for its modulus of elasticity, or the weight of 24,920,000 lb. on the square inch, while cast iron has 5,895,000 feet or 18,421,000 lb. Of other metals, the modulus of elasticity is probably smaller, but has not yet been well ascertained.

2. *Longitudinal Compression.*—The compression which any column suffers, is at first equal to the

dilatation occasioned by an equal and opposite strain, being in both cases proportional to the modulus of elasticity. But while the incumbent weight is increased, the power of resistance likewise augments, as long as the column withstands inflexure. After it begins to bend, a lateral disruption soon takes place. A slender vertical prism is hence capable of supporting less pressure than the tension which it can bear. Thus, a cubic inch of English oak was crushed by the load of only 3860 lb., but a bar, of an inch square and 5 inches high, gave way under the weight of 2572 lb. It would evidently have been still feebler if it had been longer. On the other hand, if the breadth of a column be considerable in proportion to its height, it will sustain a pressure greater than its cohesive power. Thus, though the cohesion of a rod of cast iron, of the quarter of an inch square, is only 300 lb., a cube of that dimension will require 1440 lb. to crush it.

In general, while the resisting mass preserves its erect form, the several sections are compressed and extended by additional weight, and their repellent particles are not only brought nearer, but multiplied. This repulsion is likewise increased, by the lateral action arising from the confined ring of detrusion. The primary resistance becomes hence greatly augmented, in the progress of loading the pillar.

The most precise experiments on this subject are those of Mr Rennie. The weights required to crush

cubes, having the dimension of the quarter of an inch of certain metals, are as follow:

Iron cast vertically,	11,140 lb.
Iron cast horizontally,	10,110 lb.
Cast Copper,	7,318 lb.
Cast Tin,	966 lb.
Cast Lead,	483 lb.

Cubes of an inch are crushed by the weights below:

Elm,	1,284 lb.
White Deal,	1,928 lb.
English Oak,	3,860 lb.
Craigleith freestone,	8,688 lb.

Cubes of an inch and a half, and consequently presenting a section of $2\frac{1}{4}$ times greater than the former, might be expected to resist compression in that ratio. They are crushed, however, with loads considerably less.

Red brick,	1,817 lb.
Yellow baked brick,	2,254 lb.
Fire brick,	3,864 lb.
Craigleith stone, with the strata,	15,560 lb.
Ditto, across the strata,	12,346 lb.
White Statuary Marble,	13,632 lb.
White-veined Italian Marble,	21,783 lb.
Purbeck Limestone,	20,610 lb.
Cornish Granite,	14,302 lb.
Peterhead Granite,	18,636 lb.
Aberdeen Blue Granite,	24,536 lb.

These facts show the comparative firmness of different materials; but it is to be regretted, that such results are not of much practical use, since they are confined to a very narrow scale, and applicable only to cubical blocks. While the breadth remains the same, the resistance appears to depend on some unascertained ratio of the altitude of the column. Nay, the absolute height itself has probably a material influence on the effect. Thus, from some experiments made in France, it appears, that prisms of seasoned oak, two inches square, and two, four or six feet high, would be crushed by the vertical pressures of 17,500 lb., 10,500 lb., and 7,000 lb.; but, if four inches square, and of the same altitudes, they would give way under loads of 80,000 lb., 70,000 lb., and 50,000 lb. In the first set of trials, the mean cohesive power amounts to 130,000 lb., and in the second to 520,000 lb. The vertical support is therefore greatly inferior to these limits. When the length of the pillar exceeds 36 times its breadth, the resistance to longitudinal compression appears to be diminished 18 times. Mr Rennie estimates, that the granite which composes the great arch of the new London Bridge, would be capable of supporting four tons upon every square inch of its upper surface.

To ascertain the form of a vertical column, the best fitted to support a load, is a problem in archi-

tecture of very considerable difficulty, which seems to involve the peculiar structure and internal composition of the material. If the pressure were applied to the extremity, and in the direction of the axis, the particles under it would become condensed perpendicularly, and likewise accumulated towards the sides. The repulsive energy opposed to the incumbent weight would come to be augmented by a certain oblique or lateral action. The column is therefore made to swell out below, the curve of an ellipse forming the preferable outline, though the enlargement of the diameter at the bottom seldom amounts to a fourth part. Practice has varied, however, according to the different perceptions of the beauty of form.

If the centre of pressure should act beyond the axis, it must evidently have a tendency to bend the column. But the flexure having once begun, the deranging influence will, from the property of the lever, be exerted with increasing energy. The altitude of a column thus only contributes to its weakness. For the same reason, the walls of a house should taper towards the top, forming a sort of inverted wedge, as the chief strain is accumulated near the bottom.

3. *Transverse* or *Lateral Pressure*. In estimating the strain occasioned by a load resting upon a horizontal beam, this may be viewed as composed either of longitudinal fibres, which oppose a greater

resistance, if set on its edge, than when laid on its face. Suppose the beam AB (fig. 126.) to have one end firmly implanted in a wall GH, while a vertical pressure is applied at the other end. This beam, sinking under the load at B, may be conceived to turn on the lowest point at A as a fulcrum; consequently the particles of the vertical section AC will be forced into the oblique position AD, each of them being turned aside through a space proportional to its distance from A. The strain exerted at the end AC will therefore be the result of the aggregate dislocations of all the particles of the section. When the breadth and length of the beam remain the same, this accumulate strain must evidently be proportional to the area of the triangle CAD, and consequently to the square of the depth AC. But when the breadth of the beam is taken likewise into account, the triangle of tension becomes converted into a wedge, and the strain hence follows the direct ratio of that breadth. Omitting the weight of the beam, and assuming its depth and breadth as constant, the tension of any particle at C may be considered as acting against the short arm AC of the rectangular beam CAB, and withstanding the load suspended at B. The weight thus resisted by the cohesion of the beam, is therefore inversely as the length AB. Combining all the circumstances now together, we may conclude, that the strength of a beam firmly inserted in a wall, or its power to resist a pressure

at its remote extremity, is compounded of the direct ratios of its breadth and of the square of its depth, and the inverse ratio of its length. Thus, a beam having the same length and breadth as another, but twice the depth, is four times stronger; and a beam of the same depth and breadth, and double the length, is only half as strong. Hence also, a beam, whose depth is triple its width, will sustain a load three times greater. For the same reason, a square prism will have its strength inversely as its length and the cube of its thickness.

In general, the resistance of a beam of any form, but of a given length, to a cross strain, will be the same as if the whole power exerted were collected in the centre of gravity of each section. Thus, the strain of a triangular prism may be conceived as concentrated in a point at one-third of the distance of the perpendicular from the vertex to the base. Such a prism is therefore twice as strong set on its edge as when laid on its side.

This simple investigation, which we owe to the illustrious Galileo, though partly hypothetical, may be regarded as a near approximation to the truth. It is not only of essential service in improving the practice of carpentry, but sheds a clear light over the economy of nature in the structure of animals and vegetables. Reeds and other herbaceous plants derive their power of resisting the force of the wind from the subdivision of their length into moderate intervals by hard knots. But they acquire still great-

er strength from their hollow or tubular form; for the matter which they contain, being thus removed to a greater distance from the fulcrum, exerts its cohesion with proportionally more effect, in withstanding any lateral impulsion. The bones of animals are likewise rendered much stronger by their fistular structure, and their partition into short members connected by large compact joints. Hence, in the construction of fine mechanical and astronomical instruments, hollow brass cylinders are now preferred, on account of their stiffness and lightness, to solid pillars.

If a beam be supported horizontally at both ends, and loaded in the middle, the pressure will be equally shared between the props, and the effect must evidently be the same as if it had been fixed at the middle, and each end pulled upwards by half its load. The breaking weight is consequently double of what would be required to tear a beam, of half the length, implanted in a wall. According to the principle of Galileo, therefore, this limit is inversely as the length of the beam, and directly as the breadth and the square of the depth.

This result is, on the whole, confirmed by the numerous experiments which the celebrated Buffon performed between the years 1738 and 1746. Thus, reducing all the quantities to English measures, an oaken beam, 4 inches square and 10 feet long, broke

under the weight of 4015 lb.; and another beam of the same wood 8 inches square and 20 feet long, was broken by a load of 16,700 lb. The latter, being twice as thick, should have been eight times stronger with the same length; but the length being doubled, again reduced the excess to four times. Now, $4 \times 4015 = 16{,}060$, or very nearly 16,700. Beams of 5 inches square, and of 7, 14, and 28 feet long, were broken with loads of 16,060 lb., 7460 lb., and 2472 lb., being nearly inversely as the respective lengths. In general, if a denote the depth, b the breadth, and l the length of a beam of oak in feet, those experiments will be represented with tolerable accuracy by the formula $\left(\frac{a^2b}{l}\right)$ 1,200,000 lb. If, instead of a load exerted, the weight of the beam itself be supposed to act at half the distance, then, 52 lb. being the weight of a cubic foot of oak, $\frac{a^2b}{\frac{1}{2}l} = 1{,}200{,}000 = 52abl$, or $l^2 = 46923.a$ and $l = 217 \sqrt{a}$. A horizontal plank of oak, 3 inches deep, and 108 feet long, would hence sink under its own weight.

The same agreement may be remarked in Mr Rennie's experiments on the transverse strain of cast-iron bars 32 inches long, and $9\frac{1}{2}$ lb. weight. Thus, a bar of an inch square, resting on horizontal props at its end, bore a load of 1086 lb. at the middle, yet the half of it supported 2320, or a little more than double. Another bar of the same weight, but

2 inches deep, and half an inch thick, sustained, at the same interval between the props, likewise the double, or 2186 lb. Now, this bar had its strength quadrupled by doubling the depth, but reduced again to the double by the bisection of its breadth. A bar of still the same section, but having a depth of 3 inches combined with a breadth of one-third of an inch, supported, as might be expected, three times the load borne by a square inch bar, or 3588 lb. A bar which had an equilateral triangle of an inch for its section sustained the weight of 840 lb. when planted on its angle; but when rested on its base, it bore, as theory would indicate, 1437 lb., or very nearly the double.

If the dimensions of a rectangular bar of cast iron be expressed in feet, a, b, and l, denoting as before the depth, breadth and length; those experimental deductions will be represented with sufficient precision by the formula, $\left(\frac{a^2b}{l}\right)$ 5,000,000 lb.

From these principles, we derive the solution of a useful problem in carpentry,—to cut the strongest rectangular beam out of a given cylindrical tree. This, it may be shown, will be effected, by making the square of the depth of the beam double the square of its breadth. An easy construction is hence obtained. Let AB (fig. 127.) be an oblique diameter of a circular section; trisect it in the points C and

D, draw the opposite perpendiculars ED and FC, and complete the rectangle AEBF, which will represent the end of the beam. For it may be proved, that, of all the rectangles which could be inscribed in the given circle, the one now determined gives the solid $EG^2.GH$, expressing the strength of any section, a *maximum*.

Suppose a beam AB (fig. 128.) laid in a horizontal position upon two props, had a load applied at an intermediate point C. From what was demonstrated in Statics, the pressure must be shared between those props in the inverse ratio of its distance from them. The pressure exerted at B may therefore be represented by AC, and the effect is just the same as if the extremity B were pushed upwards by a force AC. Hence the strain at C, communicated by the lever BC, is as the rectangle under AC and BC, the two segments. If a semicircle were described upon AB, the square of the perpendicular CD would thus be proportional to the strain at C; or if AB be bisected in O, the excess of the square of OA above OC would be proportional to that strain.

When a uniform horizontal beam AB (fig. 128.) bends merely under its own weight, the strain exerted at any point C is the same as what would be produced, by half the weight of one segment BC acting at the end of a lever equal to the other AC. It is obvious, that the beam must press equally at

both ends, and that the loads of the segments AC and BC may be viewed as concentrated at their middle points I and K. Suppose the beam were sustained at C by an opposite pressure from below; it would now be held in equipoise against this fulcrum by vertical forces exerted at I and K, which are therefore inversely as CI and CK, or directly as BC to AC. The weight of the segment BC, acting at AI half the length of AC,—or half the weight of BC acting at the whole length AC, will hence occasion half the strain at C, which results from the aggregate pressure of the beam itself, the other half being furnished by the equal effort of half the weight of the segment AC acting at the distance BC.

In the hypothesis of Galileo, the prism or beam is conceived to be absolutely inflexible, and to give way only at the section of fracture. But all materials are, at least within certain limits, really elastic, and bend progressively under an increasing load, till they finally break. To determine with accuracy, therefore, the effect of lateral or transverse pressure against a bar, it becomes necessary to take the influence of incurvation into the estimate.

The principles of Dynamics show, that any weight or vertical pressure, acting upon a small portion of an elastic plate, is to the longitudinal strain which it occasions, as the length of that portion to the radius of curvature. Thus, if D*d* (fig. 129.) the element

of the curve, represent the share of the weight which this minute part sustains, the normals DO and *d*O being drawn, the common distance DO of the centre of osculation will express the direct tension exerted at C. Any inflected beam would consequently form itself into an arc of a circle, if the action of the pressure were conceived to be equally distributed over it. In that case, the length of the arc would be to the radius of its circle, as the whole weight borne is to the uniform longitudinal tension.

Suppose a thin bar (as in fig. 129.) to have one end firmly fixed in a wall, and some weight attached to the other. The action of this power in bending the plate, will evidently not be diffused equally over its length. The inflecting energy must, from the property of the lever, be here proportional to the horizontal distance of any point. Consequently, the curvature or degree of incurvation at D will vary with the ratio of DE. The plate, therefore, bends at first quickly, then more slowly, and approaches to a straight line at its extremity A. Such is the nature of the *Elastic Curve*, which has its radius of curvature DO always inversely as the corresponding ordinate DE.

The Elastic Curve at its origin coincides nearly with an Harmonic Curve; but if its inflexure be small, it may be viewed as a Cubic Parabola, the distance DE or FB being always proportional to the cube of DF or BE.

If the projecting plate, however, be only bent by its own weight, the effect of this pressure, at any point, will evidently be the same, as if half the weight of the remote portion were collected at the end. Wherefore, the tension exerted at D, which marks the incurvation, being produced by a weight DB or DE acting on the lever DE, may be considered as proportional to the square of DE. In the curve thus formed, the ordinate BE will hence be as the fourth power of the absciss BE. It might also be shown, that the extreme depression caused by this diminishing incurvation is, in the case of an appended load, the third part; but in the case of its own weight, only the sixth part of what an extension of the same curvature from the fixed end would have occasioned.

When a horizontal beam ABC (fig. 128.) is freely supported at both ends, each portion of it, though pressing equally downwards, must yet produce a vertical stress, proportional to the rectangle under the corresponding segments, and consequently the radius of curvature at C will be inversely as AC×CB, or the square of CD. Find CE a third proportional to some multiple of the diameter and CD, and connect the several points E by a curve. This curve will represent very nearly the form which the beam would spontaneously assume. The depression CE, being hence proportional to the square of CD, will express the effort of an uniform tension to with-

stand the lateral stress. This same line CE will nearly mark the curvature at E, which thus continually diminishes or flattens from the middle point F to the extremities A and B. The curve hence traced approximates to the Harmonic Curve, which likewise has an equable tension, its graduating curvature being proportional to the ordinates.

If we view this curve as only slightly bent from its horizontal base, and estimate the amount of its successive inflexions, it will follow, that the depression OF of the middle of the bar is equal to the five-sixth parts of what would have obtained, had the same curvature extended through the whole length. From the actual depression, therefore, the radius of osculation is easily computed. Thus, in fig. 130, if the incurvation of the beam were uniform, the radius of its circle would be equal to the square of HB, divided by double the corresponding depression. But the real depression HK being only five-sixths of that quantity; wherefore the radius of osculation at H $=\frac{5}{6}\times\frac{HB^2}{2HK}=\frac{5HB^2}{12HK}=\frac{5AB^2}{48HK}$, or equal to five times the square of the length of the beam, divided by forty-eight times its middle depression.

In the case of an horizontal beam supported at both ends, but depressed by its own weight, the upper surface becomes concave, and the under surface convex. The particles of the upper surface EF (fig.

130.) are therefore mutually condensed, while those of the under surface CD are distended; in a certain intermediate curve AB, the particles are not affected longitudinally, though bent from their rectilineal position. This curve of neutral action may be assumed in the middle of the beam. The attractive effort of the particles stretched over the under surface CD to approximate and regain their usual position, produces a vertical thrust measured by the radius GO. On the other hand, the repulsion exerted by the particles compressed along the upper surface EF, occasions a perpendicular detrusion marked by the radius IO. The excess GI of the former force above the latter, therefore, indicates the weight which would be sustained at I, by the predominating action of the lower surface.

If M denote the modulus of elasticity, it is evident, that the longitudinal tension of the fibre GD will be $M.\frac{GL}{LD} = M.\frac{GH}{OH} = M.\frac{48GH.HK}{5AB^2}$; but the vertical thrust produced by the strain is $M.\frac{48GH^2.HK}{5AB^2}$. Of this, the variable part GH^2, or the square of the distance from the neutral position H, represents the element of the momentum of the triangle GHL, which may be considered as accumulated in the centre of gravity. The whole amount then of the expression, taking the action of the fibres equally distant on either side from H, is

$M.\frac{32GH^3.HK}{5AB^2}$. Now, this effort is equal to the pressure which would be produced by the weight of HD, or the fourth part of the beam, acting at a distance HB; whence $M.\frac{32GH^3.HK}{5AB^4}=GH.HD^2$, or $M.\frac{32GH^2.HK}{5AB^4}=HD^2$, and doubling GH and HD, $M.\frac{32GI^2.HK}{5AB^2}=AB^2$; wherefore, $M=\frac{5AB^4}{32GI^2.HK}$. The modulus of elasticity may thus be found, *by dividing five times the fourth power of the length of a beam, by thirty-two times the product of its spontaneous depression into the square of the depth.*

In the *Experimental Inquiry into the Nature of Heat*, it is incidentally observed, that a white deal 138 inches long, and .45 of an inch deep, suffered a depression of $2\frac{1}{2}$ inches, by its own weight. Here $M.=\frac{5(138)^4}{32(.45)^2(2.5)}$, which in round numbers is 111,936,000 inches, or 9,328,000 feet. Hence the spontaneous depression of any horizontal beam is directly as the fourth power of its length, and inversely as the square of its depth. Thus, a fir plank of 10 feet long, and 1 inch deep, will bend $\frac{3}{10}$th of an inch; another of the same depth, but 20 feet long, would bend 4.8 inches; while a third beam, of still the same depth, but 30 feet, would sink no less than 24.3 inches. If this beam had the depth of 3 inches,

the depression would be diminished nine times, and would still have been three times less than the proportion of the general dimensions. The depression of a beam hence increases faster than its length.

It likewise follows, that the radius of spontaneous curvature is directly as the square of the depth, and inversely as the square of the length. Similar beams, therefore, assume always the same curvature.

The spontaneous depression being in most cases very small, the Modulus of Elasticity may be computed more easily and correctly, from the augmented depression occasioned by suspending a load at the middle of the beam. Let λ denote the length of a bar of an inch square that weighs one pound, l expressing the length of the beam, and a its depth in inches. The weight of the beam corresponding to the breadth of an inch, is hence $= \frac{al}{\lambda}$, and the fourth part of this, or $\frac{al}{4\lambda}$, applied at the middle, would produce the same strain and incurvation as the mere pressure of the beam. Putting, therefore, $\omega = \frac{al}{4\lambda}$, or $l = \frac{4\lambda\omega}{a}$ in the equation $M = \frac{5l^4}{32a^2d}$, where d indicates the depression, and it becomes $M = \frac{5l^3.4\lambda\omega}{32a^3d} = \frac{5l^3\lambda\omega}{8a\ d}$. Let $P = \frac{M}{\lambda}$ express the

value of the modulus of elasticity in pounds, or the weight of a bar of an inch square, and of the altitude of that modulus; then $P = \frac{5l^3w}{8a^3d}$.

When the length and depth of the beam remain the same, the depression being inversely as the radius of curvature, will evidently be directly proportional to the load supported by the strain. The formula now given, in strictness, though applicable only to the case where the weight was equal to that of the fourth part of the beam, is hence quite general. As an exemplification, an experiment of Mr Ebbel's may be taken. A bar of cast iron exactly an inch square, and supported at the interval of 3 feet, suffered a depression of $\frac{3}{16}$ths of an inch, from a load of 308 lb., suspended at the middle. Here $P = \frac{5(36^3)(308)}{8.\frac{3}{16}}$ equal in round numbers to 47,900,000 lb. Another bar of the same dimensions, but supported at the interval of 34 inches, seems to have been carefully observed by Mr Tredgold. With a load of 80 lb., it bent $\frac{1}{20}$th of an inch, with 180 lb. $\frac{1}{10}$, and with 380 lb. $\frac{1}{5}$; the depressions being thus very nearly proportional to weights, till the elasticity began to give way. Here $P = \frac{5(34)^3(380)}{8(.2)} = 46,674,000$ lb.

Since P is constant in bars of the same materials, it follows, that the depression is directly as the cube

of the length, and inversely as the cube of the depth. When the breadth is increased, a proportionally greater weight is required to produce the same degree of depression, which is therefore inversely as the breadth of the bar. These results are sufficiently confirmed by the experiments of Buffon, and the recent observations of Dupin and Barlow. Thus, of oaken beams of 5.35 inches square, one of $7\frac{1}{2}$ feet bent 2.67 inches, under a weight of 12,400 lb.; another of 15 feet long, bent 8.7 inches, under 5.700 lb.; and a third of 30 feet long, bent 21.7 inches, under 1910 lb. Now, if the second beam had been loaded as much as the first, the depression would have augmented eight times, and amounted to 17.36; but diminished again in the ratio of 12,400 to 5,700, gives 8 inches, differing very little from 8.7. Again, the third beam, if loaded as much as the second, would have sunk 8×8.7 or 69.6 inches; but this quantity is reduced to one-third or 23-2 inches, from the inferior weight sustained. The discrepancy is inconsiderable, and owing to that irregularity which precedes the rupture of cohesion.

Another example may suffice. A beam of 4.28 inches square, and 12,86 feet long, was bent 7.5 inches under a load of 3,228 lb., while another of the same length and thickness, but double the breadth, was bent 3.16 inches under 12,068 lb. The depression under the same weight should have been only .9375 inches, or eight times less, and this,

augmented in the ratio of 3,228 to 12,068, gives 3.8, not materially different from 3.16.

When the load depressing a horizontal beam is increased till the under side becomes overstrained, or its particles are distended beyond a certain limit, their elasticity is then destroyed, and a fracture ensues. This disunion commences at both the surfaces, being produced equally by the tearing of the particles or fibres of the under layer, and by the crushing of those of the upper one. Let the limit of cohesion be the n^{th} part of the length of any line or chain of particles; the radius of curvature OH, at the moment of disruption, must hence be equal to n.GH, or $\frac{n}{2}$.GI. But OH $= \frac{5AB^2}{48HK}$, and consequently $\frac{5AB^2}{24HK} = n.GI$, or $5AB^2 = 24n.GI.HK$. Hence, likewise, $n = \frac{5AB^2}{24GI.HK}$. Thus, in the example already quoted, where a bar of cast iron of an inch square, bent one-fifth of an inch, and then broke, $n = \frac{5(34)^2}{24.\frac{1}{5}} = \frac{25(34)^2}{24} = 1204$; or the cohesion of the upper and under surfaces was destroyed, when their contraction or distension came to exceed this portion of the whole length. Now, though n may vary in different instances, even from 150 to 1500, it is always constant in reference to the same material. Hence the square of the length of a beam

is in the compound ratio of the depth, and of the quantity of depression which precedes fracture. Hence, likewise, when the length remains the same, this final depression is inversely as the depth of the beam. Thus, Mr Barlow found a fir batten, 2 feet long and an inch deep, to break with a depression of 1.25 inches, while another of the same depth, but 3 feet long, sunk 2.6 inches before it broke. Now, the square of 2 is to that of 3, or 4 to 9, as 1,25 to 2.8, differing very little from 2.6. Again, the depression preceding the fracture of a fir batten, 3 feet long and 2 inches deep, was only 1.12, being nearly the half of 2.6.

But the breaking load, or the transverse pressure which would overcome the strength of a beam, may be derived from the same principles. Resuming the former notation $\frac{5l^3w}{32a^3d}=P$, and substituting $24n.ad$ for l^2, the expression becomes $P=\frac{l24n.ad.w}{32a^3d}=\frac{3lw}{4a^2}$. Wherefore, $w=\frac{4}{3}.\frac{a^2}{l}.P$, and since P is constant, the strength of the beam, or the weight which it is capable of sustaining, is directly as the square of its depth, and inversely as its length. This power of resistance must also be directly proportional to its breadth. The conclusions are hence the same as those of Galileo.

Suppose the beam were a square prism, having a for its side and l for its length; the strength would

then be denoted by $\frac{4a^3}{3l}$. P, being thus as the cube of its thickness. From the same principles, it might be shown, that the transverse strength of a cylinder is only two-thirds of that of a square prism of the same thickness, and is consequently denoted by $\frac{8a^3}{9l}$. P. In the case of a hollow cylinder, let a' express the diameter of the internal cavity, and the resistance of the fistular column will be represented by $\frac{8P}{9l}(a^3—a'^3)$. If the matter were condensed into a solid cylinder, the strength would be reduced to $\frac{8P}{9l}\left(a^2—a'^2\right)^{\frac{3}{2}}$. Suppose the hollow part to have nine-tenths of the diameter of the cylinder, the strength would be diminished in the ratio of 1 to $1—(.9)^3$ or .271; but had this tube been formed into a solid rod, the strength would have amounted only to .08. A cylinder having half its core hollowed out, should be rendered an eighth part weaker, which agrees with an experiment of Barlow.

The lateral strength of any beam thus depends mainly on the distance and cohesion of the upper and under surfaces. Whatever stiffens the exterior layers, contributes greatly to strengthen the whole. A small incision drawn across the under side weakens a bar essentially; while a notch cut near the middle of the upper side will not impair the strength,

but, if filled up with a harder material, will even sensibly augment it. Thus, Duhamel found, that a bar of willow, cut through one-third its depth, the cut being filled up with a thin slip of hard wood, was thereby rendered about one-sixth part stronger than before. It was even remarked, that the incision could be carried much farther, without injuring the strength of the bar.

The last application of the strength of any material, consists,

4. *In its resistance to the effort of twisting or torsion.* A cylindrical body suspended vertically, but having its upper end fixed, may be wrenched or turned round through any angle, by the exertion of some lateral force; and if its elasticity be not thus impaired, it will, after the deranging influence has ceased, return to its former position, and perform this retrocession in a certain time. Many substances may be considered as only bundles of parallel fibres, which by twisting exert an augmented longitudinal force. It will be more accurate, however, to view materials in general as composed of particles equally distributed through the mass. We may hence conceive any cylinder to consist of a series of thin discs, as represented in fig. 131. When twisted, each successive disc will make a small angular advance, till this accumulation, at a certain distance, amounts

to a complete circuit, and such revolutions will be repeated at every like interval.

Let A denote the total angle of torsion, and h the height of the cylinder; the angular derangement of any disc, as in fig. 132, will be expressed by $\frac{A}{h}$. If d represent the diameter of this disc, d^2 will mark the number of particles, and consequently d^4 must indicate their force of gyration. The torsion corresponding to the angle A and the height h, is hence denoted by $\frac{Ad^4}{h}$: *It is thus directly proportional to the angle of deviation and the fourth power of the diameter of the cylinder, and inversely as its height.*

This result is confirmed by nice experiments on the torsion of slender metallic cylinders or wires. It hence appears, that a wire of half the thickness is twisted to the same angle with the 16th part of the force, and a wire of the third of the thickness is twisted to the same extent with only the 81st part of the force; and if the length were tripled, 243d part only of such force would be required to produce the effect. From a comparison of several experiments, it appears, that the weight acting at the distance of a foot, required to wrench asunder a cylinder of an inch diameter, and 3 inches high, is 200 lb., and, consequently, the power of any other

cylinder to resist torsion is denoted by $\frac{d^4}{h}.600$. To twist an iron wire of the tenth of an inch diameter, and 10 inches long, complete through a revolution, would require the action of a weight of 80 grains at the distance of an inch.

A fine wire will bear repeated circumvolutions, and yet return to its original position. But if the exterior particles be distrained beyond certain limits, then cohesion is impaired and finally dissolved. Within those limits, however, the elastic force evolved is quite regular and directly proportional to the angle described. Hence, under the same circumstances, all the oscillations of a contorted wire are perfectly isochronous.

The power of torsion being denoted by $\frac{d^4}{h}$, and the quantity of matter in each disc by d^2, the accelerating force is $\frac{d^2}{h}$; whence the time of vibration, when the wire carries merely its own weight, will be $\frac{\sqrt{h}}{d}$. But if the wire be loaded, by a cylinder whose diameter is constant, the weight W. of the mass to be now moved will greatly diminish the accelerating energy. Omitting, therefore, the weight of the wire itself, as inconsiderable, the force of acceleration will be denoted by $\frac{d^4}{hW}$, and hence the

time required for performing an oscillation is $\frac{\sqrt{hW}}{d^2}$. From the observed oscillations again, the power of torsion is, by an inverted process, readily computed.

The principle of torsion was happily applied by the ingenious Coulomb to the construction of an exquisite balance, for detecting and measuring with accuracy the smallest forces. By help of this very delicate instrument, he investigated the laws of magnetical attraction and repulsion; and a similar combination enabled our illustrious countryman Cavendish to determine, in the seclusion of his apartments, the density of our globe.

All machines are impelled, either by the exertion of Animal Force, or by the application of the Powers of Nature. The latter comprise the potent elements of Water, Air, and Fire. The former is more common, yet so variable as hardly to admit of calculation; it depends not only on the vigour of the individual, but on the different strength of the particular muscles employed. Every animal exertion is attended by fatigue; it soon relaxes, and would speedily produce exhaustion. The most profitable mode of applying the labour of animals, is to vary their muscular action, and revive its tone by short and frequent intervals of repose.

The decrease of animal power from continued exertion is exemplified in the race-horse. At his

greatest speed, he has been known to go over three miles and a half in six minutes,—at the rate, therefore, of 35 miles an hour, or 50 feet each second. But he could not travel more than 20 miles in the course of an hour, or perhaps 100 miles during a whole day. This gradual tendency to exhaustion is more precisely ascertained in the case of expert couriers. A swift runner may spring over the space of 150 yards, at the rate of 20 miles an hour, or 28 feet each second; but his speed is reduced to 21 feet in passing over a quarter of a mile, and to $18\frac{1}{2}$ feet in completing the whole mile. If he runs over six miles, his average rate of going will only be 11 feet in a second; and to travel 600 miles, he will take ten days, which, reckoning half the time to be spent in rest, gives an average pace of $7\frac{1}{3}$ feet, or about the fourth part of the swiftness with which he could start.

The ordinary method of computing the effects of labour is, from the *weight* which it is capable of elevating to a certain *height*, in a given *time*, the product of these three measures expressing the absolute quantity of performance. But these measures have evidently a mutual dependency, which in all cases brings the final result nearer to equality. Thus, great speed will occasion a waste of force, and abridge the period of its continuance. It was reckoned by Daniel Bernoulli and Desaguliers, that a man could raise two millions of pounds averdupois one foot

in a day. But our civil engineers have gone much farther, and are accustomed, in their calculations, to assume, that a labourer will lift ten pounds to the height of ten feet every second, and is able to continue such exertion for ten hours each day, thus accumulating the performance of 3,600,000. But this estimate seems to be drawn from the produce of momentary exertions, under the most favourable circumstances; and it therefore greatly exceeds the actual results, as commonly depressed by fatigue, and curtailed by the unavoidable waste of force.

Coulomb has furnished some of the most accurate and varied observations on the measure of human labour. A man will climb a stair, from 70 to 100 feet high, at the rate of 45 feet in a minute. Reckoning his weight at 155 lb., the animal exertion for one minute is 6,975, and would amount to 4,185,000 if continued for ten hours. But such exercise is too violent to be often repeated in the course of a day. A person may clamber up a rock 500 feet high by a ladder-stair in twenty-minutes, and consequently at the rate of 25 feet each minute; his efforts are thus already impaired, and the performance reaches only 3,875 in a minute.

But, under the incumbrance of a load, the quantity of action is still more remarkably diminished. A porter weighing 140 lb. was found willing to climb a stair 40 feet high 266 times in a-day; but he could carry up only 66 loads of fire-wood, each

of them 163 lb. weight. In the former case, his daily performance was very nearly 1,500,000; while, in the latter, it amounted only to 808,000. The quantity of permanent effect was hence only about 700,000, or scarcely half the labour exerted in mere climbing. In the driving of piles, a load of 42 lb., called the *ram*, is drawn up 3½ feet high 20 times in a minute; but the work has been considered so fatiguing as to endure only three hours a-day. This gives about 530,000, for the daily performance. Nearly the same result is obtained, by computing the quantity of water, which by means of a double bucket, a man drew up from a well. He lifted 36 lb. 120 times in a-day from a depth of 120 feet, the total effect being 518,400. A skilful labourer working in the field with a large hoe, creates an effect equal to 728,000. When the agency of a winch is employed in turning a machine, the performance is still greater, amounting to 845,000.

In all these instances, a certain weight is heaved up, but a much smaller effort is sufficient to transport a load horizontally. A man could, in the space of a day, scarcely reach an altitude of two miles by climbing a stair; though he will easily walk over thirty miles on a smooth and level road. But he would in the same time carry only 130 lb. to the fourth part of that distance, or 7½ miles. Assuming his own weight to be 140 lb., the quantity of horizontal action would amount to 42,768,000,

or twenty-eight times the vertical performance; but the share of it in conveying the load is 20,961,780, or about thirty times what was spent in its elevation. The greatest advantage is obtained by reducing the burthen to 120 lb., the length of journey being augmented in a higher ratio.

These results are apparently below the average labour performed in England, which is not only most vigorous, but, in many cases, quite overstrained. Moderate exertion of strength, joined to regularity and perseverance, would be more conducive to robust health, and the comfortable duration of human life. It is painful to remark, that English labourers are sooner worn out than those of most other nations.

A porter in London is accustomed to carry a burthen of 200 lb. at the rate of three miles an hour. In the same metropolis, a couple of Irish chairmen continue at the pace of four miles an hour, under a load of 300 lb. These exertions are greatly inferior, however, to the labour performed by the sinewy porters in Turkey, the Levant, and generally on the shores of the Mediterranean. At Constantinople, an Albanian will carry 800 or 900 lb. on his back, stooping forward and assisting his steps by a short staff. At Marseilles, four porters commonly carry the immense load of nearly two tons, by means of soft hods passing over their heads, and resting on their shoulders, with the ends of poles, from which the goods are suspended.

According to some experiments of the late Mr Buchanan, the exertions of a man in working a pump, in turning a winch, in ringing a bell, and in rowing a boat, are as the numbers 100, 167, 227, and 248. But those efforts appear to have been continued for no great length of time. The Greek seamen in the Dardanelles are esteemed more skilful and vigorous in the act of rowing, than those of any other nation. Even the Chinese, by applying both their hands and their feet, are said to surpass all people in giving impulsion to boats by sculling.

The several races of men differ materially in strength, but still greater diversity results from the constitution and habits of the individual. The European is, on the whole, decidedly more powerful than the inhabitants of the other quarters of the globe; and man, reared in civilized society, is a finer, robuster, and more vigorous animal than the savage. In the temperate climates, likewise, men are capable of much harder labour, than under the influence of a burning sun. Coulomb remarks, that the French soldiers employed on the fortifications of the Isle of Martinique became soon exhausted, and were unable to perform half the work executed by them at home.

The most violent and toilsome exertion of human labour is performed in Peru, by the *carriers* or *cargueros*, who traverse the loftiest mountains, and clamber along the sides of the most tremendous

precipices, with travellers seated on chairs strapped to their backs. In this manner, they convey loads of twelve, fourteen, or even eighteen stones; and possess such strength and action, as to be able to pursue their painful task eight or nine hours, for several successive days. These men are a vagabond race, consisting mostly of mulattoes, with a mixture of whites, who prefer a life of hardship and vicissitude to that of constant, though moderate labour.

When a man stands, he pulls with the greatest effect; but his power of traction is much enfeebled by the labour of travelling. A valuable set of experiments on this subject, made by Schultze of Berlin, seem to confirm the second formula proposed by Euler. It may hence be stated generally, that if v denote the number of miles which a person walks during an hour, the force which he exerts in dragging a load, by means of a rope passed over his shoulders, will be expressed in pounds averdupois by $2(6-v)^2$. Thus, when standing still, he pulls with a force of about 72 lb.; but if he walks at the rate of two miles an hour, his power of traction is reduced to 32 lb.; and if he quicken his pace to four miles an hour, he can draw only 8 lb. There is consequently a certain velocity which procures the greatest effect, or when the product of the traction by the velocity becomes a *maximum*. This takes place when he proceeds at the rate of two miles an hour. The utmost exertion which a man walking might conti-

2

nue to make in drawing up a weight by means of a pulley, would amount, therefore, in a minute, only to 5,632; but if he applied his entire strength, without moving from the spot, he could produce an effect of 12,672.

The labour of a horse in a day is commonly reckoned equal to that of five men; but he works only eight hours, while a man easily continues his exertions for ten hours. Horses likewise display much greater force in carrying than in pulling; and yet an active walker will beat them on a long journey. Their power of traction seldom exceeds 144 pounds, but they are capable of carrying more than six times as much weight. The pack-horses in the West Riding of Yorkshire are accustomed to transport loads of 420 lb. over a hilly country. But, in many parts of England, the mill-horses will carry the enormous burthen of 910 lb. to a short distance. The action of a horse is greatly reduced by the duration of his task. Though not encumbered at all with any load or draught, he would be completely exhausted perhaps by a continued motion for 20 hours in a day. The rate in miles each hour, which a good horse could thus travel, will be represented nearly by the formula $\frac{1}{25}\left(20-t\right)^2$, where t denotes the number of hours during which he trots or walks. Thus, though the horse might start with a celerity of 16 miles, this would be reduced in 4 hours to $10\frac{1}{4}$, and in 8 hours to $5\frac{3}{4}$. Hence the great advantages

resulting from short stages, lately adopted for the rapid conveyance of the mail.

The force which a horse exerts in pulling is diminished likewise by the quickness of his pace. A strong horse may have his power of traction expressed, in pounds averdupois, by the formula $(15-v)^2$, where v denotes the velocity in miles during an hour. But $(12-v)^2$ will represent more nearly the ordinary draught. Thus, a horse beginning his pull with the force of 144 lb., would draw 100 lb. at a walk of two miles an hour, but only 64 lb. when advancing at double that rate, and not more than 36 lb. if he quickened his pace to six miles an hour. His greatest performance would hence be made with the velocity of four miles an hour. The accumulated effort in a minute will then amount to 22,528.

This formula gives results somewhat greater than the performance of the ordinary horses, employed in tracking along the English canals. They generally go at the rate of two miles and a half in an hour, with a pull of only 81 lb. But in several places, and particularly between Manchester and Liverpool, they move at only half that pace, and therefore exert a force of 104 lb. The measure generally adopted in computing the power of steam-engines is much higher, the labour of a horse being reckoned sufficient to raise, every minute, to the elevation of one foot, the weight of 33,000 lb. But this estimate is not only greatly

exaggerated, but should be viewed as merely an arbitrary and conventional standard.

Wheel carriages enable horses, on level roads, to draw, at an average, loads about fifteen times greater than the power exerted. The carriers between Glasgow and Edinburgh transport, in a single-horse cart, weighing about 7 cwt., the load of a ton, and travel at the rate of 22 miles a-day. At Paris, one horse, in a small cart, conveys along the streets, half a cord of wood, weighing two tons; but three horses yoked in a line are able to drag 105 cwt., or that of a heavy cart loaded with building stones. The Normandy carriers travel, from 14 to 22 miles a-day, with two-wheeled carts, weighing each 11 cwt., and loaded with 79 cwt. of goods, drawn by a team of four horses.

The French draught horses, thus harnessed to light carriages, are more efficient perhaps than the finer breeds of this country. They perform very nearly as much work as those in the single horse carts used at Glasgow, and far greater than those heavy animals which drag the lumpish and towering English waggons. The London dray-horses, in the mere act of ascending from the wharfs, display a powerful effort, but they afterwards make little exertion, their force being mostly expended in transporting their own ponderous mass.

Oxen, on account of their steady draught, are in many countries preferred for the yoke. They

were formerly employed universally in the various labours of husbandry. The tenderness of their hoofs, however, makes them unfit for pulling on paved roads, and they can work only with advantage on soft grounds. But they want all the pliancy and animation, which are the favourite qualities of the horse.

The patient drudgery of the ass, renders him a serviceable companion of the poor. Though much inferior in strength to the horse, he is maintained at far less cost. In this country, an ass will carry about two hundred weight of coals or limestone, twenty miles a-day. But, in the warmer climates, he becomes a larger and finer animal, and trots or ambles briskly under a load of 150 pounds.

The mule partakes of the distinct qualities of the ass and of the mare. This cross-breed is much esteemed in Italy and Spain, being equally fit for draught and burthen. It is a stronger and a hardier animal than even the horse, performs longer tasks, is less subject to disease, and lives to double the age.

In the hotter parts of Asia and Africa, the ponderous strength of the elephant has been long turned to the purposes of war. He is reckoned more powerful than six horses, but his consumption of food is proportionally greater. The elephant carries a load of three or four thousand pounds,—his ordinary pace is equal to that of a slow trot,—he travels easily over forty or fifty miles in a day, and has been

known to perform, in that time, a journey of an hundred and ten miles. His sagacity and intelligence direct him to apply his strength according to the exigency of the occasion.

The camel is a most useful beast of burthen in the arid plains of Arabia. The stronger ones carry a load of ten or twelve hundred weight, and the weaker ones transport six or seven hundred; they walk at the rate of two miles and a half in an hour, and march regularly about thirty miles every day. The camel travels often eight or nine days, without any fresh supply of water; when a caravan encamps in the evening, he is perhaps turned loose, for the space of an hour, to browze on the coarsest herbage, which serves him to ruminate during the rest of the night. In this manner, without making any other halt, he will perform a dreary and monotonous journey of two thousand miles.

The dromedary, a small species of camel, though less fitted for bearing loads, travels with great speed, at a very hard trot, which is extremely fatiguing to the rider. Yet a Bedouin Arab, mounted upon one of those animals, lately conveyed an express from Cairo to Mecca, a distance exceeding 750 miles, in the space of five days.

Within the Arctic Circle again, the rein deer is a domesticated animal, not less valuable. He serves to feed and clothe the poor Laplander, and transports his master with great swiftness, in a cover-

ed sledge, over the snowy and frozen tracts. The rein deer subsists on the scanty vegetation of moss or lichens, and, though very docile, he is not powerful. Two of them are required to draw a light sledge: but so harnessed, they will run fifty or sixty miles on a stretch, and perform sometimes a journey of a hundred and twelve miles in the course of a day. Such exertions, however, soon wear them out.

A sort of dwarf camel was the only animal of burthen possessed by the ancient Peruvians. The Lama is indeed peculiarly fitted for the lofty regions of the Andes. The strongest of them carry only from 150 to 200 pounds, but perform about fifteen miles a-day over the roughest mountains. They generally continue this labour during five days, and are then allowed to halt two or three days, before they renew their task. The Paco is another similar animal, employed likewise in transporting goods in that singular country; it is very stubborn, however, and carries only from fifty to seventy pounds.

The stag-hound is capable of running three or four hours on a stretch, and at the rate of 15 miles an hour. Mastiffs are sometimes, in our large towns, made to assist in drawing light carts. The rough shaggy dog of the Esquimaux is a most hardy and powerful animal. He is early trained by those forlorn tribes to carry burthens, and generally to drag their loaded sledges over the hard but uneven surface of the snow. The dogs are harnessed in a line,

sometimes to the number of eight or ten, and they perform their task with great rapidity, steadiness, and perseverance. They can draw a heavy sledge to a considerable distance, with the swiftness of 13 or 14 miles an hour; but they will travel long journeys at half that rate, each of them pulling the weight of 130 pounds.

Even the exertions of goats have, in some parts of Europe, been turned to useful labour. They are made to tread in a wheel which draws up water, or raises ore from the mine. In Holland, the young goat, gaily caparisoned, is yoked to the ornamental miniature chariots of the children of the wealthy burghers. Though a very light animal, the goat is nimble, and climbs at a high angle. Supposing this soaring creature, though only the fourth part of the weight of a man, to march as fast along an ascent of 40 degrees, as he does over one of 18 degrees,—the sine of the former being double that of the latter,—it must yet perform half as much work.

VI. HYDROSTATICS

explains the Equilibrium and Pressure of Fluids. It is, indeed, only the application of the principles of Statics to the peculiar constitution of Fluids. Every substance appears to be composed of an assemblage of atoms, connected together by a system of mutual attraction and repulsion. In solid bodies, these integrant molecules affect a certain arrangement, and resist every change of figure. But fluids are distinguished, by the loose aggregation of their particles, which yield to the smallest external impression.

This indifference of the particles to arrangement or configuration, is, however, insufficient alone to constitute *Fluidity*. The disintegration of a Solid approximates it to such a property, but without conferring at all the distinctive character of a Fluid. When a solid body is reduced to a very thin plane, it will easily fold up in one direction; if it be divided into slender filaments, it will bend every way freely, and still betray no disposition to fracture. Crumbled to dust or powder, the aggregate particles now give way to any external force, yet without diffusing this impression through the mass.

Among such detached portions of matter, there exists no sympathy or mutual concatenation.

A solid body, subjected to a compressing force, undergoes a proportional contraction, but recovers its volume after this contraction is withdrawn. But if the compression be urged beyond a certain limit, the substance will be crushed under the load, and will suffer a complete dissolution. A fluid, however, inclosed in the chamber of a very strong metallic vessel, is found to be capable of withstanding the greatest pressure we can command, while its contractions are comparatively more extensive; and it always returns with unimpaired vigour to its former condition, the moment that such external force has ceased to act. The constitution of a fluid remains unaltered under the most enormous loads, and its several portions at all times separate and reunite with extreme facility. Whether the fluid has a liquid, or assumes a gaseous, form, it can equally bear any compression, the corresponding contraction in the latter being only much greater than in the former.

If the minutest subdivision of a Solid really contributes nothing to Fluidity, neither will any supposed smoothness or globular shape of the particles confer that character. Exact spherules might procure lubricity on a plane, but, among one another, they will become implanted, and affect certain configurations, as manifested in the common piling of round

shot. The absolute contact of the particles is besides inadmissible, since all bodies whatever seem to be capable of indefinite condensation.

But Fluidity occurs in very different degrees. Any disturbing impression is more quickly obeyed by one liquid than by another; by water, for instance, than by oil or treacle. All the possible shades of softness, in fact, might often be traced, from a solid to a fluid substance. The application of heat generally promotes fluidity. Thus, honey in winter is a candied or friable solid; in the spring it melts down; but, as the summer advances, it gradually loses its viscidity, and flows more freely. Oil seems affected in the same way, and hence the practice in Italy of depositing it in casks for exportation during the winter season, when it is thickest and least penetrating. But water itself, and other liquids, not excepting mercury, have their fluidity likewise augmented by heat, as evinced in their quickened flow through capillary tubes.

If an angular bit of glass be held in the flame of a blowpipe, it will become gradually rounded, as the heat penetrates and softens the mass. In like manner, a fragment of sealing-wax loses its rough exterior, and assumes a regular curved surface, on approaching it to the fire. But the centre of curvature is the point to which the combined attractions of the outer range of particles must be directed. Since this point retires, therefore, from the surface

in the progress of Softness to Fluidity, the mutual connection of the integrant molecules must be exerted over a much greater extent in Fluids than in Solids. This inference appears to afford likewise an explanation of the facility of their internal motion, so conspicuous in Liquids. When the sphere of activity is confined, as in solid substances, to a very narrow spot, a few particles only are connected, by their sympathetic tendencies. Any internal dislocation will in this case require the approximation of certain particles and the recession of others, which must hence occasion the exercise of corresponding repulsive and attractive forces. But these forces, being directed to a small number of centres, will constitute a very unequal group, incapable of gaining a smooth and graduated equilibrium. Every change of inclination would produce a violent effort to attain a new position of repose. On the other hand, when the sphere of activity embraces a multitude of particles, as in the composition of Fluids, the slightest mutual derangement will be sufficient to accommodate every variation of external impression. The most minute deviation of each particle, would, by such prodigious repetition, amount to any required change of direction. In their dislocations, the particles are almost unconstrained, and need scarcely approach or recede; the repulsive and attractive forces evolved become, therefore, extremely small, and by their multitude produce in every po-

sition nearly a perfect counterbalance. A Fluid is hence fitted to obey any impression with the utmost facility.

But whatever may be the system of forces that connects the fluid atoms, the properties of the compound are deducible, from the absolute facility with which those ultimate particles receive and transmit external impressions. Suppose a Fluid, either gaseous or liquid, to be inclosed in a very broad and extremely shallow cylinder of glass or metal AB (fig. 135), at the top of which is inserted a tall narrow tube CD of the same material, having a plug or piston E nicely fitted into it. On pushing down this piston, the fluid particles immediately under it will at first give way, and in receding, must approach closer to each other. They will, in consequence, display a repulsive force proportional to their mutual approximation. Now, it may be proved that points or atoms can never be ranged through space in perfect straight lines; wherefore, the pressure communicated, will not be confined to the vertical column, but will insensibly diverge in every direction. The particles at F will repel obliquely those at G or at H, and these again will distribute the oblique impression to other adjacent particles. The whole stratum of fluid, from the orifice C to the boundaries A and B, must hence continue to recede from the piston, and contract its volume, till the integrant particles have all of them attained the same

mutual distance, and exert the corresponding repulsive energy. An uniform condensation must thus be diffused through the compressed mass, before an equilibrium can take place in it. This condition obtains equally in liquids and in gaseous fluids, only the contraction produced by a compressing force, which is so visible in the former, can seldom be distinguished by ordinary perception in the latter. But a very minute alteration of the volume of a liquid evolves as much force, as an extensive change in the mass of a gaseous fluid.

The pressure exerted by the piston E at the orifice C is hence diffused equally through the whole of the fluid, every particle of which acquires the same intensity of repulsion. Each point on the surface of the vessel must therefore sustain an equal effort. If a wide cylinder IK, fitted likewise with a piston, were inserted at I, the compression opposed would be proportional to the space of action or the circle of the orifice. On the supposition that the surface of the piston I were ten times greater than that of E, it would likewise support a load ten times greater than the pressure applied at E or C.

Let the piston I be now removed, while the piston E is conceived to act as before. The liquid must evidently rise in the cylinder IK (fig. 136), till its weight becomes equal to the pressure exerted at the orifice I. In like manner, if another cylinder LM were inserted, the liquid would rise till its weight was equal to the pressure at L. But the

pressures at I and L being proportional to those orifices, or the circular sections of the cylinders IK and LM, the altitudes IK and LM of the columns themselves must be equal. If a cylinder NO were inserted obliquely, the liquid would still rise to the same level; for in this case, the column being partly sustained by the under side, its weight would, from the property of the inclined plane, be to its pressure at N, as the length ON to the perpendicular OP.

Let the piston E itself be withdrawn, and the liquid will mount in the cylinder CD, till its weight becomes equal to the pressure applied at C, and consequently will attain the same altitude as in the other communicating cylinders. Suppose the cylinders to be enlarged and brought nearer to each other, and still the same level will be maintained among them. Conceive they were even united, so as to compose a single cylindrical vessel, and the contained liquid would always assume a level surface. Such is the distinguishing character of fluids.

It hence follows, that the condensation accumulated at any point of an open liquid mass, and therefore the actual pressure exerted there, is proportional to the altitude of the incumbent column of a supposed vertical tube, or to the depth of the point below the surface of the fluid. The pressure of water against the perpendicular sides of any cistern must thus increase regularly, from the top to the bottom.

In a cubical vessel, the pressure borne by each side would be just half the weight supported at the bottom, and consequently the pressure sustained by all the four sides would be double of this weight.

That the pressure of a fluid is exerted equally every way in proportion to its depth, may be confirmed by various experiments. Thus, having fastened a long narrow glass tube to the neck of a thin bladder, fill this with water till it stand perhaps an inch higher, and plunge the whole in a tall jar of water; the liquid will be seen to rise in the tube, and maintain the same altitude, exactly in proportion as the bladder descends. Again, if a tall glass tube, spreading below into a wide funnel-mouth, to which a loaded plate of brass has been ground and closely fitted, were let down and held in a body of water, at the depth where a cylindrical column of fluid, incumbent upon its broad base, has a weight equal to that of the plate, this would remain supported. But if a hole were pierced in the side of the tube admitting a small portion of the water to fill up the funnel, its load would quickly be precipitated to the bottom. On the other hand, if the tube had its funnel-mouth turned upwards, and fitted with a thin brass plate surmounted by a very thick cylinder of cork; the buoyancy of this cover would be overcome at a certain depth below the surface of the water. But, on letting water into the funnel, the pres-

sure now exerted under the plate would immediately float it up.

The fundamental principle, that a fluid compressed in a close shallow vessel exerts the same effort upon every equal portion of the confining surface, was first distinctly stated by the famous Pascal, who even proposed it as a new mechanical power of great efficacy and ready application. If the piston I (fig. 135.) were, for instance, an hundred times larger than the piston E, the force of one man pushing down the former would be sufficient to withstand the action of an hundred men exerted upon the latter. Nor did another feature of resemblance escape this acute philosopher, that, as in all other mechanical combinations, what is here gained in power is lost in celerity, or, in other words, that the height to which a load is raised is still inversely as the purchase. Thus, while the piston E descends through one inch, the piston I ascends only the hundredth part of an inch. This consideration is essentially the same as the principle of *virtual velocities*, which affords another demonstration of the equality of pressure diffused through a fluid. For, if one pound depress the piston E one inch, the piston I will lift an hundred pounds over the hundredth part of an inch, the momentum of the weight at E being still equal to the opposite momentum of the load incumbent at I. The pressure exerted by the fluid upon every point

of the surface of the shallow vessel is hence the same.

The project originally started by Pascal, of applying this capital property of fluids to the construction of a powerful mechanical engine, has been reduced to convenient practice in our own time. The progress of the arts has now begun to realize all the delicacy of theory. Watt employed, to a certain extent, the soft compression of air, as the chief agent in his Coining Engine; and Bramah has most successfully availed himself of the constrained energy of water, in the composition of his Hydraulic Press.

It will be easy, from the principles already stated, to determine what share of the weight of a liquid contained in a vessel of any form, is supported by the sides, and what part rests upon the bottom. Suppose a wide-spreading vessel, of which CABD (fig. 137.) represents a vertical section, to be filled with some liquid. The pressure, which a very narrow column of fluid $FEef$ exerts against the side of the vessel at E may be decomposed into a force acting vertically, and another acting horizontally. But those three forces, being proportional to perpendiculars to their several directions, are as the lines Ee, $e\varphi$, and $E\varphi$, and consequently, are as the elementary rectangles $Ee \times EF$, $e\varphi \times EF$, $E\varphi \times EF$. Now, the whole pressure exerted upon the portion Ee of the side of the vessel, is evidently, as the depth EF of

the liquid film multiplied into Ee the breadth of its base; and therefore this base will sustain a *vertical* pressure denoted by $e\phi \times EF$, or the weight itself of the elementary column $EFfe$, and will also resist a *horizontal* thrust expressed by $E\phi \times EF$. But, for the same reason, the other side of the vessel at G supports the load of the elementary column $GHhg$, and pushes horizontally with the force $GH \times G\gamma$, which is equal to the opposite thrust $EF \times E\phi$. These horizontal forces are hence mutually balanced or extinguished; while the sides of the vessel sustain the whole weight of the series of vertical columns, from C to AI, and from D to BK. The same property belongs to every parallel section, and, consequently, the bottom of the vessel supports only its incumbent prism or cylinder IABK, while the rest of the surrounding liquid is upheld wholly by the reaction of the sides.

The pressure exerted by a fluid against the bottom of a vessel is thus nowise augmented by its spreading shape; nor shall we find it at all diminished, on the other hand, by any degree of contraction. Let CABD (fig. 138.) represent such a tapered form of vessel. Assuming a vertical section as before, the perpendicular thrust against the element Ee of the side must be compounded of FE, the depth under the surface of the fluid, and the base Ee. But this force may be decomposed into a vertical and a horizontal pressure, in the ratio of Ee to $e\phi$ and $E\phi$, or as $FE \times Ee$ to

$FE \times e\varphi$ and $FE \times E\varphi$. The space Ee is therefore pushed outwards horizontally by a force denoted by $FE \times E\varphi$, and directly upwards by the force $FE \times e\varphi$, or the weight of an exterior column $FEef$. In the same manner, it is shown that the pressure exerted against the element Gg of the opposite side of the vessel may be resolved into a horizontal thrust $GH \times G\gamma$, and a thrust $GH \times g\gamma$ from below, in the direction GH, which is equal therefore to the weight of the exterior column $GHhg$. But those horizontal thrusts $EF \times E\varphi$ and $GH \times G\gamma$, being evidently equal and opposite, are mutually extinguished. The vertical thrusts $EF \times e\varphi$ and $GH \times g\gamma$ from below are resisted by the elasticity of the elementary spaces Ee and $G\gamma$ of the vessel; or, what is the same thing, these spaces press downwards the columns $LEel$ and $MGgm$, by forces equal to the weights of the exterior columns $EFfe$ and $GHhg$. The conjoined pressures exerted at Ll and Mm are hence represented by the columns $LFfl$ and $MHhm$. The pressure of the whole section upon AB is, therefore, the same as that of the rectangle IABK. Collecting then all the sections, their total pressure upon the bottom will be equal to the weight of an uniform prismatic or cylindrical column IABK.

This very singular and important conclusion, that the pressure of a liquid upon the bottom of any vessel depends merely on its altitude and the surface of its base, might be derived from simpler but indirect

considerations. Suppose the portion of the liquid, which in fig. 137. encircles the vertical cylinder IABK, to be frozen or converted into a solid substance, but without changing its density; the pressure of this congealed mass would evidently continue the same as before, and therefore the bottom has only to sustain the weight of the cylindrical column which rests immediately upon it. If the lateral ice were again melted, it would only resume its horizontal thrust, which could in no degree alter the vertical pressure of the liquid.

In like manner, if IABK in fig. 138. were a cylinder of liquid resting upon a circular base; conceive the portions IAC and KBD about the sides to be rendered solid or congealed without alteration of density, and the remaining portion ACDB will evidently exert the same pressure as before. The incurved sides AC and BD of this tapering vessel hence produce an effect analogous to the load of the supposed frozen masses IAC and KBD.

The property now demonstrated is commonly termed the *Hydrostatic Paradox*. The smallest portion of a liquid is thus capable of producing a pressure equal to that of the largest mass. The fact is confirmed and elucidated by a variety of striking experiments. In all water-works, it forms an essential consideration. But the principle extends its influence likewise to remoter objects. No masonry is safe which neglects it. If the smallest

quantity of water should lodge to a considerable height in the gravel, sand, or loose earth close behind a wall or embankment, it would exert a lateral pressure sufficient to push the solid materials from their base. Hence, a sudden shower often occasions great devastation. The thinnest vein of water, collected in a perpendicular crevice, will split the hardest rock, and hurl its fragments down the precipice. This hydrostatic pressure is the chief agent which nature employs, in the silent and gradual demolition of mountains.

Suppose any interior portion of a liquid to become solid; it would evidently remain in the same state of indifference or equilibrium as before. It must therefore be borne up by the vertical pressure of the fluid with a force just equal to its weight, or the weight of the liquid whose place it occupies. Conceive this congealed mass to have its gravity augmented or diminished; it will be pulled downwards or upwards by the difference between this force and the weight of an equal bulk of the liquid. Substitute any solid body instead of this block of ice, and *the loss of weight by the immersion it sustains will be equal to that of the volume of fluid which it displaces.*

But this fundamental property, first detected by the genius of Archimedes, may be demonstrated by a stricter process of reasoning. Let a cylindrical body FEGH (fig. 139.) be plunged vertically in a

vessel CABD, filled with water or any other liquid; the lateral pressure, acting equally around the axis, will evidently produce a complete balance of efforts. But the cylinder will be pushed upwards by a column of fluid having IE for its altitude, and the circle EG for its base, and pressed downwards again by the circular column IFHK. The solid is, therefore, buoyed up by the excess of the former force above the latter, or by the weight of an equal cylinder FEGH of the liquid.

Suppose a solid of any shape were immersed in a liquid, and let fig. 140. represent a vertical section. The pressure exerted perpendicular to the element of the surface Ee is resolved into a horizontal and a vertical force, and these three forces are as the lines Ee, $e\varphi$ and $E\varphi$, or as the rectangles $GE \times Ee$, $GE \times e\varphi$, and $GE \times E\varphi$. Wherefore, the horizontal impression at Ee is represented by $GE \times e\varphi$. Draw the horizontal lines EH and eh, and the corresponding verticals HI and hi. It is evident that the portion of the perpendicular pressure against Hh acting in the horizontal direction HE will be denoted by $IH \times hi$, which is equal to the opposite exertion $GE \times e\varphi$. In every horizontal space $EHhe$, therefore, the lateral efforts are balanced, and hence the whole section has no tendency towards either side. This must be the case likewise in all the collective sections which compose the submerged solid, which has no disposition to move to the one side of the vessel or to the other. But the section is pushed

directly upwards at E*e* by a force represented by GE × E*e*, or the elementary film of liquid GE*eg*, while it is pressed downwards only by the film GF*fg*. It is consequently buoyed up by their difference, or by the film FE*ef*, and the buoyant effort of the aggregate parcels, therefore, is equal to the whole section. The collective sections, again, form the solid body, which hence loses, by immersion, just as much weight as that of an equal volume of the surrounding fluid.

On this principle is founded the method of ascertaining the density of a body, or the relation of its weight to its bulk, which, in reference to some common standard, is termed its *specific gravity*. Water, at its state of greatest contraction, is preferred as the most convenient unit of comparison, the density of other bodies being reckoned in decimal parts. The *Hydrostatic Balance* is generally used for this purpose. The substance to be examined may be either liquid or solid. The specific gravity of liquids is easily determined, by means of a ball or pear-shaped lump of glass or crystal, either partly hollow or loaded. This poise, suspended by a hair or fine thread, is weighed *in vacuo* or air, then in pure water, and next in the particular liquid; the loss of weight which it suffers in water is to its loss in the fluid under trial, as unit to the specific gravity of this fluid. The calculation is sometimes carried to five places of decimals; though it is seldom safe or expedient to push them beyond three figures. If the

lump of glass were ground to such a size as to lose exactly a thousand or ten thousand grains in distilled water, no computation would be required, its loss of weight in the liquid indicating at once the specific gravity.

In the case of solid substances, they may be either denser than water or rarer, and they may be insoluble in it or capable of solution. The mode of determining their specific gravity will accordingly be different.

1. Insoluble solid bodies, denser than water, are weighed in *vacuo*, and then in distilled water; and the loss of weight, which they suffer by immersion, is to their whole weight, as unit is to their specific gravity.

2. Insoluble solid bodies, lighter than water, require to be joined to some heavier substance, in order to make them sink in that liquid. The weight of the ballast, when immersed alone, must be previously ascertained. This weight, diminished by the weight of the compound after immersion, will, therefore, give the buoyant power, or the weight of a mass of water equal to the bulk of the rare substance. The weight thus corrected is hence to the weight of the substance in *vacuo*, as unit is to its specific gravity.

3. When the solid substances to be examined are saline, or liable to solution in water, they may be gently heated and covered with a thin coat of melted bees-wax. Thus defended, they may now be plunged without any risk in distilled water. A slight allowance should be made for the buoyant influence of

the film of wax itself, which, however, must be very minute, since wax has very nearly the density of water itself. This mode of finding specific gravity is evidently very imperfect, and quite inapplicable to substances that have a powdery form. The instrument which I term a *Coniometer*, founded upon the dilatation of air, may supply the defect.

A solid substance, rarer than the fluid medium, must evidently sink, till it displace an equal weight of the fluid. The submerged part of the solid hence always marks the volume of this equiponderant mass. If the floating body have a globular shape, terminated by a long slender stem, its depression in any liquid will measure the smallest differences of specific gravity. The stem may be made exactly cylindrical, for instance, and divided into portions which correspond to the thousandth parts of the bulk of the ball. Such is the general construction of the *Hydrometer*, a very convenient instrument for examining readily the densities of different liquids. The stem will scarcely bear more than an hundred distinct subdivisions; but the range can be easily enlarged, by attaching, as circumstances may require, loads answering to 100, 200, 300, &c.

One of the easiest and simplest methods of determining the densities of different liquids, is by a set of small glass beads, previously adjusted, and numerically marked. Thrown into any liquid, the heavier balls sink, till they approach the required density and become gradually buoyant, and the one which first rises to the surface indicates, in thousandth

parts, the specific gravity of the fluid. These balls are adapted for examining liquids, whether lighter or heavier than water.

But the most accurate and concise mode of ascertaining the density of liquids, is to employ a small glass measure with a very short narrow neck, and adjusted to hold exactly a thousand grains of distilled water. The vessel being filled with any other liquid, the weight of it is observed, and thence its relative density to water may be found, by merely striking off three decimal places. At each operation, the glass must be carefully rinsed with pure water, and again dried, by heating it, and then sucking out the humified air, for a few minutes, by help of a slender inserted tube.

If fluids of various densities, and not disposed to unite in any chemical affinity, be poured into a vessel, they will arrange themselves in horizontal strata, according to their respective densities, the heavier always occupying a lower place. This stratified arrangement of the several fluids will succeed, even though a mutual attraction should subsist, provided only that its operation be feeble and slow. Thus, a body of quicksilver may occupy the bottom of a glass vessel, above it a layer of concentrated sulphuric acid, next this a layer of pure water, and then another layer of alcohol. The sulphuric acid would scarcely act at all upon the mercury, and a considerable time would elapse before the water sensibly penetrated the acid, or the alcohol the water. Bodies of different densities might remain suspended in those

strata. Thus, while a ball of platinum would lie at the bottom of the quicksilver, an iron ball would float on its surface; but a ball of brick would be lifted up to the acid, and a ball of beech would swim in the water, and another of cork might rest on the top of the alcohol.

Hence the reason of various natural phænomena. Thus, the fresh water discharged by a river into the sea continues to float upon the surface of the denser mass of salt water, till the several strata become intermixed by the action of winds, or the commotion of tides and currents. For the same reason, the brackish water collected near the mouth of a great river must rise proportionally above the general level. Hence, likewise, certain wells and springs near the coast are observed to swell and sink regularly with the flow and ebb of the tide, from the mere variation of hydrostatic pressure of the adjacent sea acting on the veins of fresh water. Fountains occur sometimes on the very summits of detached rocks and islets. The same principle appears to furnish the simplest and most probable explication of the gradual subsidence of the Baltic Sea, which has not at present the fifth part of the saltness of the Ocean. It must therefore stand at a higher level in proportion to its depth; but if we suppose its waters to have worked into certain submarine beds of salt, they must contract and sink down, as they dissolve it and become denser. To verify this hypothesis, it is only requisite to ascertain whether the Baltic be actually growing salter.

It may be convenient here to state, merely in round numbers, the specific gravities of the more remarkable substances.

Platinum, purified,	19.50	Plumbago,	1.86
——— hammered,	20.34	Alum,	1.72
——— laminated,	22.07	Asphaltum,	1.40
——— drawn into wire,	21.04	Jet,	1.24
Gold, pure and cast,	19.26	Coal, from 1.24 to	1.30
—— hammered,	19.36	Sulphuric acid,	1.84
Mercury,	13.57	Nitric acid,	1.22
Lead, cast,	11.35	Muriatic acid,	1.19
Silver, pure and cast,	10.47	Equal parts by weight of water and	
—— hammered,	10.51	alcohol,	.93
Bismuth, cast,	9.82	Ice,	.92
Copper, cast,	8.79	Strong alcohol,	.82
——— wire,	8.89	Sulphuric æther,	.74
Brass, cast,	8.40	Naphtha,	.71
—— wire,	8.54	Sea water,	1.03
Cobalt and Nickel, cast,	7.81	Oil of Sassafras,	1.09
Iron, cast,	7.21	Linseed oil,	.94
—— malleable,	7.79	Olive oil,	.91
Steel, soft,	7.83	White sugar,	1.61
—— hammered,	7.84	Gum Arabic and honey,	1.45
Tin, cast,	7.30	Pitch,	11.5
Zinc, cast,	7.20	Isinglass,	1.11
Antimony, cast,	4.95	Yellow amber,	1.08
Molybdænum,	4.74	Hen's egg, fresh laid,	1.09
Sulphate of barytes,	4.43	Human blood,	1.05
Zircon of Ceylon,	4.41	Camphor,	.99
Oriental ruby,	4.28	White wax,	.97
Brazilian ruby,	3.53	Tallow,	94
Bohemian garnet,	4.19	Pearl,	2.75
Oriental topaz,	4.01	Sheep's bone,	2.22
Diamond,	3.50	Ivory,	1.92
Crude manganese,	3.53	Ox's horn,	1.84
Flint glass,	2.89	Lignum vitæ,	1.33
Glass of St Gobin,	2.49	Ebony,	1.18
Fluor spar,	3.18	Mahogany,	1.06
Parian Marble,	2.34	Dry oak,	.93
Peruvian emerald,	2.78	Beech,	.85
Jasper,	2.70	Ash,	.84
Carbonate of lime,	2.71	Elm, from .80 to	.60
Rock crystal,	2.65	Fir, from .57 to	.50
Flint,	2.59	Poplar,	.38
Sulphate of lime,	2.32	Cork,	.24
Sulphate of soda,	2.20	Chlorine,	.00302
Common salt,	2.13	Carbonic acid gas,	.00164
Native sulphur,	2.03	Oxygen gas,	.09134
Nitre,	2.00	Atmospheric air,	.00121
Alabaster,	1.87	Azotic gas,	.00098
Phosphorus,	1.77	Hydrogen gas,	.00008

One hundred cubic inches of chlorine, carbonic acid gas, oxygen gas, atmospheric air, azotic gas, and hydrogen gas, weigh respectively 76.2, 46.5, 33.9, 30.5, 29.6, and 2.1 grains Troy. One cubic inch of distilled water, at the ordinary temperature of 62 degrees Fahrenheit, weighs in air 252.5 grains. Hence the weight of a cylindrical inch of water is 198.3 grains; but the same measure of quicksilver would weigh 2691 grains, which affords a ready method for ascertaining the diameters of very narrow or capillary glass tubes.

The averdupois pound is now enacted to contain 7000 grains Troy, and must hence be equal to the weight of 27,724 cubic inches of water. Wherefore, a cubic foot of water weighs 62.331 pounds, or almost exactly one thousand ounces averdupois. The weight of an Imperial gallon is fixed at ten such pounds. A cubic foot of water seems to have anciently corresponded to the weight of a bushel or firlot *(fourthlet)* of wheat, four of which make a boll, eight times this a ton, and the double of this again a chalder. The ton is therefore 2000 lb., or more generally reckoned twenty hundred weight, each hundred consisting of 112 lb.

The weight of a cable of 720 feet, or 120 fathoms length, worked up from strands 180 fathoms long, is found, by multiplying the square of the girth or circumference in inches by 19½ lb. The square of half the diameter will hence nearly express in pounds the weight of a foot of cord. A chaldron of coal at Newcastle is reckoned equal to 53 cwts., or 5936 lb.;

but in London the chaldron contains 36 bushels, each weighing 84 lb.

A hollow glass cylinder, open at top, and fitted below with a loaded plate of brass, being immersed in a vessel of water, till the quantity of liquid thus displaced shall have a weight equal to that of the compound, the plate will not only float, but sustain the incumbent glass. If a solid cylinder were pushed vertically into the water, it would press down the vessel by a load equal to the weight of an equal bulk of the fluid, which is readily indicated by suspending the vessel from one of the ends of a balance.

A glass tube of considerable length being laid horizontally, with both ends bent upwards, and now filled with any liquid, such as water or quicksilver, the opposite terminating surfaces must always keep the same level. If small sights with cross wires were hence set to float in the vertical tubes, this combination might serve as a levelling instrument. Such a construction has accordingly been sometimes employed, though it seems not susceptible of much accuracy.

The *Spirit Level* is far more delicate. This valuable instrument consists of a short glass tube having a very slight curvature, and almost filled with alcohol, leaving but a small bubble of air. Alcohol is preferred to every other liquid, because it moves readily, and is not liable to freeze. The horizontal line is indicated by a tangent to the upper and convex side of the tube, at the middle of the floating bubble.

If the two vertical branches of a tall recurved tube be filled with liquids of different densities, the opposite surfaces will no longer maintain the same elevation. The lighter fluid will rise proportionally higher, so that their altitudes will be inversely as their respective densities. Such a form of tube might therefore serve as an imperfect Hydrometer.

Since the pressure of any fluid is proportional merely to the depth below the surface, the strain borne by a sluice or the sides of a canal must increase uniformly from the top to the bottom. The centre of pressure is hence not in the middle, but at one-third of the entire altitude. To this point, therefore, if more strength be wanted, the additional prop should be applied.

If water be confined in a canal or basin by a wall or embankment, the thickness of the dike must increase regularly in proportion to its depth. The adhesion of any materials or their resistance to a horizontal thrust, may be estimated as a certain proportion of their weight, commonly the half or the third part. Let AB (fig. 141.) be the height of the wall, and make its density to three times that of water, as AB to BC, and join AC, which will represent the proper slope. If the dike be composed of stones or bricks, the base BC must be at least equal to the altitude AB; but if it consist of earth, BC should be one half more. When the embankment is formed of earth, its side must not be perpendicular; it should form an inclined plane, not exceeding 35 degrees,

or the angle of repose, lest the softened parts should slide down; the outside, being more solid, may be steeper.

In canal navigation, a boat is raised from a lower to a higher level by a series of locks, a portion of the water somewhat exceeding the length of the vessel being inclosed at the sides by walls, and at both ends by masonry, and opposite flood-gates. As soon as it passes these gates, they are shut behind it; and a small lateral or superior sluice being opened, the water rushes into the inclosure, and quickly mounts to the higher level, thus enabling the vessel again to proceed. A similar operation is performed at each successive lock. In descending the canal, the procedure is exactly reversed, the water contained in the series of inclosures being allowed to flow out, and thus lower by degrees the level of the boat.

The flood-gates are contrived to shut at a certain angle. If this angle be very acute, they sustain too great a pressure, and yet close feebly. Let AC and BC (fig. 142.) represent the flood-gates of a canal, which are opened by help of the extended arms AE and BF. When shut, the gate AC is pressed at right angles by the water, with a force as AC itself, which, from the principle of the lever, must exert a perpendicular effort at the end C, as the square of AC. The thrust thence produced in the direction AB will be as AC×CD, and will encounter an equal and opposite thrust from the gate BC. These two forces constitute the power which

closes the gates. The force with which they are made to cohere, thus increasing with AC and CD, must augment rapidly when the angles BAC and ABC are enlarged, or their mutual inclination ACB becomes diminished.

If the angle ACB were very obtuse, those conjoined gates would, like a low roof, occasion a great thrust against the walls of the canal, or the centres of the gates at A and B. The thurst in the direction CA might be shown to be $\frac{AC^2 \times AD}{CD}$. It will hence be easy to determine the angle ACB, with the centres of the flood-gates which suffer the smallest strain; for AD being constant, $\frac{AC^2}{CD}$ must be a *minimum*. But this quantity is evidently the diameter of a circle circumscribing the triangle ACB; and since the least circle is that described about the point C, the angle ACB, at which the gates lock, should be a right angle.

Water may be contained in circular basins or cylindrical vessels, and conveyed in pipes, which therefore bear a lateral pressure proportioned to the altitude of the column. But this pressure will, in consequence of the curvature, occasion likewise a longitudinal strain or distension. Let fig. 143. represent a section of the tube; and assume, in the interior circumference, the proximate points B and C, equally distant on either side from A. But three forces may be conceived to act at these points; the perpendicular thrust of the water against A, and

longitudinal tensions at B and at C. These forces being proportional to the sides of a triangle perpendicular to their several directions, are consequently as the lines BC, BO and CO. But the pressure exerted by the fluid at A, is as the little space BC; wherefore the tension of the ring at the points A, B, C must be expressed by the radius BO. The same distending energy is hence exerted around the whole internal circumference. The lateral pressure of the water against each ring of the cylinder, must thus produce the same effect, as a longitudinal force applied to it, equal to the weight of a prism of the fluid, resting on a base which has the breadth of that ring, with the radius for its thickness. The strength of the cylinder must consequently be in the compound ratio of the altitude of the water, and of its own radius or diameter. Similar pipes will therefore bear equal pressures, their thickness being proportioned to the diameter. Thus, a pipe of only a foot in diameter, and half an inch thick, will withstand the thrust of the same altitude of water, as a pipe of the same materials two feet wide and a whole inch in thickness. But since lead has only the tenth part of the tenacity of cast-iron, a pipe of that soft metal will require, in similar circumstances, to have ten times greater thickness in proportion to its diameter, than one of cast-iron. Such is the case with a pipe of elm, while one of free-stone would still need to have its thickness doubled.

Let h denote the height of the water in inches, d the diameter of any pipe, t its thickness, and

c the longitudinal cohesion of a square inch bar of its material expressed in pounds averdupois. The longitudinal strain occasioned by the internal pressure against each ring of an inch breadth, is in cubic inches $\frac{hd}{2}$, which corresponds to $\frac{hd}{2}.\frac{1}{27{,}7}=\frac{2\,hd}{111}$, in pounds averdupois. Wherefore, $\frac{2\,hd}{111}=ct$, and $h=\frac{111c}{2}.\frac{t}{d}$; or, if H express the altitude in feet, $H=\frac{37\,c}{8}.\frac{t}{d}$. Substituting the observed cohesion of cast-iron, and $H=\frac{37.19096}{8}.\frac{t}{d}=88319\,.\frac{t}{d}$.

But this result must exceed the actual strength of such pipes. In the first place, they will evidently give way, as soon as the elasticity becomes impaired, and the smallest fissures begin to open. This happens in malleable iron, when only half its whole cohesion is exerted, and probably occurs sooner in cast-iron. But, in the next place, the longitudinal tension occasioned by the perpendicular pressure of the water being proportional to the radius of curvature, must increase, with the thickness of the pipe, from the inner to the outer surface. The particles are hence not pulled by equal and parallel forces, as in the case of a vertical bar of iron; but must be distrained by an oblique action, which will accelerate their separation. We may therefore estimate the actual cohesion as reduced nearly one-half, which

gives in round numbers $H = 50,000 \, . \, \frac{t}{d}$, or it will be safer in practice to reckon the co-efficient 30,000, or even 25,000 feet. Thus, for a cast-iron pipe of the very best quality a foot in diameter, and 3-4ths of an inch thick; $H = 30000 \, . \, \frac{\frac{3}{4}}{15} = 1500$ feet. Again, let a pipe of the same diameter have a thickness of an inch and quarter; and $H = 30,000 \, . \, \frac{\frac{5}{4}}{15} = 2500$ feet. Such are the dimensions of the cast-iron pipes now laid, for conveying water from the Pentland Hills to Edinburgh; and such was the strength of some of them which had been formed carefully of the best materials, ascertained by means of a large forcing pump, it being sufficient, however, that the pipes should withstand the pressure of a column of water respectively of the altitudes of 400 and 800 feet.

Following the same analogy, the former leaden pipes laid at Comiston, which were but four inches and a half wide and three-fifths of an inch thick, could have sustained only the thrust of 400 feet in height of water: For $3000 \, . \, \frac{\frac{3}{5}}{4\frac{1}{2}} = 3000 . \frac{6}{45} = 400.$ *Drawn* leaden pipes are, on the principle of case-hardening, considerably stronger. Thus, Mr Jardine has found that such a pipe 2 inches wide, and 2-7ths of an inch thick, bears a pressure equal to that of a vertical column of water of 600 feet; but stretched about the three hundredth part of an inch under that of 650, and finally burst after the columnar pressure

had been gradually raised to 1200 feet. But the formula for cast lead would here give only 428 feet.

According to Tredgold, the lateral cohesion of oak is 2316 lb. for each square inch. Hence a pipe formed of this tough wood, 15 inches in diameter and 2 inches thick, would sustain a column of water of the height of only $\frac{37}{8} . 1500 . \frac{2}{15} = \frac{37}{4} . 100$, or 875 feet. It would have required twice that thickness to enable a pipe of larch to bear the same strain. Elm, which is more commonly used for this purpose, approaches to the strength of oak.

The cohesion of Portland stone being only 857 lb., a pipe formed of that material, and one foot in diameter and an inch and half thick, would be sufficient to bear a pressure of water, of no greater altitude than $\frac{37}{8} . 500 . \frac{1\frac{1}{2}}{12} = \frac{37}{64} . 500$, or 288 feet. Those elegant pipes lately cut out of a block of free stone, in a series of cores, by the application of a circular saw, have hence failed, from their weakness and their disposition to chip and crack.

The same principles regulate the strength of a circular basin confining water. The perpendicular pressure against the wall depends indeed merely on the altitude of the fluid, without being affected by its volume; but the longitudinal effort of the thrust, or its tendency to open the joints of the masonry, is measured by the radius of the circle. To resist that action, in very wide basins, the range or course of

stones, along the inside of the wall, must be proportionally thicker. On the other hand, if any opposing surface present some convexity to the pressure of water, the resulting longitudinal strain will now be exerted in closing the joints and consolidating the building. Such reversed incurvation is hence generally adopted in the construction of dams, the bend inwards of the arc being about the eightieth part of the length of its chord, while the exterior boundary is made rectilineal.

If a solid body, not subject to solution, but placed in a liquid denser than itself, it will sink till the pressure which it encounters from below becomes sufficient to support its entire load, which is then just equal to the weight of the portion of fluid displaced. While the body floats, therefore, it is at the same time drawn downwards, and pushed directly upwards, by the action of two equal and opposite forces. The incumbent weight may be considered as collected in its centre of *Gravity*, and the sustaining efforts as united in the centre of *Buoyancy*, which is evidently the same as the centre of gravity of the water displaced, or of the immersed portion of an uniform solid. To these two points, therefore, the antagonist forces are directed; and the line which joins them, called the *Line of Support*, will have constantly a vertical position, in the case of equilibrium.

The centre of gravity of the whole mass, about which it turns in the water, must evidently continue

invariable; but the centre of buoyancy will change its relative place, according to the situation of the immersed portion of the solid. If those two centres should coincide, the body will float indifferently in any position. It will likewise float, as often as a vertical line, drawn from the centre of buoyancy, shall pass through the centre of gravity. But this will obtain whenever the line of support becomes perpendicular to the horizon. The equilibrium, however, may be either *permanent* or *instable*. It is *permanent*, if, on pulling the body a little aside, it has a tendency to redress itself, or to recover its original position; it is *instable*, when the body, on being slightly inclined, tumbles over in the liquid, and assumes a new situation. These opposite conditions will occur in a body of irregular form, when the centre of gravity occupies the highest or the lowest possible position; for, though the volume of immersion remains the same, the solid will evidently be less or more depressed in the fluid medium, according to the width of its section or *water-line*.

If the centre of buoyancy stand higher than the centre of gravity, the floating body will, in every declination, maintain its *stability*, and regain its perpendicular position; for, though made to lean towards either side, the vertical pressure exerted against that variable point, will soon bring it back again into the line of support. But the elevation of the centre of buoyancy above that of gravity, is by no means an essential requisite to the stability of flotation; on the contrary, it falls in most cases con-

siderably below the centre of gravity about which the body rolls. The buoyant efforts may be considered as acting upon any point in the vertical line, and, consequently, as united in the point where this line crosses the axis of the floating body. If the point of concourse thus assigned should stand above the centre of gravity, the body will float firmly, and will right itself after any small detrusion; if it coincide with the centre of gravity of a homogeneous body, this will continue indifferent with regard to position; but if the vertical should meet the axis below the centre of gravity, the body will be pushed forwards, its declination always increasing till it finally oversets.

To begin with simple and regular bodies: Let a wooden sphere, of uniform consistence, be set to float in water. It will sink, till the weight of the fluid displaced by the immersed portion HFI (fig. 144.) shall be equal to its own load. The centre of gravity of this body is obviously C, the centre itself of the sphere. But the centre of buoyancy B must be the centre of gravity of the volume of immersion HFI, and will therefore lie below C, in the axis KCF perpendicular to the water-line or plane of flotation HI. The ball is hence pressed down by its own weight collected at C, and pushed up, in the opposite direction, by an equal force combined at B, both of the forces, however, concurring in the same point C. Wherefore, being always held in equilibrium by those

antagonist forces, it will remain still in any position which it happens to occupy. But this indifference to floating will obtain only, when the sphere is perfectly homogeneous, and its centre of gravity coincides with the centre of magnitude; for otherwise the former, descending as low as possible, would always assume a determinate position.

Suppose next a mere segment of the sphere to float in water. The centre of gravity G (fig. 145.) of this homogeneous body, will now lie below the centre of the sphere, and in the axis AF at right angles to its base DE; but B, the centre of buoyancy, or the centre of gravity of the immersed segment HFI, must, in every situation of the floating mass, occur in a perpendicular bisecting the water-line HI, and consequently passing through the centre of the sphere. In the case of equilibrium, this perpendicular must have a vertical position, or the inverted base of the segment must form a horizontal plane. Suppose the body to be drawn aside into the position D′F′E′; it will then be pressed down by its own weight collected at G, and pushed upwards by an equal buoyant power exerted at B, in the opposite direction BC. But this force may be conceived to act upon any point in the line BC, and therefore at the centre C, the concourse of the two axes CF and CF′. The buoyancy transmitted to C hence pushes the axis CF′ obliquely, the greater part of it heaving the point C in the direction FC, and an-

other small part pressing C perpendicular to CF′ or CG′, and making the body turn about its centre of gravity G, from E′ towards E. Every derangement is thus corrected by a restoring energy, which maintains a permanent equilibrium.

Suppose an oblate homogeneous spheroid (fig. 146.) were now substituted for the entire sphere. Since its transverse sections are proportional to the corresponding sections of a sphere described about the shorter axis, it will sink to the same depth AF as before, and carry the centre of buoyancy B into a like position. The declination of the axis from CF to CF′, occasions that point to shift its relative place from B to B′. The change, however, is produced, by the wedge EAE′ being joined to the one side of the immersed segment, and the wedge DAD′ taken away from the other side. Wherefore the bulk of the half segment AFE is to that of the accrescent wedge EAE′, as AO, the distance of the centre of gravity of the former, to BB′, the lateral evagation of the centre of buoyancy. That proportion between the segment and its wedge is the same, both in the sphere and the spheroid; but the distance AO, being increased by the oblateness of the figure, the variation BB′ is augmented in the like ratio. Now the triangles BcB′ and EAE′ being similar, it follows that Bc must be greater than BC, and that the effort of buoyancy is exerted at a point above the centre of gravity of the spheroid. This verti-

cal thrust tends consequently to redress the floating body, and to secure its stable equilibrium.

On the other hand, let a prolate spheroid (fig. 147.) be immersed in water or any other liquid. Its plane of flotation, and its centre of buoyancy will each have the same height, as in a sphere described on the longer axis. But the shifting of that point from B to B′ will be diminished in proportion to the narrowness of the spheroid. Wherefore the vertical B′*c* will meet the principal axis below the centre of gravity of the solid, and will push it still more aside, till the spheroid falls, and extends its longer diameter in a horizontal position. It may then roll indifferently upon that line, as the sphere itself turns about its centre.

But to investigate the subject more generally, let HAIF (fig. 148.), a solid of any form, not abruptly irregular, be set to float in water. The principal axis AF will divide it into correspondent equal portions, and will cross the plane of flotation HNI at right angles. Conceive the body to be now slightly inclined, the water-line being moved into the position H′NI′, and the corresponding vertical into AF′: The change AB′ of the centre of buoyancy or of the centre of gravity of the submerged part HFI, will be to the distance NO of the lateral wedge INI′, as the contents of this wedge is to the capacity of the segment NIF. Wherefore BB′ is proportional to the sine of the angle INI′ or of

BNB′, and consequently BM is proportional to the sine of the angle BB′M or to the radius, and must thus be constant. While the floating solid thus varies its inclination, and the centre of buoyancy shifts its place, the point M in the axis where the effort to redress the body is exerted remains unaltered, like the centre of gravity itself. Such, at least, is its constancy in all moderate oscillations; and, from the relative position it holds, all the properties of floating bodies are easily derived. That characteristic point, standing always above the centre of gravity of the mass, and limiting its greatest elevation in the case of permanent stability, was hence called by Bouguer, to whom we are indebted for all this fine theory, the *Metacentre.*

Let the floating body be a homogeneous parallelopiped (fig. 149). If placed vertically in the liquid, it will evidently sink, till the immersed part NF shall be to its whole height AF, as its density is to that of the fluid. The centres of gravity and of buoyancy will evidently be the points C and B, in the middle of the axis AF and of its depressed portion NF. Conceive the solid to be now inclined to one side, shifting its water-line from the position HNI into H′N′I′. The centre of buoyancy, in making a corresponding change from B to B′, will describe a small arc of a circle; for B′ will be raised in relation to the altitude OP of the centre of gra-

vity of the triangle INI′, as the area of the rectangle NIF to that of the triangle, while B′ is carried laterally in the same ratio. But the rectangle NIF is to the triangle INI′, as NF to $\frac{1}{2}$II′, and therefore NB=$\frac{2}{3}$NI; whence NF : $\frac{1}{2}$II′ : : $\frac{2}{3}$NI : BB′, and BB′ $= \frac{\text{NI.II}'}{3\text{NF}}$. But, from similar triangles, BB′ : BM : : II′ : NI, and consequently the height BM of the metacentre is $\frac{\text{NI}^2}{3\text{NF}}$ or $\frac{\text{HI}^2}{12\text{NF}}$. Let AF, the altitude of the parallelopiped, be denoted by a, its breadth or thickness HI by b, and its density by x. When the metacentre coincides with the centre of gravity, and the solid floats passively and indifferent to its position, BM is equal to BC or to $\frac{\text{AF}-\text{BF}}{2}$, and therefore $\frac{b^2}{12ax} = \frac{a-ax}{2}$. Whence, by reduction, we obtain $2b^2 = 12a^2x - 12a^2x^2$, and the quadratic equation $x^2 - x = -\frac{b^2}{6a^2}$. Of which, the two roots are $\frac{1}{2} \pm \sqrt{\left(\frac{3a^2-2b^2}{12a^2}\right)}$. If the parallelopiped become a cube, then $a=b$, and the two densities of indifferent flotation are $\frac{1}{2} \pm \sqrt{\frac{1}{12}}$, or expressed in approximate numbers $\frac{71}{90}$ and $\frac{19}{90}$. Between these limits, there can be no stability; but above, or below them, the floating again acquires permanence. Hence a cube of beech will float erect in water,

while one of fir or cork will overset; yet all these three cubes will stand firmly when set upon the surface of mercury.

Let the radical part of this expression vanish, and $3a^2=2b^2$, or, in approximate numbers, $11a=9b$. Hence a parallelopiped, of half the density of water, and having 9 inches for its altitude and 11 inches for the side of its square base, would float indifferently. But if its density were either increased or diminished, it would gain stability. Thus, if the density were two-thirds of that water, the metacentre would stand $\frac{13}{72}$ parts of an inch above the centre of gravity, and $\frac{15}{36}$ above it, were the density reduced to one-third. A parallelopiped, with such proportions, might therefore in every case continue erect.

It might likewise be proved, that if the parallelopiped were set upon water with one of its solid angles upwards, the stability would be limited within the densities of $\frac{9}{32}$ and $\frac{23}{32}$. In short, if the specific gravity were greater than $\frac{9}{32}$, or less than $\frac{23}{32}$, the solid would permanently float in that position; but if this were either less than the former, or greater than the latter, it would overset.

A cylinder will, according to its density and the proportion of its diameter and altitude, exhibit the three features of a floating body,—indifference, instability, or permanence of equilibrium. When this uniform solid is placed with its axis horizontal, it

will sink always to the same depth in the water, and the centre of buoyancy will occur likewise at a given depth in a perpendicular from the middle of that line to the place of flotation, and therefore in the vertical passing through its centre. The cylinder thus levelled may hence repose in a horizontal position, the opposite forces being constantly balanced.

Let the cylinder be now set perpendicularly upon its end in the water. It will evidently sink, till its depth NF (fig. 150.) of immersion shall be to its whole height AF, as its density is to that of the fluid. The point C in the middle of the axis will be the centre of gravity of the cylinder, and B, the middle of the immersed portion, will mark the centre of buoyancy. Suppose the floating body to be pressed to one side, so that the water-line shall change its position from HNI to H′NI′; the centre of buoyancy, from the accession of the circular wedge INI′ and the defect of H′NH′, must likewise proportionally shift its place, from B to B′. But it may be shown that, π denoting the circumference of a circle of which the diameter is unit, $\frac{\pi}{8} \cdot NI^3 \cdot II'$ will represent the product of the contents of the circular wedge INI′ into the distance of its centre of gravity from N. Consequently the quotient of this expression by $\frac{\pi \cdot NI^2 \cdot NF}{2}$, the solidity of the

half cylinder NFI, will give the evagation BB′ of the centre of buoyancy, which is therefore $=\frac{NI.II'}{4NF}$. But the triangles INI′ and BMB′ being evidently similar, $II' : NI :: BB' : BM$. Whence, by substitution, the altitude BM of the metacentre is $\frac{NI^2}{4NF}$ or $\frac{HI^2}{16NF}$. Let the height of the cylinder be denoted by h, its diameter by d, and its specific gravity by x; then $NF = hx$, $BC = \frac{h-hx}{2}$, and $BM = \frac{d^2}{16hx}$. Wherefore an equilibrium of indifference will obtain, when $\frac{h-hx}{2}$ is equal to $\frac{d^2}{16hx}$, or $8h^2x - 8h^2x^2 = d^2$, and hence the quadratic equation $x^2 - x = -\frac{d^2}{8h^2}$. Of which, the two roots are $x = \frac{1}{2} \pm \sqrt{\left(\frac{2h^2-d^2}{8h^2}\right)}$. If $2h^2 = d^2$, the part affected by the radical sign vanishes, and these two roots become equal. When the density of the cylinder, therefore, is .5, and the ratio of its diameter to its altitude is that of 1 to $\sqrt{2}$, the solid will float indifferently in its vertical position. Suppose, for the sake of round numbers, the height of the cylinder to be 12 inches, and the diameter of its base 17 inches: It will sink 6 inches in the water, and con-

sequently the centre of buoyancy will lie 3 inches below the centre of the mass. But the metacentre will meet in the same point, for $\frac{(17)^2}{16.6} = \frac{289}{96} = 3$. If the density were increased to .75, the depression would be 9 inches, and the distance of the centre of buoyancy from the general centre of gravity only $1\frac{1}{2}$ inches. Now $\frac{(17)^2}{16\times9} = \frac{289}{144} = 2$; so that the metacentre would stand half an inch above the centre of the cylinder, and would therefore secure its equilibrium. Were the density of the solid reduced, however, suppose to one-third, the depression would be only four inches, leaving the centre of buoyancy likewise 4 inches below the centre of gravity. But $\frac{(17)^2}{16\times4} = \frac{289}{64} = 4\frac{1}{2}$ = the distance of the metacentre. The cylinder will hence, with this diminished density, yet maintain a stable equilibrium.

Let the diameter and altitude of the cylinder be equal; then $x = \frac{1}{2} \pm \sqrt{\frac{1}{8}}$, and the two values of the density, expressed in approximate fractions, are $\frac{17}{20}$ and $\frac{3}{20}$. Any intermediate density will be attended with instability. Thus, suppose the density of the cylinder were three-fifths, its height being 20 inches, it would sink 12 inches, and bring the centre of buoyancy 6 inches below the general centre

of gravity. But $\frac{(20)^2}{16\times12}=\frac{400}{192}=2\frac{1}{12}$, the altitude of the metacentre, which therefore lies below the centre of gravity, and must, from its position, overset the cylinder. On the other hand, if the density transgress those limits, whether in excess or defect, a permanent equilibrium will result. Supposing the density to be .9, the depression is 18 inches, and $\frac{(20)^2}{16\times18}=\frac{400}{288}=1\frac{7}{18}$; so that, the metacentre stands $\frac{7}{18}$ of an inch above the centre of gravity. Again, let the density be .1, the corresponding depression is 2, and $\frac{(20)^2}{16\times2}=\frac{400}{32}=12\frac{1}{2}$; the metacentre being now $3\frac{1}{2}$ inches above the centre of gravity. Hence a wooden cylinder of equal height and diameter, which cannot stand erect in water, will yet maintain a vertical position when set upon quicksilver.

Suppose, lastly, the floating body to be a parabolic conoid, (fig. 151.) with its base upwards and its axis vertical. The part immersed having only half the contents of the corresponding portion of the cylinder, the height BM of its metacentre is $\frac{NI^2}{2NF}$, or half the parameter of the generating parabola. But the centres of gravity of the whole and of the submerged part occur at two-thirds of the respective altitudes, while the bulks or weights are as the

squares of those altitudes. Wherefore, assuming the former notation, it follows that the equation $\frac{2}{3}(h-h\sqrt{x})=\frac{d^2}{8h}$ will correspond to the case of indifferent equilibrium. Now, by reduction,
$16h^2-\sqrt{x}\,.\,16h^2=3d^2$, and $\sqrt{x}=1-\frac{3d^2}{16h^2}$ or
$x=\left(1-\frac{3d^2}{16h^2}\right)^2$. When $3d^2=16h^2$, the value of x vanishes; but so long as the ratio of $\sqrt{3}$ to 4, or that of 13 to 30 nearly, subsists between the altitude and the diameter of the solid, its equilibrium will remain firm at every given density. If the altitude and diameter become equal, the specific gravity which would occasion indifference of flotation is $\frac{169}{256}$ or .66. Any increase of density beyond this limit would procure stability. Let p denote the parameter of the generating parabola, and $4ph=d^2$, or $ph=3d^2$; whence $1-\frac{3p}{4h}=\sqrt{x}$. Thus, while the parameter continues invariable, the altitude of the parabola may be made to procure indifference of floating, for every degree of density. The corresponding depth of immersion is xh, or the portion of the axis above the water-line is expressed by $\frac{8h-3p}{4h}\,.\,3p$.

These remarks will explain the cause of the over-

setting of the huge masses of ice which float within the Arctic Circle. Such enormous blocks are generally of a columnar shape, approaching to the form of a parallelopiped or of a cylinder, though perhaps occasionally contracted below. The upper surface thaws slowly, from the action of the atmosphere, as the summer advances; the under side likewise melts at first, but becomes soon protected by a stratum of fresh water of the same temperature, consisting of the dissolved portion of the ice which is upheld by the superior density of the surrounding medium. The principal waste of the icy mass taking place along its submerged sides, the current of melted water continually rises upwards, and leaves a new surface to the attack of a warmer current. After the width of the vast column has become thus diminished beyond a certain ratio to its height, it tumbles over upon its side.

To determine this limit, suppose the floating block to be first a parallelopiped. Let the height a be denoted by h, and the side b of the base by hy; since the density of ice compared with that of sea-water is .89, the depth of submersion will be .89 h. Wherefore the metacentre will coincide with the centre of gravity, when $\frac{h^2y^2}{12\times .89h} = h\times .055$. Consequently, by reduction, $y^2 = .5874$, and $y = .766$. Hence the icy parallelopiped will overset, the moment its breadth approaches to three-fourths of its

altitude. Thus, if the whole height of the mass were 1000 feet, 890 feet would sink, leaving 110 feet to tower above the surface. The elevation of the centre of gravity beyond that of buoyancy would hence be 55 feet, which is the limit of the metacentre after the base of the column has been reduced to a breadth of 766 feet.

Again, suppose the ice to have a cylindrical form. Assuming the same data as before, we shall have $\frac{a^2y^2}{16\times.89a}=\frac{a-.89a}{2}$, and hence $y^2=16\times.89\times.055$ $=.7832$. Wherefore $y=.885$. A cylindrical mass of ice 1000 feet altitude would sink 889 feet in sea-water; but when the diameter of its base was reduced to nearly the same quantity, or 885, it would overset. The instability of the cylinder thus begins sooner than that of the parallelopiped, taking place when the width below becomes eight-ninths instead of three-fourths of the whole height. But the greater extension of the summit, owing to its slow waste, may in every case hasten the period of overwhelming the icy column.

Lastly, conceive the block of ice to be wasted and rounded below into the shape of a parabolic conoid. At the moment when its equilibrium fails, we have seen that $\sqrt{x} = 1-\frac{3d^2}{16h^2}$. But the density of ice in relation to sea-water being .89, it follows that the

square root of this, or .9435 = $1 - \frac{3d^2}{16h^2}$, and hence $\frac{3d^2}{16h^2} = .0565$; wherefore $\frac{d^2}{h^2} = .30133$, or $\frac{d}{h} = \sqrt{.30133} = .549$. Whenever the mass has its base reduced in the ratio to its depth of about 11 to 20, it will suffer a total inversion, its lowest point being whelmed uppermost. This form of a body of ice would, therefore, suffer a greater previous waste; but its balance is at last more completely destroyed. Stability in every case becomes precarious, after the breadth of a block is inferior to its depth.

To investigate generally the conditions of a floating body, let HAIF (fig. 148.) represent the vertical section, the point C being the centre of gravity of the whole, and situate in the principal axis ANF, while B indicates the centre of gravity of the immersed portion HIF, or the centre of buoyancy. When the solid, declining from its vertical position, changes the water-line from HNI to H′NI′, the centre of buoyancy shifts from B to B′. It is evident that the area of the segment HIF will be to the areas of both the accrescent triangle NII′ and the deficient triangle HNH′, as NO, the distance of the centre of gravity of the former, is to BB′. But the areas of those triangles being equal to NI × II′, and the distance NO equal to two-thirds of NI; it follows that the

area of HIF is to NI × II, as $\frac{2}{3}$NI to BB′. Again, from the similar triangles IAI′ and BNB′, II′ is to NI as BB′ to BM; wherefore, by compounding those analogies, the area of HIF is to the square of NI, as $\frac{2}{3}$NI is to BM, the height of the metacentre above the centre of buoyancy. Hence, the area of the section HIF being denoted by A, and the width of the water-line HNI by b; $A : \left(\frac{b}{2}\right)^2 :: \frac{b}{3} : BM$, or $A : \frac{b^2}{4} :: \frac{b}{3} : BM = \frac{b^3}{12A}$. *The Flotation will hence be stable, only when the quotient of the cube of the breadth HI by twelve times the area of the segment HIF exceeds the interval BC between the centre of gravity and that of buoyancy.* The stability of a ship is hence greatly augmented by increasing the dimension of the water-line.

The area of any section, and consequently the position of its centre of gravity, may be determined, to any degree of accuracy, by the method proposed by Newton, and improved by Stirling and Simpson, of drawing a line of the parabolic kind through any given system of points. If the parallel sections were all equal, the calculation of the altitude of the metacentre would be concise and direct. When those sections are unequal, the area of each is to be multiplied into the corresponding height of the metacentre, and the sum of the products divided by the aggregate contents of the immersed part of the solid. In

the case of a merchant ship, it may furnish a tolerable approximation to take the section near to the prow, where the girth is commonly largest.

It will not differ much from the truth, to assume the cross section of the hull of a ship as of the form of a parabola. Wherefore, the height of the metacentre above the centre of buoyancy, or BM (fig. 152.) is equal to the cube of HI, the breadth of the water-line, divided by twelve times the area of HFI, or of two-thirds of the rectangle under HI, and the depth of immersion NF. Consequently, $BM = \frac{HI^3}{8HI.NF} = \frac{HI^2}{8NF} = \frac{NI^2}{2NF}$. BM is hence equal to half the parameter of the parabola, or to the double of its focal distance. But FB, the altitude of the centre of buoyancy, is equal to $\frac{3}{5}$NF, and therefore FB, the whole height of the metacentre above the keel, is $\frac{3}{5}NF + \frac{NL}{2NF} = \frac{6NF^2 + 5NI^2}{10NF}$.

Such is the height of the metacentre above the keel, on the supposition that the vertical sections of the hull are all equal, as may be nearly the case in long track-boats. But the figure of the keel in most vessels fitted for sailing approaches to a semi-ellipse, which is likewise the general form of an horizontal section. Owing to these conjoined modifications, the metacentre is on the whole depressed by one-fourth part, and consequently its altitude above the centre

of buoyancy will be expressed by $\frac{3}{4} \cdot \frac{HI^2}{8NF} = \frac{3NI^2}{8NF^2}$. Suppose, for the sake of illustration, a ship whose water-line is 40 feet wide, and its depth of immersion 15 feet. Here $\frac{3(20)^2}{8.15} = \frac{1200}{120} = 10$ feet; but the centre of buoyancy, or the centre of gravity of the immersed portion of the hull, falls 6 feet, or $\frac{2}{5} \times 15$ below the water-line, and hence the metacentre stands four feet above it. The ship will therefore float securely, so long as the general centre of gravity is kept under that limit, or less than four feet above the plane of flotation. In loaded vessels, the centre of gravity has been found to be commonly higher than the centre of buoyancy, by about the eighth part of the extreme breadth. Wherefore, in the present instance, the centre of the whole mass would lie still one foot below the surface of the water, or five feet lower than the metacentre, leaving ample space for maintaining the stability of the ship.

Such is the position of that metacentre situate in the vertical plane at right angles to the longitudinal axis, and which regulates the *rolling* of a vessel from side to side. But there is another similar point in the plane of the masts and keel, which determines the *pitching*, or the movement of alternate rising and sinking of the prow. The altitude of this metacentre is derived from the same formula, by

merely substituting the length for the breadth of the vessel. Thus, let the keel measure 180 feet, and $\frac{3(90)^2}{8.15} = 202\frac{1}{3}$ feet. With such powerful stability, therefore, in the direction of its course, a ship can scarcely ever founder in consequence of pitching at sea.

The formula now given for computing the height of the metacentre above the centre of buoyancy, may, with some modification, be deemed sufficiently accurate in practice. It is best adapted, however, for cutters or frigates, and will require to be somewhat diminished in the case of merchant vessels. The late Mr Atwood performed a laborious calculation on the hull of the Cuffnells, a ship built for the service of the East India Company, having divided it into 34 transverse sections, of five feet interval. The result was, that the metacentre stood only 4 feet 3 inches above the centre of buoyancy. But that ship, being designed chiefly for burthen, appears from the drawings to have been constructed after a very heavy model, its vertical sections approaching much nearer to rectangles than parabolas. To suit it, the formula above given would have required to be reduced two-thirds, or $\frac{NI^2}{4NF}$. Now, the breadth of the principal section was 43 feet and 2 inches, and its breadth 22 feet 9 inches. Whence

$\frac{(21.6)^2}{91} = 5.1$ feet, differing little from the conclusion of a stricter but very tedious procedure.

Since the height of the metacentre is inversely as the draught of a vessel, and directly as the square of its breadth, its stability depends mainly on its spreading shape. This property is an essential condition in the construction of life-boats. But the lowering even of the centre of gravity has been found to be sometimes insufficient to procure stability to new ships, which, after various ineffectual attempts, were rendered serviceable, by applying a sheathing of light wood along the outside, and thus widening the plane of flotation.

It is not very difficult to determine the centre of buoyancy, by guaging the immersed part of the hull. A cubic foot of sea-water weighs 64 lb. averdupois, and thirty-five feet, therefore, make a ton. The load of the vessel corresponding to every draught of water may be hence computed. But to find the place of the centre of gravity is more difficult; since this depends less upon the figure of the hull than upon the disposition of its internal burthen. Conceive a horizontal plane to touch the bottom of the vessel, and let the weight of each part of the timbers and of the cargo be multiplied into their height above it; the sum of all these products, divided by the accumulated load, will give the altitude of the

centre of gravity. Again, suppose another plane at right angles to the longitudinal axis of the ship, to touch its bow; the perpendiculars drawn to this, from every part of the hull and cargo, are to bé multiplied into the several weights, and the quotient of the total amount by the general load will give the distance of the common centre of gravity. Such a calculation, however, would involve a tedious multiplicity of details.

The height of the metacentre above the centre of gravity in a loaded vessel, may be discovered by a simple observation. Let a long, stiff, and light beam be projected transversely from the middle of the deck, and a heavy weight suspended from its remote end, inclining the ship to a certain angle, which is easily measured. Thus, if NL (fig. 151.) represent this lever, P the weight attached, M the metacentre, and CMQ the declination produced, C being the centre of gravity, and CR a perpendicular drawn from it to the vertical LP. The power of the weight P to redress the vessel will be expressed by $P \times CR$; but, W denoting the entire load, the effort exerted at the metacentre to keep the mast erect, will be represented by $W \times CQ$, or $W \times CM \times \sin CMQ$. Wherefore $P \times CR = W \times CM \times \sin CMQ$, and consequently the elevation CM above the centre of gravity is expressed by $\frac{P}{W} \cdot \frac{CR}{\sin CMQ}$. Now CR may, without any sensible error, be assumed as equal to the length LM

of the beam from the middle of the deck. Supposing the height of the metacentre to be 3 feet $10\frac{1}{4}$ inches above the centre of gravity, a weight equal to the two hundredth part of the burthen or tonnage of the ship, and acting on a lever of 50 feet in length, would occasion a declination of five degrees. If the experiment were performed in a wet-dock, or on a smooth calm sea, such small angle could be measured with sufficient accuracy. In calculating the effect of this disturbing influence, it is easy to perceive, that half the weight of the beam should be annexed to P. The distance MB of the metacentre from the centre of buoyancy, having been ascertained by a previous computation, the height BC of that unvarying point above the centre of gravity is thence deduced. A trifling correction may be likewise made, for assuming CR as equal to NL; by diminishing NL first, by its product into the versed sine of the inclination, and next augmenting it, by the product of CN into the sine of that angle. The same result might be obtained, in ships of war, by a more complex calculation, indeed, from the ordinary process of *heeling*, when the guns are all run out upon one side.

A similar observation might discover the altitude of the longitudinal metacentre of the ship, above the common centre of gravity. But, acting in this direction, a greater load will be required to produce a sensible depression. Let such a load be carried to the prow of the vessel, and again transferred to the

stern. The intermediate place of the centre of gravity is hence determined, for its distances from those opposite points of pressure must evidently be inversely as the corresponding angles of inclination. The small aberration of the centre of gravity occasioned by the interchange of these loads, may likewise be computed. Finally, therefore, the product of either load into its distance from the centre of gravity, being divided by the product of the whole burthen of the ship into the sine of the inclination, will give the height of the metacentre of the longitudinal section on which depends the motion of pitching.

When a floating body has its equilibrium disturbed, it brings into action a redressing force, which is always proportioned to the quantity of derangement. It is hence made to oscillate like a pendulum about its general centre of gravity; and, for the same reason, its alternate movements, whether of less or greater extent, are still performed in equal times. But any ship is liable to three distinct kinds of oscillations—the *vertical*, the *transverse*, and the *longitudinal*, or what is called *heaving*, *rolling*, and *pitching*. Thus, if a vessel were lifted a little above its seat of flotation, and then allowed to sink into the water, it would continue for some time afterwards alternately to rise and fall. If the vessel were suddenly pushed to one side it would roll upon its longitudinal axis; but, if it were depressed at the head or the stern, it would begin to pitch or spring about the transverse axis.

1. The alternate heaving and subsiding of a ship in the vertical direction, is occasioned by the fluctuating excess or defect of the buoyant effort of the water. The time of oscillation is therefore the same as that of a pendulum, whose length is equal to the mean depth of immersion, or to the quotient of the capacity of the submerged portion of the hull, by the horizontal surface of flotation. This medium draught of water may be reckoned at about two-third parts of the extreme depth of the keel. Thus, a ship which draws 20 feet may have 13 feet allowed for its mean depth, which is four times the length of a second's pendulum. Each oscillation upwards and downwards of such a vessel will hence take two seconds.

2. The time required for the rolling of any ship, though performed about its general centre of gravity, must be the same as that of its vibration, if it had been suspended like a pendulum from its metacentre. From the principle investigated in p. 99, the distance of this centre of gravity from the centre of oscillation, is equal to the quotient of the momentum of rotation, or of the sum of the products of all the different bodies into the squares of their several distances from the longitudinal axis, divided by the product of the whole mass into the interval between their common centre of gravity and the metacentre. It hence follows, that while the distribution of the cargo remains the same, the time of rolling, or of

lateral oscillation, will be inversely as the square of the elevation of the metacentre above the centre of gravity. Again, those alternating motions may be rendered proportionally slower, by removing the various articles of the cargo to a greater distance from the longitudinal axis, above and below the centre of gravity, to both the sides of the vessel. If the lateral distances were doubled, the oscillation would be rendered twice as slow; and if tripled, they would become three times slower.

Suppose the floating body were an homogeneous parallelopiped, of which the height is a, and the breadth b: It may be shown that the momentum of rotation about the centre of gravity, is expressed by $\frac{a^3b+b^3a}{12}$, and consequently the distance of the centre of oscillation below the centre of gravity is equal to $\frac{a^2+b^2}{12}$ divided by the relative elevation of the metacentre. Again, if the transverse section of the floating body were considered as a parabola, of which the height and breadth are denoted as before by a and b: It may be computed that the momentum of rotation is then equal to $\frac{144a^3b+105ba^3}{3150}$; whence, dividing this by $\frac{2ab}{3}$, or the area of the parabola, the quotient $\frac{48a^2+35b^2}{700}$ will express the pro-

duct of the distances of the common centre of gravity from the metacentre above, and from that point below it, which represents the centre of oscillation.

Suppose, for example, a parallelopiped 80 feet broad and 48 feet deep, of an uniform density, but three times less than that of water: It would evidently sink 16 feet, and then float. The centre of buoyancy lies hence 8 feet below the water-line, while the centre of gravity stands 8 feet above it. But the height of the metacentre above the centre of buoyancy being $\frac{(80)^2}{12.16} = 33\frac{1}{3}$ feet, it must exceed by $17\frac{1}{3}$ feet the altitude of the centre of gravity. Now the momentum of rotation, divided by the weight of the floating body or $\frac{80^2+48^2}{12} = \frac{8704}{12} = 725\frac{1}{3}$ feet; and the quotient of this, by $17\frac{1}{3}$ feet, gives $41\frac{11}{13}$ feet, for the length of an isochronous pendulum. The floating parallelopiped would consequently roll or oscillate, in 3.58″, or in about $3\frac{1}{2}$ seconds.

Conceive a body of the same density and dimensions, but of a parabolic form, to float in water. Its depth of immersion will be 23.08 feet, and therefore the centre of buoyancy will fall 9.23 feet below the plane of flotation, while the common centre of gravity stands 5.720 feet above it. The breadth of the water-line being now 55.47, the altitude of the metacentre above the centre of buoyancy is

hence $\frac{(55.47)^2}{8 \times 23.08} = 16.86$ feet; so that the metacentre is only 1.91 feet higher than the centre of gravity. Now $\frac{48(48)^2 + 35(80)^2}{700} = 478$ feet, the quotient of the momentum of rotation by the weight of the parabolic section; this number, again, being divided by 1.91, gives 250.26 feet, the length of a pendulum which would oscillate in concord with the rolling of the body. Those vibrations are hence performed very slowly, each taking 8.76″, or about $8\frac{3}{4}$ seconds.

In the cases now computed, the transverse sections are all equal. But, to determine the rolling of a ship, it would be requisite to combine the measures of different sections of the hull, and thence deduce a medium result. To abridge this calculation, however, it may perhaps be sufficiently correct, if we consider the distance of the general centre of oscillation as equal to two-thirds of the distance which corresponds to the greatest cross section of the vessel.

On the stowage of a ship's cargo, depends chiefly the character of the rolling. If the goods be placed nearer the longitudinal axis, the momentum of rotation being diminished, the rolling will in consequence be quickened. But if the various articles of loading be removed, as far laterally as possible from the position of the centre of gravity, the oscillations

will be rendered proportionally slower. When the cargo consists of light goods of the same kind, the hold is quite filled up, and no room left for skilful stowage. But if it include many ponderous articles, the rolling may be damped, and the motion of the vessel rendered easier, by bringing those loads near to its sides. In ships of war, such a change of trimming, by the removal of the guns and shot, is occasionally practised, with the greatest advantage.

3. The time of oscillation about the transverse axis, or that of *pitching*, may be found by a similar calculation. The various loads in each vertical section parallel to the keel, are to be multiplied into the squares of their several distances from the centre of gravity; and the quotient of the collective aggregate by the whole burthen of the vessel, will express the product of the distances of the general centre of gravity from the longitudinal metacentre and the centre of oscillation. It may be sufficiently near the truth, to take two-thirds of the result of a computation grounded on the chief section in the plane of the masts and of the keel. The momentum of rotation of the longitudinal section is to that of the cross section, nearly as the square of the length of the ship is to the square of the breadth; but the altitudes of the corresponding metacentres being likewise approximately in that ratio, the times of rolling and pitching will in all cases of uniform stowage approach to an equality. The disposition of a ship to

violent plunging may often be corrected, by conveying the heaviest part of the cargo towards the head and the stern. The remoteness of the lading from the common centre of gravity, serves materially to retard and soften the oscillatory movements, both about the longitudinal and the transverse axis.

Let the parallelopiped of 80 feet wide and 48 feet deep be resumed as an example, its length being now considered as 480 feet. Here the altitude of the longitudinal metacentre above the centre of buoyancy is $\frac{(480)^2}{12.48} = 400$ feet, and consequently its height above the common centre of gravity must be $400 - 16 = 384$ feet. But the quotient of the momentum of rotation of each longitudinal section divided by its weight, is $\frac{(480)^2 + (48)^2}{12} = 19392$ feet, which being divided by 984, gives *50.5* feet for the length of an isochronous pendulum. The time of oscillation is hence 3.985″, or very nearly 4 seconds. It is obvious that such problems could easily be reversed, and that the height of the metacentre above the centre of gravity, could be computed from the different oscillations in smooth water.

While the shape and the lading of a vessel continue the same, the height of its centre of buoyancy and the derivative position of the metacentre must likewise remain unaltered. By lowering the places of the more ponderous articles, the common centre

of gravity may be depressed or brought nearer to that of buoyancy; and by dispersing them towards the sides and extremities of the hull, the oscillations of rolling and pitching may be retarded and rendered smoother. Such are the only internal changes which can be made; but it is of great importance in many cases to lessen the draught of water, or to lower the elevation of the centre of gravity. This can only be effected, however, by supplying externally the requisite additional buoyant force.

Since a body submerged in water is pushed upwards by the effort of the volume of fluid which it displaces, its whole power of buoyancy must be equal to the excess of that pressure above its own weight. So long, therefore, as the bulk remains unaltered, this power will increase with every diminution of the weight. Hence the buoyancy procured to a floating body by the application of inflated bladders or bottles of skin, empty casks or hollow chests. Bladders are sometimes employed to assist swimmers, and they support the fishermen's nets. Bags of goats skins, covered with boards, have from the earliest times been used as floating bridges among rude nations for crossing rivers. The Egyptians have been accustomed in all ages to descend the Nile on slender rafts, carrying their produce, though supported only by empty earthen jars. A hollow wooden box, of a conical shape, and attached by an iron chain, serves at present as a buoy, for marking the position of sunken rocks or sand banks. Vessels which have

been stranded are often raised up again, by fastening at ebb tide, a row of empty casks along each side. Long boxes or chests, lightened after their fixing, by having the water pumped out, have likewise been employed to lessen the draught of ships of burthen. In this way, the Hollanders, in the year 1672, when their commercial prosperity was at its height, dispatched to various climes, numerous heavy laden vessels, thus conducted from the harbour into deep water. But the raising of the hull in this method was found to occasion an overstraining of the timbers, the external pressure of the water being suddenly withdrawn from the bilging sides. But in the year 1688, Bakker, an ingenious burgomaster of Amsterdam, obviated that objection, by a very useful contrivance, which, from its property of transporting immense bodies, was termed a *Camel.* It consisted of two huge chests, so formed as to embrace closely under water the hull of the largest ships: Its length was 127 feet, its breadth at the one end 22, and at the other 13 feet. These chests were securely fastened by ropes passed under the keel, and stretched by horizontal windlasses. Oblique props or stays were then applied, and wedged firmly, to support the ship's sides. The water was now vigorously pumped out of the hold of these chests, which were divided into several compartments, for the greater convenience of adjusting, during that operation, the balance of the ship. Thus lightened, an Indiaman which drew 15 feet water,

had its draught reduced to 11 feet. The largest vessels were hence enabled to effect the passage of the Pampus, between two sand banks opposite the mouth of the river *Y*, twenty-five miles below the city of Amsterdam. This simple but valuable invention, though scarcely known in England, has long been adopted at Venice and other parts of the Continent. It has likewise been introduced into the rivers and ports of Russia, where some of those Camels are constructed of the enormous dimensions of 217 feet long, and 36 feet broad.

From their aptitude to receive and propagate every impression, fluids derive the capital property of maintaining the same level in any system of connected vessels. But this general principle is subject to certain modifications. Thus, water is observed always to stand somewhat higher in narrow glass tubes than in those of greater width. Alcohol manifests a similar disposition, inferior only in degree. Neither of these liquids, however, appears to rise above the level in the finest metallic pipes, but rather betrays an opposite tendency. Quicksilver, on the other hand, suffers some depression in glass tubes of narrow bores; yet recovers its elevation, or even mounts higher, when the inside is lined with the thinnest film of bees-wax or tallow.

This occasional rise of water, and depression of mercury, in very narrow tubes, must evidently proceed from the operation of some peculiar attractive

or repulsive force existing among the particles of the fluid, or between them and the surface of the glass. As the effect appears the most conspicuous, when the width of the bore is so small as to resemble that of a hair, the cause of the phænomenon has been termed *Capillary Action*. The popular mode of explaining the fact, is to refer the suspension of the slender column of water to the attraction of the interior ring of glass immediately above it. But why should not the ring just below the summit of the column attract it equally downwards? And such opposite forces producing a perfect equilibrium, the water would merely preserve its level, and show no disposition to rise in the tube.

The chief obstacle in explaining the mode of capillary action, comes from the prejudice, that a *vertical* attraction is necessary to account for the elevation of the liquid. Yet such undoubtedly is not the primary direction of the force evolved; for the action of the glass being evidently confined within very narrow limits, this virtue must be diffused over the internal surface of the tube, and must hence exert itself *laterally*, or at right angles to the sides. Nor is it difficult to conceive how a lateral action may yet cause the perpendicular ascent. It is a fundamental property of fluids, that any force impressed in *one* direction, may be propagated equally in *every* direction. The tendency of the fluid, then, to approach the glass, will occasion it to spread over

the internal cavity of the tube, and consequently to mount upwards.

But to view the matter a little more closely: Suppose a drop of water were laid upon a clean horizontal plate of glass; it will change its globular form, adhere to the glass, and spread out, till it has covered the whole surface with a thin aqueous film. What is the cause of this result? Surely not the mere incumbent weight of the water, for it would have been insufficient to surmount even the mutual adhesion of the particles of the fluid, or their natural tendency to conglomerate. But the same effect will take place, if the drop be applied to the under side of the plate. The water, therefore, diffuses itself over the glass, just in the same manner as if it were urged by the pressure of a column of its own substance acting against that surface. Its attraction to the glass produces the lateral motion of the fluid, since the remoter particles cannot approach without spreading themselves and extending the film. The cohesive power will consequently augment with the gradual approximation, till this has attained the term of closest union.

Let the plate be dipped perpendicularly in a bason of water, and the film will suffer a very considerable modification. A new portion of liquid will greedily attach itself to the film, and draw it downwards by this additional load. A fluid margin is formed at the line of junction, with a depressed incurvation, extending to the distance of about the

sixth part of an inch. Suppose next that two glass plates, held parallel and adjacent, were set perpendicular to a body of water, the liquid would mount in the included space to a certain altitude. The aqueous protuberance being now confined, the ascent of the column must be proportionally greater; and this effect is also doubled, by the joint action of the two opposite surfaces. Each surface acts only upon its proximate thin film; but the force being spent in supporting the ulterior particles which adhere to this, the weight of the aqueous column suspended on both sides must remain constant, and hence its altitude will be inversely as its thickness, or the distance between the two plates. The power of efficient suspension corresponding to each inch square of surface may be estimated at $2\frac{1}{2}$ grains, or the hundredth part of the weight of a cubic inch of water. Suppose the interval between those plates to be the 300th part of an inch; then each column may be considered as acting against a column of the thickness of the 600th part of an inch, and hence the corresponding ascent must be six inches. In general, if d denote the interval of the plates, $\frac{1}{50d}$ will express the height of the column suspended. This result corresponds perfectly with observation. If two rectangular plates of glass, having their edges joined in a vertical line, be opened at a very acute angle, and set upright in a shallow basin of water, the liquid will rise between the converging surface, and trace out the boundary

of a rectangular hyperbola, referred to its asymptotes. For perpendiculars being inversely to the surface of the water as the thickness of the inserted column, or the separation of the plates, must likewise bear the same inverse ratio to their distances from the centre, which is a distinguishing property of that curve.

In capillary tubes, the attraction of the internal surface is exerted on a thin circular lining; but this force is dilated and consumed by the pressure of the cylinder which adheres to the film, and occupies the interior cavity of the tube. Now, the area of a circle is equal to the rectangle under its circumference and half the radius; and therefore the attractive power of the glass will produce the same effect as if it acted merely against a column whose thickness is one-fourth part of the bore of the tube.

The measure of that force being represented by $\frac{1}{100}$, if d denote the diameter of the cavity, the altitude at which the water will be suspended in the tube is expressed by $\frac{1}{100\times\frac{1}{4}d}=\frac{1}{25d}$. A tube with the 150th part of an inch bore would hence be capable of elevating water six inches. The altitude of suspension in capillary tubes is thus the double of what obtains with parallel glass plates, which have their mutual distance equal to the diameter of the bore. This altitude is likewise in general inversely as the width of the tube.

The suspension of a fluid in capillary tubes must depend entirely on the smallness of the upper orifice. If the bore should swell out below, the water will not rise indeed spontaneously within the cavity; but, by plunging the tube into the bason till the liquid reach its capillary part, the whole included mass may be lifted up to the former elevation. The central column of the water, which has the same width as the bore, being sustained by its adhesion to the film at the top, the pressure of the parallel columns of fluid surrounding it below can have no influence in disturbing the equilibrium. The same effort of pressure extends through each horizontal stratum. The centre column of the fluid, being wholly supported by attraction, does not press against the bottom; but the particles in the higher parts of it are actually pushed by the excess of that force above their weight, and thus bear the load of the lateral mass. The lower and wide part of the vessel may consist of metal or any other substance different from glass. It is sufficient that the cavity terminate above in a fine capillary tube. By this arrangement a very large body of water may be kept suspended.

If the capillary tube be dipped in another narrow tube holding water, this will evidently stand at a lower altitude than before, since the opposite action of the outer cavity, though much inferior, is exerted in pulling back the liquid. Let d and D denote the diameters of the interior and exterior bores, and

$\frac{1}{25}\left(\frac{1}{d}-\frac{1}{D}\right)$ will express in inches the ascent above the under surface.

If a capillary tube be lifted from the bason of water in which it was dipped, a drop will adhere to the lower orifice, and the column will remain at the same height in the bore. But if this drop be absorbed by the contact of bibulous paper, the column will subside; for the tendency of the water to agglobulate then counteracts, in some degree, its capillary diffusion within the tube. Having covered the level surface of a piece of glass with a fine film of water, bring the tube with its charge to touch it, and the fluid will immediately desert the cylindrical cavity, and spread over the film. The attraction which the cylindrical column of water, joined to its weight, bears to the expansive horizontal film, overcomes that of the narrow film which lines the inside of the tube. This may be viewed as an extreme case; but the mutual attraction of the particles of water or other fluids must always diminish their ascent in capillary tubes; for such a force, tending to concentrate and agglobulate the mass, will evidently oppose any filamentous ramifications of the fluid. Thus, into a wide cylinder glass, terminating in a fine capillary orifice, mercury may be poured to the height of several inches, without betraying any disposition to run out. But, if the projecting point be likewise immersed in mercury, the moment a junction or contact takes place, the whole of the fluid mass will

hasten to subside. Mercury is, in the same manner, upheld to a considerable altitude in wide glass tubes, fitted with bottoms of iron or box-wood that have delicate slits.

The depression of mercury in capillary tubes may be estimated at $\frac{1}{68d}$, if d denote the diameter in inches. Thus, when the width of the bore is one-seventeenth part of an inch, the mercurial column stands a quarter of an inch below its level; it would sink even five inches, if the bore were contracted to the 340th part of an inch. The convexity of the surface of the mercury in the tube indicates here the excess of the mutual attraction of its particles above their adhesion to the sides of the glass. If that surface should ever appear flattened or hollow, it only betrays the impurity of the mercury, or the soiling of the inside of the tube. The notion which has sometimes prevailed, that such convexity marks a tendency of the mercurial column in the barometer to fall, is merely a vulgar error. Great caution becomes requisite, however, to boil mercury in those tubes, lest a slight and scarcely visible oxydation should line their inner surface.

It hence follows that, if the product of the elevation into the corresponding width of the bore expressed in inches, should, at any part of the tube, be less than $\frac{1}{25}$ or .04, the water will ascend still higher. Suppose the bore were tapered, therefore, like the spindle formed by the revolution of a rectangular

hyperbola about its asymptote, when the diameter exceeds not four-tenths of an inch, the fluid would remain suspended indefinitely at any altitude. If the upper orifice were sufficiently small, it is obvious that capillary action might even surpass the power of atmospherical pressure. But every practical effect is produced within this limit. Assuming it as equal to a column of 34 feet of water, or nearly 400 inches, the width of bore answering such an ascent would be only the ten thousandth part of an inch.

Capillary action is not confined to glass tubes; but is exerted among all substances which are perforated by pores, or subdivided by fissures or interstices. On this power, depend chiefly the functions of the excretory vascular system in plants and animals; and hence likewise the ascent of humidity through the shivered fragments of rocks, broken potsherds, gravel, earth, and sand. Thus, if the pores of the human skin were no finer than the three thousandth part of an inch in diameter, they would yet be sufficient to support lymph to an altitude of 120 inches, or 10 feet, or much higher indeed than is required for any individual. The rejection of the perspirable matter from those external mouths, must occasion a continued flow of the liquid from the lower and wider trunks of the capillary vessels, aided no doubt by a connected chain of alternate contractions and dilatations, extending through their mus-

cular structure. The pores in the leaves of trees and tall plants must be still finer, seldom perhaps exceeding the ten thousandth part of an inch. As fast as the humidity is exhaled into the atmosphere, it is constantly supplied by the ascent of sap from the roots.

The ingenious Dr Stephen Hales, whose experiments throw so much light on the vegetable economy, attempted to measure the *force* with which the sap mounts in the ramifying vessels of plants. He cemented a living branch into a glass tube filled with water and planted in a small cistern of mercury, and he exposed the whole to the action of the sun and air. As the process of vegetation advanced, the water continued to rise in the tube, and was followed by the quicksilver. In the space of two or three days, a column of several inches was often drawn up, though the effect varied considerably in different circumstances. But this experiment, however striking, was entirely fallacious. It did not indicate, as Dr Hales imagined, the power of the vegetating principle; for a dead branch would have given the very same results, had the evaporation from its extreme surface been sufficiently copious.

The vast energy which capillary action derives from the combination of numerous and very minute orifices, is illustrated and confirmed in a striking manner, by an application of the *Atmometer*, an instrument designed to measure with accuracy the

quantity of evaporation in a given time. A thin ball of porous earthen-ware, about three inches in diameter, is cemented by its narrow neck to a glass tube, a quarter of an inch wide, and three feet long. The whole cavity of the ball and of the tube being now filled with recently distilled water, is inverted, and set upright in a small cistern of quicksilver. The surface of the ball soon loses its glistening humid appearance; but the evaporation continues just as at first, and the water rises gradually in the tube, to supply the incessant consumption. The quicksilver follows the water, and mounts, with greater or less rapidity, according to the slenderness of the tube, and the dryness of the encircling air of the room, from 2 to 5 or even 8 inches every day. This ascension is nearly uniform for some time, till the quicksilver has gained an elevation of perhaps 20 inches. The water having then its atmospheric pressure lessened, yields a portion of its contained air, which, collecting within the ball into diffuse vapour, checks by its elasticity in some degree the rise of the water and quicksilver in the tube. However, this ponderous fluid still continues to mount, though more slowly than before; and, in the space perhaps of ten days, it stands scarcely an inch below the level of the barometric column. In this situation it remains, till the whole of the water within the ball has evaporated, and the pores are again opened to the admission of the external air. When this

event takes place, the mercury in the tube suddenly falls down into its cistern.

The attraction of the very fine pores of the Atmometer is thus more than sufficient to support a load of mercury equal to that of 400 inches of water. Those pores are hence smaller than the ten thousandth part of an inch. Hollow balls made of coarser clay would no doubt have less effect, and a gradation of power might probably be traced among them.

We may hence conceive the rise of water through successive strata of gravel, coarse sand, fine sand, loam, and even clay. If the gravel were divided into spaces of the 100th part of an inch, the water would ascend more than four inches; but supposing the particles of the coarse sand to be the 500th part of an inch, it would mount through a bed of sixteen inches of this material. Assuming the fine sand to have interstices of the 2,500th part of an inch, the humidity would be drawn up through a new stratum of seven feet thick; and if the pores of the loam were only the 10,000th part of an inch, it would gain the farther height of twenty-five feet and a half through that soft mass. The clay would retain the moisture at a greater altitude, and in this way each stratum of finer pulverization might successively raise the moisture still higher. But though the extreme subdivision of the particles may carry the elevation almost to unlimited extent, yet will it

also retard the insinuation of the water. Hence the use of clay in *puddling* or choking up the grosser pores, which might favour the efflux from a dam.

Hitherto we have considered the action of the interior surface of the tube upon the proximate film of liquid, which occasions this to spread laterally, and therefore to mount upwards, as counterbalanced by the mere weight of the adherent and included column of water. But the mutual cohesion of the particles of the fluid brings another force into play, exerted in producing precisely the opposite effect. The column contained within the tube, being thus entirely detached from the general fluid mass, wants the lateral attraction which held it before in equipoise. This deficient power must hence occasion the same result as if an equal and opposite pressure from the sides of the tube had come into action, causing the column to retreat and subside towards the body of the fluid.

It is hence not the whole attraction of the glass to water, but merely its excess above half the cohesion subsisting among the particles of the liquid itself exerted internally, or on the one side, that is employed in producing the capillary ascension. If this attraction were equal to its antagonist cohesion, the fluid would remain balanced at the common level. On the other hand, if the attraction of the sides of the tube be less than the corpuscular cohesion of the

fluid, it will sink proportionally below the general surface. The depression of mercury in capillary tubes is not occasioned therefore by any supposed repulsion of the glass, but is the necessary result of the preponderating cohesion of its own particles.

Hence dew or rain collects into globules, or spreads into rounded heaps, on the glossy surface of a cabbage leaf. Water likewise runs into such drops, like the globules of quicksilver, when thrown upon a red-hot surface of glass, or brick, or metal, the vapour forming around it a sort of atmosphere, which prevents its contact and adhesion. In this way the surface of copper is congealed into a thin plate, which being removed is immediately succeeded by another concretion derived from the same affusion of water. In like manner, quicksilver affects a globular shape, when poured upon a table of wood or marble. A ball of crude platinum having a specific gravity of 14.3, or little more than that of mercury, will even float upon it, being indirectly sustained by the cohesion and retreat of the liquid medium. When this floating mass, however, is once pressed down, the mercury closes over it, and prevents it from rising again.

To the powerful cohesion of their particles, is owing the property possessed by all liquids, of remaining heaped up to a sensible height above the brims of the vessels which contain them, whether formed of glass or of metal. Small bits of cork and

other light substances set to float in water are observed gradually to approach; a fact which has often been vaguely ascribed to the distant attraction of those bodies. This very minute force, however, is scarcely appreciable, and the tendency of the corks to unite is really caused by the portion of water adhering to their sides, which presents a continuous sinuosity, pressing the prominent liquid to a mutual junction.

The contour of the top of the included column likewise indicates the relations of those opposite forces. When they are equal, it has no curvature at all; but if the attraction of the glass be superior to half the cohesion of the particles of the fluid, the upper surface becomes concave; and if this cohesive force prevail, it will assume a convex outline. The curvature of the boundary has, at its middle point, a radius proportional to the width of the tube. This external curve may be considered, when the bore is small, as merely an arc of a circle; and consequently its margin will, in every change of dimensions, meet the inner surface at the same angle.

The attraction which fluids have to solids, differs little in general from half the cohesion of their mobile particles. Hence, of those forces, a very small portion is ever exerted in elevating or depressing the fluid columns within capillary tubes. The attraction to a solid is ascertained, by observing the weight required to detach a given circular disc from the ho-

rizontal surface of the fluid. Thus, I find, that, at the ordinary temperature, a piece of plate-glass adheres to water, with a force equal to 40 grains for every square inch. But the mutual cohesion of the aqueous particles in the same space must amount to 75 grains. Water is hence pressed upwards in a capillary tube, by the excess of 40 above the half of 75, or $2\frac{1}{2}$ grains for each inch of internal surface, as already stated. I have likewise found, that a force equal to 144 grains for each square inch is required to detach a plate of glass from a surface of mercury. The mutual attraction of the particles of this fluid, must therefore be measured by $313\frac{5}{7}$ grains. The mercury is hence depressed in a capillary tube, by the defect of attraction, or the force of $12\frac{5}{7}$ grains, which corresponds to the 267th part of a cubic inch.

The attraction between solids and fluids is greatly diminished by heat. Thus, at the temperature of 48° by Fahrenheit's scale, water adheres to glass with the force of 42 grains for each square inch, but at 97° with only 32.9 grains. The attraction of mercury to glass at 51° amounts to 148.4 grains, and at 81° only to 130 grains. A similar alteration, differing only in proportion, is manifested by other fluids. Thus, the attraction of sulphuric acid to glass at the temperature of 48° is 48.2 grains, but reduced to 40.1 grains at 140°.

It may be hence inferred, that the mutual cohesion of the particles of fluids is likewise affected by

change of temperature. But what appears very remarkable, the capillary action is not subject to any sensible influence from that cause. The alterations which heat induces, on those powers of attraction and repulsion, must therefore be such as to leave always the same differences, whether of excess or defect.

On these principles we may explain why a fine needle will swim on the surface of water, which always forms a small pit for it. The attraction among the particles of the fluid occasions the same effect as an actual repulsion, pressing upwards with a force inversely proportional to the breadth of the needle or the separation of the water. Hence the various slender insects which commonly frequent pools easily walk on the surface without being wetted, the water retreating from the contact of their glossy and filamentous limbs. The same property defends the feathered tribe against the injury of the rain; and the dust of the butterflies wings affords those slender membranes a similar protection. We may remark that the needle can hardly be made to swim on alcohol or strong ardent spirit. The only way of succeeding is, to set the first needle afloat on a small body of water in a glass, and afterwards introduce the alcohol gently by means of a tube at the bottom.

It is difficult to measure directly the cohesion of fluids; but an approximation may be derived from the magnitude of drops and the thickness of liquid

sheets, heaped upon an horizontal surface. In this view, let us trace the formation of a drop of water, as it slowly collects at the end of a capillary syphon. The mutual attraction of the particles always rounds the under part of the pendant fluid, which continues to lengthen till its accumulating weight begins to overcome the cohesion of the particles. But this force being 75 grains for each horizontal square inch, while a cubical inch of water weighs 252.5 grains, must correspond to the pull of a cylinder of .18 inch high, which will influence also the breadth of the pendant liquid. Beyond this limit a separation will ensue, when the cylinder merges into a sphere a little wider, or about two-tenths of an inch diameter.

The cohesion among the particles of alcohol and of sulphuric acid, being respectively the fifth part of 215 and of 460 grains, the weight of a cubic inch of each of those fluids, a drop of them should measure .17 of an inch in diameter.

Again, since the cohesive power of quicksilver, at the ordinary temperature, amounts to 312 grains on each horizontal inch, while a cubic inch of that ponderous liquid weighs 3424 grains, the drop must separate when its mean depth approaches to the nine-hundredth part of an inch.

These results agree sufficiently with observation. They give 91, 23, 40, and 76 grains, for the several weights of an hundred drops of water, of al-

cohol, of sulphuric acid, and of mercury. In the higher temperatures, the drops of all liquids must, from their decreasing cohesion, be considerably diminished. A drop, or, in the language of the apothecaries, a *gutt*, is hence incorrectly assumed as, in all cases, equivalent to the weight of a grain. Independent of the influence of heat, we have seen that they differ very considerably, and that a drop of alcohol, the most ordinary solvent in tinctures, is not only much lighter, but even smaller, than one of water.

VII. HYDRODYNAMICS

Consists in the application of the principles of Dynamics to the impulsion and flow of water and other liquids: It therefore explains all the various motions of such liquids, and of fluids in general. This important branch of science has, in reference to the construction and performance of water-works, been commonly termed Hydraulics; but it should then comprise, in strictness, the joint operation both of air and of water.

The theory of Hydrodynamics is entirely of modern origin, and still wants that perfection which, in unfolding the simple pressure of fluids, Hydrostatics has attained. The forces evolved in the internal motion of such a system of particles are so excessively complicated, that it becomes necessary to adopt some auxiliary hypothesis or abbreviation, in order to obtain even approximate results. Theory must therefore, as in the practice of the mechanical arts, be rectified, by a constant appeal to experience. In this way, the most important accessions have been lately made to the science of Hydrodynamics.

The fundamental problem in the motion of fluids is to determine the efflux from a small aperture in the bottom or the sides of the containing vessel. Suppose a cylinder ABCD (fig. 152.) of the thin-

nest materials, holding water, to have a very minute hole opened at F, giving passage to the liquid. The whole body of the water may be conceived to remain at rest, while each particle in succession, as it reaches this opening, suffers the momentary pressure which causes its expulsion. The action of the accelerating force which incites the motion, is hence confined to the mere interval between the several particles. The force itself is evidently proportional to the number of particles arranged in the vertical line EF: Each particle is at first urged equally on every side; but when it comes within the limits of the aperture, the pressure in front ceases, and it is pushed forwards, by the whole weight of the incumbent row. But, from the principles of dynamics, the square of the velocity generated, is proportional to the product of the accelerating force into the space. If h denote the height of the cylinder of water, and n the number of particles in the perpendicular range; then $\frac{h}{n}$ will be the mutual interval, and $\frac{h}{n} \times n$, or h, will express the square of the velocity of ejection. But $1 \times h$, or h, represents likewise the square of the velocity, which a particle would acquire by its mere gravity, in falling through the whole altitude of the fluid. The velocity of expulsion in feet each second is therefore expressed by $8\sqrt{h}$. If the aperture be enlarged beyond a mere point, each particle in its parallel access will

evidently escape with the same celerity, and hence the quantity of discharge will be proportional to the surface of the section. The effect must be likewise the same, whether the aperture be made at the bottom, or in the side of the vessel, or be placed with any obliquity; since the impelling force is ultimately directed at right angles to it. If the jet be thrown exactly vertical, it ought evidently to mount to the level of the cistern.

The accuracy of this general conclusion, however, is affected by several assumptions. It supposes the aperture to be suddenly opened, and indicates only the first ejaculation of the water. But the whole mass of fluid is, in consequence of this act, put in motion; the particles advance slowly to the orifice, but are not urged now in their exit by the same vigorous pressure. The mutual attraction of the liquid particles, besides, greatly impedes their separation and escape, especially in the case of a very small aperture. But, what more than any other cause diminishes the quantity of efflux, is the tendency of the various streamlets towards a common centre, which occasions them to shoot beyond the edge of the orifice, and therefore to contract the channel of the current. The effect of these different distributing causes can only be deduced, from a comparison with actual experiment.

A simple consideration leads to the same result: Suppose a narrow vertical tube were inserted in the

orifice at the side of the cylinder. The water would mount to the elevation of the opposite surface; but the addition of the tube could have no effect whatever in promoting the ascent of the fluid, since the resistance of its perpendicular sides could furnish only an horizontal pressure. If the tube were annihilated, the column of water would still remain the same as before. That pressure which sustained the fluid at rest, might be expected to raise it into the like position.

This reasoning seems plausible, but is perhaps not very conclusive. But the same inference can be drawn from a stricter view of the question. Conceive the jet to be directed perpendicularly: The motion of the water is then exactly reversed, and its descent is followed by a corresponding ascent. Hence the common centre of gravity must rise just as much in the second instant, as it fell during the first. Thus, while the surface of the water in the cylinder sinks from AB (fig. 153.) into the position GH, an equal portion IKLM is projected upwards. The centre of gravity of the mass first sinks from O to P, and then springs up again to O. Hence OP is half the depth of subsidence AG, and the product of whole ABDC into OP is equal to the product of the portion IKLM into its elevation above O; wherefore the product of AC into OP is equal to the product of AG into the altitude of the projected portion of fluid above the common centre of gra-

vity. This altitude is consequently the half of AC. The jet should therefore attain the height of the head AB.

A similar conclusion is deduced from the principle of the *Conservation of Living Forces*, which is peculiarly applicable to such an elastic body as water. In subsiding from AB to GH, the fluid mass describes a space AG, with an accelerating velocity, the square of which must be proportional to AG. But the portion of water projected upwards from the orifice must have risen with a retarding celerity, the square of which, at its commencement, being proportional to the altitude obtained. Now $AC \times AG = IK \times AC$, and therefore the jet should spring to the height of the surface, AB.

It is obvious that pressure might be substituted for the weight of the incumbent cylinder of water. The jet thus produced will rise to the altitude of an equiponderant column of fluid. If the density of the compressing mass be different from that of the projected liquid, the velocity of discharge must likewise vary. Thus, a column of mercury acting upon water, will expel it with a celerity nearly four times greater, or what is due to an altitude repeated $13\frac{1}{2}$ times. On the contrary, the inferior action of a cylinder of water upon mercury, could only raise this to the $13\frac{1}{2}$ part of its own height.

Suppose water to spout at right angles from a small aperture E (fig. 154.) in the side of an up-

right cylindrical or prismatic vessel ABCD. It will be projected with a velocity which would carry it horizontally through double the space BE, in the time that a body takes to fall from B to E. But it now descends with an accelerated motion to the level of DC, while it continues its uniform horizontal flight, thus describing a parabola which meets the ground in G. Wherefore $\sqrt{BE} : \sqrt{EC} :: 2BE : CG$, and $BE : EC :: 4BE^2 : CG^2$; consequently $CG^2 = 4BE \times EC$, and having, upon the diameter BC, described a semicircle, $CG^2 = 4EF^2$, and $CG = 2EF$. Thus, the horizontal range of the jet is double of the ordinate EF. This range is hence the greatest when the aperture is in E, the middle of the altitude BC being then equal to the altitude itself. In other cases there are two apertures, L and H, equidistant from the middle, which give the same range $CK = 2LM = 2HI$.

Let the aperture be placed at the side close to the bottom of the vessel, but directed at different elevations. It is evident, from the theory of projectiles, that the extension of the level AB (fig. 155.) will define the directrix of all the parabolas described by the jets. If a circle be described from the centre C with the radius CB, it will trace the foci. Let the tangent CE indicate the elevation, and draw CF making the angle ECF equal to BCE; let fall the perpendicular FH, and take HG equal to it; the dis-

tance CG will mark the range of the jet. But $CG=2CH=2CF\times \cos FCH = 2CB \times \sin 2ECH$. Hence, when the elevation is 45°, the focus of the parabola falls on the horizontal line at F', and the range CK is then the greatest possible, being double of the altitude CB.

Since the velocity of emission from any orifice is $8\sqrt{h}$, the square of it is $64h$. If a parabola, therefore, be described upon the altitude BC (fig. 156.) as an axis, and with a parameter of 64 feet, the ordinate GH will express, in feet each second, the flow from the aperture GC. Suppose the side of the vessel were slit down its whole height, the efflux of water would then be denoted by the area of the parabola. But if an opening equal to that slit were made at the bottom, the discharge of fluid would be represented by the circumscribing rectangle BIHC. Wherefore the discharge by a perpendicular slit, is only two-thirds of what is effected by an horizontal aperture of the same dimensions.

The efflux from an orifice at the bottom of a vessel being always as the square of the altitude, it may, by the accumulate elevation of the fluid, be brought to correspond with any given measure. When a stream of water flows into the cistern just as fast as it is again discharged, the fluid must evidently maintain a constant level, from which the celerity is easily computed. Suppose this altitude to be three inches, the velocity of discharge would

be $8\sqrt{\frac{1}{4}}$, or four feet each second, that is 240 feet every minute. The delivery would, therefore, be at the rate of a cubic foot a minute, if the aperture were only the 240th part of a square foot, or three-fifths of a square inch. This opening corresponds to a circle of which the diameter is .874 parts of an inch.

Hence, a very convenient *modulus* or measure, for ascertaining readily the quantity of water delivered in a minute by any pipe or conduit. Let a cylinder of tin, above four feet high and one foot wide, be fitted with a float, bearing a slender divided rod, which passes through a hole in the middle of a bar stretched across the open top. This rod is marked 1, 2, 3, 4 at the distances of 3, 12, 27 and 48 inches, being the fourth parts of the squares of the first numbers in feet. The divisions may likewise be quartered by beginning with the sixteenth part of 3, 12, 27 and 48 inches; and continuing the process by multiplying the squares of 5, 6, &c., as far as 16 by 3, and dividing the products again by 16. The cylinder, having a circular hole with a fine edge cut in the bottom, being now set upright at some distance from the ground to receive the current, the water mounts gradually in the vessel, *till* it has gained an altitude sufficient to cause a discharge equal to the whole influx. The rod then indicates the number of cubic feet received every minute.

But the application of this modulus may be extended, by adapting to it a series of apertures which screw into the bottom of the cylinder. Let them have successively these diameters in inches .437 — .874 — 1.748 — and 3.496. The rod would then be square, and marked on all its four sides; the divisions on the first side corresponding to a discharge of $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and one cubic feet, those on the next to 1, 2, 3, 4, those on the third side to 4, 8, 12, and 16, and those on the fourth side intimating a flow of 16, 32, 48 and 64 cubic feet every minute. From these data, it is easy to construct other moduli suited to particular limits. The correction required in the size of the apertures, to accommodate theory with practice, will be afterwards explained.

The native troops in India are accustomed to relieve guard, on the sinking of a perforated metallic cup in a vase of water. As a converse of this, the ancients, instead of a sand-glass, employed a cistern, from which the water trickled through a small hole at the bottom, under the name of a *Clepsydra* or *water-clock*, to measure time. In a cylinder, the flow would evidently diminish, as the level of the surface is incessantly lowered. To procure an uniform descent of the water, it would be necessary to adopt the figure of a conoid of the parabolic kind, each circular section of which is proportional to the square root of the corresponding altitude: Suppose this were 24 feet, and the diameter at the top 13.28

feet, the diameters of the successive sections being always six times the fourth root of the altitude. The velocity of efflux would then be $8\sqrt{23}=39.192$ feet each second. If the water sank at the rate of a foot every hour, the width of the orifice would be to the extreme diameter or 13.28, as 1 to $60\sqrt{39.192}$. This gives an aperture of .424 parts of an inch. A conoid of such dimensions would, as represented in fig. 156, therefore, answer correctly as a Clepsydra, the equable subsidence of a float marking the series of twenty-four hours in a natural day. This float being fastened to a thread wound about a cylindrical barrel, of a foot in circumference, would carry the index of a dial regularly round.

Let it be required to find the time which water takes to flow out of a cylindrical vessel, through a hole in the bottom. Since the velocity of projection, after the level has subsided to G, (fig. 157.) is in the subduplicate ratio of the altitude GC, it may be represented by the ordinate GH of a parabola applied to the axis CB. Wherefore, the time of discharging the thin stratum Gg is expressed by $\frac{Gg}{GH}$; but, having drawn the normal HN to the curve, the elementary triangle $H\gamma h$ is obviously similar to NGH, and $H\gamma$ or $Gg : \gamma h :: GH : GN$; whence the element of the time or $\frac{Gg}{GH}=\frac{\gamma h}{GN}$. Now, the subnormal GN is constant, being equal to

half the parameter of the parabola; consequently the portions of time $\frac{\gamma h}{GN}$, taken collectively during the descent of the surface of the water from B to G, will be denoted by $\frac{KE}{GN}$, the whole time of the flow being $\frac{BE}{GN}$. Hence, if the altitude BC were divided into equal parts by a number of ordinates, the differences between these would mark the intervals of the times of descent. Suppose an extreme tangent were applied at E, meeting the extension of DC in L, from which is drawn LM parallel to the axis CB. The time of the whole discharge, if the water had continued to flow with its initial velocity, will be denoted by EM. But the tangent would cut BC produced at an equal distance beyond C, and consequently LM must bisect EB. The discharge of the fluid requires, therefore, double the time which would have sufficed, if the efflux had maintained its first intensity.

In general, let H and h denote in feet the altitudes of the column of water in two different situations, and n the ratio of the diameter of the cylinder to that of the aperture. The initial velocity is $8\sqrt{H}$, and hence the whole time of discharge from the vessel, had the flow continued uniform, would have been $\frac{n^2 H}{8\sqrt{H}} = \frac{n^2}{8}\sqrt{H}$. But the time of actual discharge

is the double of this quantity, or $\frac{n^2}{4}\sqrt{H}$. Now the intermediate time during which the surface of the water descends from B to G, being proportional to EK, the excess of the ordinate BE above GH, which are themselves in the subduplicate ratio of the corresponding heights AC and CGH; it follows that $\sqrt{H}-\sqrt{h}$ may be substituted instead of $\sqrt{H}$. The expression then becomes $\frac{n^2}{4}(\sqrt{H}-\sqrt{h},)$ which denotes, in seconds, the time of the subsidence of the level of the water in the cistern from B to G.

In these investigations, we have considered, for the sake of simplification, the aperture of projection as extremely small. But when it bears a sensible proportion to the width of the vessel, the celerity of discharge will evidently be diminished. The accelerating pressure may be conceived as only the excess of the weight of the contained water above that of the column incumbent over the orifice. The velocity of emission is hence expressed by $8\sqrt{H}\sqrt{\left(\frac{n^2-1}{n^2}\right)}$, or $\frac{8}{n}\sqrt{H}\sqrt{(n^2-1)}$. Wherefore the time of subsidence from the level H to h will be $\frac{n^2}{4}\sqrt{\left(\frac{n^2}{n^2-1}\right)}(\sqrt{H}-\sqrt{h})$.

When an aperture is made in the side of a vessel holding any fluid, the equilibrium of hydrostatic pressure must evidently be disturbed. This force,

which was exerted equally all around the inner surface, now ceases to act at the opening; or rather it expends its action there, in projecting the uncovered portion of the fluid. The reaction, or incessant recoil sustained by the general mass, will hence be the same, as the external pressure of a column of the fluid of equal altitude directed against the section of the orifice. If the vessel have therefore a cylindrical form, and turn freely about a vertical axis, while it carries under it two horizontal branches, extending both ways, each of them perforated near the end at right angles to their plane with the axis, but in opposite sides; the machine will be driven backwards by the spouting fluid, and forced to revolve as long as the action continues. The force thus exerted is equal to the weight of a column of the fluid having the general altitude, with the two apertures for its base. But the effect becomes augmented in this case, by the length of the arms or levers to which the power is applied.

This principle has been long successfully adopted in the construction of a very simple but efficient water-mill, commonly designated Barker's mill. It is capable, however, of various other useful and important applications, which remain still to be carried into execution.

The action of centrifugal force may be advantageously employed, either in raising water to a higher level, or in urging its flow from an orifice. Let a cylindrical vessel, (fig. 158.) carrying an horizontal

arm or tube DE, turned up at the end into a perpendicular branch EG, be made to revolve quickly about its axis AB. If while at rest, it contained water to the height C, this would obviously stand at the same level in the opposite tube at F. But after the circulation of the system commenced, a different arrangement would take place: The several particles in the horizontal column will be driven forward by the centrifugal effort, which, as they revolve all in the same time, is therefore in the compound ratio of their gravity and of their distance from the axis of motion. This force will consequently increase uniformly from B, along the whole range, to the extremity E. Wherefore the action exerted at this limit will be the same as the weight or pressure of a vertical column, whose altitude P is to the length of the arm BE, as the centrifugal force at the middle point O is to the power of gravitation. This incumbent load hence raises and supports an equal column above the level F, the small vertical portion FE being sustained merely by CB, and not affected by any centrifugal tendency. Let BE be denoted by r, and the time of revolution in seconds by t; the centrifugal force at O is, therefore, $\frac{5.\frac{1}{2}r}{4t^2} = \frac{5r}{8t^2}$, and the altitude of the equivalent column, or FG, is $\frac{5r}{8t^2} \cdot r = \frac{5}{8}\left(\frac{r}{t}\right)^2$. If the vertical tube EG

were shut above F, and an aperture made at that point, the rush of the fluid along the pipe DE, which corresponded to the pressure of GF, would be exerted in projecting the fluid. The velocity of expulsion would hence be $\frac{5r}{t}$. But if the orifice were opened at E, the celerity of emission would be farther augmented by the pressure of FE. In Barker's mill, the action is hence apparently augmented by the centrifugal force brought into play by the revolution of the machine. But this force being gained only at the expense of the pressure of the water in the cistern AB, cannot affect the absolute quantity of performance.

Suppose perpendicular tubes (fig. 159.) HL, IM, KN, &c. were erected at equal distances from the axis AB along the horizontal area BK. Since the time of revolution is constant, the ascents PL, AM, and RN, &c. of the fluid must be proportional to r^2, or to the squares of the several distances BH, BI, and BK, &c. PL, QM, and RN, &c., are hence as the numbers 1, 4, 9, &c. A parabola described from the axis AB through the vertex C, would, therefore, pass through the several points L, M, N, &c. Conceive the vertical pipes to be united into one mass, and their summits will trace the curve of a parabola. Obliterate the sides of those tubes altogether, and the surface of the fluid, extending (as in fig. 160.) on every side of the axis, will, by its circumvolution,

form the cavity of a parabolic conoid. The curve will not be affected by the depth of the water in the cistern, since of each perpendicular column all the particles situated below the vertex must receive the same horizontal thrust, derived from centrifugal action, to enable them to support the additional vertical pressure. The parameter of the parabola depends merely on the time of circulation, being equal in feet to eight-fifths of the square of the number of seconds. Employing the usual notation, $t=\frac{2\pi.r}{v}$ and $t=\frac{4\pi^2.r^2}{v}$, or very nearly $\frac{40r^2}{v^2}$; whence the parameter is denoted by $\frac{64r^2}{v^2}$ or $\left(\frac{8}{v}\right)^2$. Wherefore, the central depression of the water, being from the property of the curve equal to r^2, divided by the parameter, is $\frac{v^2}{64}$ or $\left(\frac{v}{8}\right)^2$. The depth of the cavity is thus independent of its width, being proportional to the square of the exterior velocity.

Next, let a reflected pipe rise obliquely, as in fig. 162. The horizontal pressure at F is denoted by $\frac{5CF^2}{8t^2}$, and that exerted at G by $\frac{5AG^2}{8t^2}$. But as long as the continuity of the water included between F and G is maintained by its cohesion, the centrifugal force directed to the middle point O may be considered as what sustains the oblique column FG,

or an equiponderant vertical column GH. Whence $\frac{5}{8t^2}.PO^2 = GH$, or $\frac{5}{8t^2}.PO^2 = FG.\sin GFH$; but $PO = CF + FG.\frac{1}{2}\cos GFH$, and therefore $\frac{5}{8t^2}(CF + FG.\frac{1}{2}\cos GFH)^2 = FG.\sin GFH$. Let FG be supposed to rise from the bottom of the axis AB, near which level the water of the cistern has likewise subsided; the expression will become $\frac{5}{8t^2}(FG.\frac{1}{2}\cos GFH)^2 = FG.\sin GFH$, or $\frac{5}{32t^2}.FG^2.\cos^2 GFH = FG.\sin GFH$. Wherefore $FG.\cos^2 GFH = \frac{32t^2}{5}.\sin GFH$, or $FG = \frac{32t^2}{5}.\frac{\sin GFH}{\cos^2 GFH} = \frac{32t^2}{5}.\frac{\tan GFH}{\cos GFH}$. The centrifugal action is thus unequally exerted along a straight tube obliquely placed.

Let it now be required to find a curve, in every part of which that force has the same efficacy. We may here exclude the influence of hydrostatic pressure, and consider smooth balls as occupying the sides of the cavity. The centrifugal force acting against the curve at C (fig. 163.) is denoted by DC, and may be decomposed into EC perpendicular to the curve, and DE parallel to it. This last is the only force exerted in sustaining a ball at C. But part of the weight of the ball being upheld by the

curve, the efficacy of the direct centrifugal action DE is proportional to DN, which must hence be constant. And such is the property of the subnormal DN in the parabola, which therefore answers the conditions of the problem. Let P denote the parameter of the curve, of which ADN is the axis; then DN $= \frac{1}{2}$P, and consequently the centrifugal force due to a radius DN becomes equal to the power of gravity, while a ball at C is just supported, or $\frac{5.\frac{1}{2}P}{4t^2} = 1$, and $5P = 8t^2$. Wherefore if t, the time of revolution, were equal to $\sqrt{\frac{5}{8}.P}$, or very nearly $\frac{4}{5}\sqrt{P}$, a ball would be upheld in any part of the cavity of a parabolic conoid, and the slightest acceleration would be sufficient to roll it up to an indefinite height. But the circumvolution of a vessel of this form would have the same effect in raising a sheet of water along its inner surface. If the parabolic conoid were perforated at the vertex, and a little immersed in a standing pool, it would at first produce no sensible effect; but after it had quickened the time of its revolution beyond the precise limit, it would draw up the water at once to the brim, and disperse that liquid in a copious horizontal shower. A machine of this kind, driven by a small wind-mill, might be applied advantageously to the draining of marshes.

If a cylindrical glass vessel, partly filled with water, be set on the middle of a whirling table, the depression of the liquid will increase rapidly as the circumvolution is accelerated. With double the velocity, the depth of the cavity will be quadrupled; and with triple the velocity, it will be rendered nine times greater. In urging this voraginous motion, the water will continue to descend in the centre, while it rises up the sides of the vessel: It will then divide at the bottom, and spread more upwards, forming always a parabolic conoid, but with a diminishing parameter. See fig. 161, which exhibits three successive states of circumvolution. If a thin ring were applied round the top of the cylinder, the water in its extreme celerity would cover the inside of the cylinder with a stratum of almost equal thickness. If a hollow glass globe be partly filled with water, and made to whirl very rapidly, the fluid will separate, leaving a vacant cylindrical space around the axis; and if a portion of quicksilver be introduced, it will rise up along the inside, mounting to the widest part of the sphere, and there forming a beautiful zone. Suppose a cylinder 6 inches wide and 16 inches high, filled with water to the altitude of 4 inches: When the circumvolution is performed in .68″, the water is depressed an inch, the central part standing at $3\frac{3}{4}$ and the sides at $4\frac{1}{4}$; but when the cylinder circulates in .24″, the cavity sinks 8 inches in the bottom, and rises at the sides to the height of 8 inches.

If the revolution were achieved in .152″, the water would mount to the lips, leaving a dry circle at the bottom of the vessel of $2\frac{1}{4}$ inches in diameter.

Hence the origin of dimples on the surface of small streams, and of whirlpools in mighty rivers and narrow seas. The effect is produced by the lateral attrition of adverse currents, which, extending their influence from the neutral point or centre of equal and opposite action, gradually convert their parallel motions into a combined system of circumvolution. Suppose each current had a velocity of 9 miles an hour, or 13.2 feet each second, and the radius of the whirlpool to be 100 feet: The time of circulation would then be 47.6″, and the depression in the centre of the gulf only 2.72 feet, the parameter of the parabola being 3672 feet.

Since the pressure of a column of water occasions a corresponding flow, every current may be viewed as originating from the action of such a force, and can therefore be determined by the altitude of the incumbent fluid. Hence the construction of Pitot's tube, a very convenient small instrument for measuring the velocity of any stream. It consists of a recurved tube of glass, of which the one branch, however, is much taller than the other; the short branch is bent at the top into a spreading funnel-shaped mouth, which receives the direct shock of the

water, and communicating this impression, causes a proportional elevation above the common level. See fig. 164. To prevent the irregular oscillation of the liquid in the syphon, it may be proper to have the bore much contracted in the whole of the under part of either branch. This object is farther promoted by covering the funnel orifice by a thin circular piece of brass with a very small hole in the centre. The pressure will be still propagated as before, but with more steady effect. The divisions on the scale are reckoned upwards from the surface of the stream. To ascertain them, let the expression $8\sqrt{h}=v$ be resumed, and $64h=v^2$, or $h=\frac{v^2}{64}=\left(\frac{v}{8}\right)^2$, where h denotes in feet the height of the column, and v the velocity each second. Hence the rise corresponding to the rate of a mile an hour would be $\left(\frac{22}{15}\right)^2\cdot\frac{12}{64}=\frac{484}{1200}$, or 4-10ths of an inch: The scale would hence be marked 1, 2, 3, 4, 5, 6, 7, 8, 9, &c. miles, at the respective heights of .4 –1.6 –3.6 –6.4 –10.0 –14.5 –19.7 –25.8 –32.7, &c. inches. Few rivers, therefore, would require the glass tube to rise six feet above the surface of the water. The instrument may be composed of a long narrow tube of brass or tin, cemented to a wide cylinder of glass, carrying the divisions.

A construction somewhat different might answer for measuring a ship's way at sea. A long tin pipe

presenting its trumpet orifice under water at the prow could be conducted along the side of the keel, and brought up into the cabin, there to be cemented into a wider perpendicular tube of glass. The scale could be made to slide along, having been adjusted to the level of the external water during a calm; or this level might be found, by placing another parallel glass tube, which has cemented to it a narrow tin pipe running out below towards the bottom of the vessel.

If the channel of any stream be suddenly contracted, the water will be forced to rise above its ordinary level. Suppose, for instance, that the flow was at the rate of a mile an hour; this might be considered as the effect of an incumbent column of four-tenths of an inch. But where the section of the current is reduced to one half, the resulting celerity of two miles an hour would correspond to the pressure of an altitude of 1.6 inches. An accumulation of the water to the height of the difference, or 1.2 inches, would hence be occasioned by this obstruction. In general, if v denote the velocity of a river in miles every hour, a the measure of its vertical section, before it suffers any impediment, and b that of its contracted course; the rise of the level will be expressed in feet by $\frac{v^2}{25}\left(\frac{a^2-b^2}{b^2}\right)$. It may, in practice, however, be more accurate to admit a slight

modification, and assume $\frac{v^2}{50}\left(\frac{3a^2-2b^2}{b^2}\right)$. This formula was remarkably exemplified in the ruinous pile of the old London Bridge, the piers of which were so massive as to reduce the breadth of the water-course from 14 parts to 3. The velocity in spring-tides is 2 miles an hour; and hence $\frac{4}{50}\left(\frac{588-18}{9}\right)=5$ feet. Such then was the fall, and this with a rapidity of 9½ miles an hour. No wonder that the shooting of the Bridge was sometimes attended with so much hazard and danger.

All the preceding investigations are grounded on legitimate theory, but some reduction is necessary in bringing the results to conform with actual practice. If a wide orifice at the side, but close to the bottom, of a tall cylindrical vessel, containing water, be covered by a thin plate of metal perforated by a fine round hole; on removing the finger from this aperture, the first emission of the fluid will exactly agree with calculation. But after the whole mass has acquired motion, the various streamlets, all directed towards the centre of the hole, bend at a little distance from its sharp edges, and escape in a contracted current. It has been ascertained by experiment, that, through such a simple aperture, the flow is only five-eighth parts of what theory would indicate. If, therefore, we substitute five instead of eight in a

preceding formula, an expression will be obtained for the quantity of real water discharged from a given orifice in any vessel. Let A denote the area of this orifice, and h the height of the fluid; then $5\sqrt{h}.A$ will express the efflux, in cubic feet every second, or 300 $A\sqrt{h}$ the quantity delivered in a minute. If, while h marks the altitude of the water in feet, d should measure the diameter of the circular hole in inches, the discharge every minute will be represented by $2.6526d^2\sqrt{h}$, which corresponds very nearly to $159d^2\sqrt{h}$ in an hour.

If a cylindrical tube, whose length is rather more than twice its width, be adapted to this aperture, the discharge will become augmented from five parts to six and a half, and will consequently amount to thirteen-sixteenths of the quantity assigned by theory. The formula in this case will therefore be $390A\sqrt{h}$, for every minute; or the diameter of this annexed tube or *adjutage* being denoted in inches by d, the discharge will be $3.41\ d^2\sqrt{h}$.

Prony has lately proposed, as a *modulus* for Hydraulic Operations, the quantity of water discharged in the space of twenty-four hours from a wide shallow cistern, kept constantly full by an ingenious mechanical contrivance. At the depth of 5 centimetres, or 1.9685 English inches below the surface, is the centre of a circular hole of 2 centimetres or .7874 inches diameter, perforated through the thin side of the vessel; and to this aperture a cylindrical

tube of only .6693 inches in length must be fitted. The measure which is thus delivered in a natural day amounts to two *moduli* or 20 cubic metres, corresponding to 711.24 cubic feet, being very nearly three-fourths, instead of five-eighths, of the quantity assigned by rigorous theory.

The addition of the projecting pipe has in all cases really the effect of extending the action of the hydraulic pressure from the inner to the outer aperture, while the cavity is kept always full by the incumbent weight of the external atmosphere, aided likewise by the adhesion of the water to the sides of the tube. Accordingly, if small perforations be made in this eductive pipe, the air will enter, and flow along the contracted surface of the current, hence occasioning a proportional diminution of the quantity of discharge. On a small scale, the contraction and separation of a jet from the sides of its tubulated orifice may be remarked under the exhausted receiver of an air-pump. The efficacy of the pipe thus proceeds entirely from its continuing always full, while the celerity of the stream is still maintained or even augmented. The quantity of discharge is likewise increased with the length of the pipe, at least to certain limits. Thus, from an orifice of an inch diameter, the quantity of water delivered, in a given time, through a spout of an inch long, is to the expenditure by one of a whole inch in length, as 6 to 7.

If the anterior part of the cylinder, which is annexed to the aperture, be contracted in the middle, so as to consist of two reversed frusta of a cone, the discharge of fluid will be still exactly the same. Suppose (fig. 165.) AB and EF to be each two inches wide, the contraction CD 1.6 inches, the length IL = 2.3 inches, and LM = 4.1 inches: The effect is found to be the same as if a regular cylinder of six inches length had been applied at AB. But if the simple tapered projection ACDB (fig. 166.) were affixed, the measure of discharge would be diminished in the ratio of 4 to 3. Now, the section CD being to EF as $(4)^2$ to $(5)^2$, or as 16 to 25, the velocity of the stream, even at the wider aperture of its exit, is consequently augmented in the ratio of 21⅓ to 25, or of 64 to 75. This compound pipe, ACE, FDB, (fig. 165.) therefore, not only accelerates the flow through its narrowed throat CD, but even quickens the emission from the wide orifice EF.

When water issues through a simple circular aperture in a thin plate, it forms a contracted vein, of a conical or rather tapering funnel shape, its section being reduced to very near five-eighth parts at a distance little more than half its diameter. If a tube of this figure be annexed, the quantity of discharge will come within the thirtieth part of the result of theory, as computed for the exterior aperture. (See fig. 167). But the adjutage being directed upwards,

the jet will rise to fourteen-fifteenth parts of the whole height of the incumbent column.

If to this tapered spout another conical tube five times longer, and opening to the original width, be joined, the discharge of water will be augmented from 21 to 38 parts. (See fig. 168.) The effect of the adjutage is now the greatest possible, the absolute flow from the anterior aperture being only 40 parts according to theory. This pipe widens at an angle of about three degrees; but when the annexed piece diverges at a greater angle, its influence becomes diminished, and appears to cease altogether at an angle of 16 degrees, as in fig. 169. The stream has now ceased to fill up the whole of the cavity, and is consequently no longer augmented by adhesion to the sides of the spout.

The lateral action of water flowing through a pipe is evinced in a more striking manner. Let a cylinder one inch wide and three inches long be adapted to an orifice at the bottom of a cistern; and on the upper side, at the distance of half an inch from its origin, let a narrow arched glass tube be inserted and carried down to a bason of water three feet lower. When the stream is projected with a velocity of nine feet in a second, it will draw up water to the height of two feet; but if the tube be shortened within that limit, the slender column will mix with the body of the current, and soon drain the contents of the bason. When a conical tube, open-

ing with a considerable angle of divergence, is substituted, a series of slender glass tubes inserted at different distances from the interior aperture will be found to raise the water to several heights, which diminish as the stream begins to separate from the sides of the spout.

This property of running water may, in various situations, be turned to useful account. By connecting the edge of the stream, for instance, by means of a small slanting pipe with a collection of water at a lower level, this will be gradually drawn up and carried away in the general current. If a swift descending rivulet be made to shoot across any small pool, it will sweep the water over its opposite bank. Venturi, to whom we are chiefly indebted for these remarks, availed himself of the rapidity and lateral draught of a mill-race, to drain a marsh situate considerably below it, near the city of Modena.

The same principle is likewise the principal cause of the action of the Hungarian Blowing Machine, which consists of a very tall perpendicular pipe, terminating below in a close wide box. A stream of water rushes down this shaft, drawing along with it the air which enters the small holes pierced along the sides, and becomes accumulated and condensed in the chamber, whence it again issues in a powerful blast on opening a cock. The blowing and dispersion of the spray on all sides from water-falls have a

like origin. A body of air is involved in the broken descending current: collected in the recess or broken cavity, it exerts its elastic efforts in every direction. The adhesion of running water to the sides of its channel is also the main cause of those eddies, which impede the general motion.

If a fluid suffers impediment while escaping at a small aperture, it encounters much greater obstruction in affecting its passage through a train of pipes, or flowing over an extended channel. This retardation, however, is quite distinct in its nature from ordinary friction. When a solid is drawn along the surface of another solid body, it is virtually made to ascend over a series of inclined planes, and consequently the impediment it meets with, may be viewed as merely equal to a certain portion of its weight, independent of the rapidity or slowness of the motion. But a fluid, in its passage over a resisting surface, needs not have its whole mass either elevated or depressed. Those particles only which come in succession to touch the solid boundary, are impeded and detained. The pressure of the incumbent fluid cannot affect the other particles, which are thence urged equally in every direction. The loss of impulse which a current sustains from the attrition of the sides of a pipe or of a canal, is occasioned by the incessant stoppage of the extreme particles. This consumption of force must hence be compounded of

the number of particles arrested in a given time and of their velocity, and is therefore proportional to the square of the velocity. But the retardation of the current must likewise depend on the extent of impeding influence, that is, on the length of the pipe, and on the relation which the interior surface bears to its whole capacity. But the square of the velocity, with which the stream first issues from the cistern, being proportional to the altitude of the incumbent column, a certain part only of this constant inciting force is employed in generating the initial velocity, while the rest is expended in renewing the velocity as fast as it expires, along the sides of the channel, or in maintaining through the mass a general uniform flow.

It results from some accurate experiments of Bossut, that water has its celerity diminished eight times, by passing through a tube of an inch in diameter and 204 feet long. Of sixty-four parts of compression, one part only must therefore have created the motion of the fluid, while sixty-three parts were required to support it. This motion is hence renewed eight times during the passage of the water, or at the interval of every 25½ feet through the whole extent. In each of these successive transits, all the central particles must be thrown towards the sides of the pipe, whence they are again drawn into the body of the stream, and there acquire new celerity. The celerity is lost and regained close to the inte-

rior surface within a very small but limited space. Wherefore, from the fundamental principle of dynamics, the square of the velocity acquired or extinguished must be as the product of the inciting force into the limit of its action. The square of the velocity hence suffers a diminution proportioned to the extent of surface compared with the capacity of the pipe, or it is directly as the length, and inversely as the diameter. But the pressure of the water has no effect whatever in causing this reduction. Thus, resuming the former example, let the pipe of an inch wide and 204 feet long be fed by a cistern of ten inches altitude, and the quantity of discharge noted. Raise this cistern now to twenty inches, and turn up, by a soft bend, the farther extremity to the height of ten inches, and the corresponding flow will be still the same. Increase the altitude of the cistern to one hundred inches, while the remote end is bent with an elbow to the height of ninety inches, leaving still the same excess or exciting force, and the quantity of discharge will be found not to vary.

Let V denote the velocity with which water * would issue from a simple orifice near the bottom of a cistern, and v the reduced velocity in consequence of passing through an extended horizontal pipe, d being the diameter of the pipe and L its length. From what has been shown, it follows that $v^2 = V^2 \left(\frac{50d}{50d+L}\right)$, and $v = V\sqrt{\left(\frac{50d}{50d+L}\right)}$. The deno-

minator $50d+L$ of the radical fraction expresses the consumption of the pressure or inciting force, the part $50d$ indicating the length of track which corresponds to the initial velocity. The absolute waste of such force follows the measure of internal surface, while the quantity of impulse must depend on the charge of the pipe; and hence the relative influence will be computed as $\pi d\,(50d+L)$ to $\frac{\pi}{4}.d$, that is, as $(50d+L)$ to $\frac{1}{4}d$, and is therefore expressed generally by $\frac{\frac{1}{4}d}{50d+L}$.

Since H denoting the height of the cistern, the primary velocity of projection is equal to $8\sqrt{H}$, or $V^2 = 64H$; whence $v^2 = 64H\left(\frac{50d}{50d+L}\right) = 64\left(\frac{50dH}{50d+L}\right)$, and $v=8\sqrt{\left(\frac{50dH}{50d+L}\right)}$. Suppose this pipe were curtailed to a mere adjutage, then $v^2 = 64.\frac{50H}{50} = 64H$. This coefficient 64 might, therefore, be adapted to the formula; or, if some allowance were made for the unavoidable expense of force in the transition of the water from the cistern to the pipe, the round number 50 may be preferred. The modified expression for the velocity of the final discharge will then become $50\sqrt{\frac{dH}{50d+L}}$, which agrees well with observation. In very long pipes,

the first part of the denominator may be omitted, and the expression for the final velocity will be simply $50\sqrt{\frac{dH}{L}}$. It may hence be sufficiently accurate in most cases to assume that V^2 is to v^2, as $64H$ to $2500.\frac{dH}{L}$, or as $64L$ to $2500d$; and supposing the coefficient 64 to be reduced to 50 in the act of entering the train of pipes, then will $v^2 = V^2 . \frac{50d}{L}$, and $v = V\sqrt{\frac{50d}{L}}$. Wherefore, of the altitude H of the incumbent column, the part $H . \frac{50d}{L}$ only is exerted in creating the flow of the water, the remainder of the inciting force being wholly consumed in overcoming the obstruction the current meets with along the internal surface. Every length of tube amounting to fifty times its diameter, thus occasions a waste of power equal to that which produced the general impulsion.

Let AB (fig. 166.) represent the altitude of the cistern, of which the small part AC causes the primary motion of the water, and the remainder CB maintains its lateral attrition through the extended horizontal pipe BD. It is evident, that if an inclined pipe CD were substituted for BD, the length would not be sensibly increased, while the inciting force CB, now diffused, will exert still the same action as before. The velocity which the fluid must

acquire in its gradual descent along CD is hence equal to what it receives from the constant pressure during a horizontal progress. If the train of pipes be sufficiently prolonged, the points C and A may be viewed as coincident, and the angle ADB considered as the slope of uniform flow. Suppose the declivity from A to D to be the n^{th} part of the distance; then $L = n\,H$ and $v = 50\sqrt{\frac{dH}{nH}} = 50\sqrt{\frac{d}{n}}$. The celerity of the flow thus depends as much on the width of the pipe as on the rate of its descent. Let $n = 1000$, and v will be $\frac{50}{31.6}\sqrt{d}$. With such a gentle slope, therefore, it would require a pipe of 1.582 of a foot in diameter, to give the flow of a single foot each second; and a pipe of one foot wide would discharge only 1.243 cubic feet of water in the same time.

Conceive a rectangular channel to have a section equal to the circle of the pipe, while its bottom and sides are equal to the circumference; the retardation which this would occasion to the current must evidently be the same. Hence, the fourth part of the diameter will be equal to the quotient of the section by the compound measure of its bottom and sides, which is called the *mean hydraulic depth*. This depth being denoted by a, therefore $4a = d$, and by substitution $v = 50\sqrt{\frac{4a \cdot H}{L}} = 100\sqrt{\frac{aH}{L}}$.

Hence much less obstruction is encountered along an open canal than within a close pipe. Let, as before, the slope be the n^{th} part of the distance; then $v=100\sqrt{\frac{a}{n}}$. If $n = 1000$, the expression will become $v = 100\sqrt{\frac{a}{1000}} = \sqrt{10\,a} = 3.1623\,\sqrt{a}$.

In these estimates of the velocity and discharge of water through pipes and conduits, the inside is presumed to be smooth, the width uniform, and every sudden bending avoided. The want of evenness of surface impedes the motion of the fluid, which is farther obstructed by any violent change of celerity or direction. Whether the channel be contracted or enlarged, the change is unavoidably attended by a proportional loss of impulsion. Any sharp flexure of the pipe or conduit will occasion a still greater waste of the inciting force. The diminution of the square of the velocity is expressed by the product of that square into the square of the sine of the angle of deflexion, divided by the constant number 270. With a deflexion of 30 degrees, the velocity would therefore lose only the 2160th part; but if the tube branched off at right angles, the retardation would amount to the 540th part. Every contraction or enlargement of the pipes, requiring a corresponding change in the celerity of the water, must likewise create an expense of force, though this effect could scarcely be reduced to calculation.

Water is subject in its motion through pipes to another impediment, owing to the air which constantly separates from it and collects in all the upper sinuosities of the train. This accumulation is most copious, whenever the supply of water happens to be insufficient to fill the whole extent of cavity. To remedy the defect, boxes of cast-iron are fixed above the principal incurvations of the pipe, to receive the compressed air, and by the operation of a valve or of a cock, gradually to discharge it, without allowing any of the water to escape. Of such air-vessels, with a cylindrical form, four feet high, and eighteen inches wide, fourteen have been made, for the pipes which are to supply Edinburgh. (See fig. 167.) These being screwed at the summit of each declivity, will be opened, every two or three days, by the surveyor of the works.

It must be observed, that the emission of fluids from a very minute orifice, or through a capillary tube, is not reducible to the foregoing principles. The obstruction is then proportionally far greater; but the peculiar quality and condition of the fluid materially derange the whole result. The internal motions of the particles of any liquid, which in this case mainly determine its flow, are extremely retarded by any tendency to a viscuous state. Heat, therefore, as it brings them nearer to the condition of perfect fluidity, promotes greatly the celerity of their course. Thus, pure water near the boiling

point is found to run about five times faster than at the verge of freezing. Alcohol, again, has its constitution so much altered by the rise of only 124° of temperature, as to flow six times quicker than before. Quicksilver is indeed less affected in this way; but it endures heat through a far wider range.

To supply the inhabitants of great cities, water is often conveyed from distant but high-seated springs, by a long train of pipes. The velocity of the flow may be determined from the formula, $v=50\sqrt{\frac{dH}{L}}$; whence the quantity of water delivered every minute is $2356d^2\sqrt{\frac{dH}{L}}$ cubic feet, or $2356\,d^{\frac{5}{2}}\sqrt{\frac{1}{n}}$. Let m denote the altitude of the source above the reservoir in hundredth parts of the whole distance, and the discharge of water in a minute will be expressed by $235.6\,d^{\frac{5}{2}}\sqrt{m}$.

Suppose a pipe were composed of two portions having different widths. Let the length and diameter of the first part, and the height and velocity of the contained water, be denoted by l, d, h and v; while these measures in the second part are represented by capitals. The whole accelerating force is therefore $=hd^2+HD^2$, and the retardation $=ldv^2+LDV^2$; whence, by substituting $V^2.\frac{D^2}{d^2}$ for v^2, we have

$l.\frac{D^2}{d}.V^2 + LD.V^2 = hd^2 + HD^2$, and $V^2 = \frac{hd^3+HD^2d}{lD^2+LDd}$, or $V = 50\surd\left(\frac{hd^2+HD^2}{lD+Ld}\cdot\frac{d}{D}\right)$. The quantity of water delivered in a minute is therefore $2356D^2\surd\left(\frac{hd^2+HD^2}{lD+Ld}.\frac{d}{D}\right)$.

The former supply of Edinburgh was brought by two trains of cast-iron pipes; one from Green Craig to the Castle Hill, 26,930 feet long, and 7 inches in diameter, under a head pressure of 404 feet; the other from Comiston to Heriot's Reservoir, 13,520 feet long, and 5 inches wide, under a load of 88 feet. The first, when fully charged, is found to deliver 46, and the second only 10 cubic feet, every minute, making together 56 feet, which furnishes a supply of 80,640 cubic feet in the space of twenty-four hours. This amounts to scarcely three-fifth parts of the quantity assigned by the formula. The deficiency must be attributed wholly to the imperfect execution of those pipes, their uneven interior surface, and their frequent abrupt and sudden bendings.

The water-works lately designed for the complete supply of Edinburgh, were conducted in a much finer style, but at vast expense. The several pieces of pipe were nicely fitted together by spiggot and

faucet, all the accidental prominences along the inside being carefully removed by chiseling. The pipes showed no visible incurvation, and were generally laid with a gentle regular slope, the ground on which they rest being lowered in some places and raised in others. From the Crawley Spring to Straiton March Fence, the distance is 18,800 feet, with a fall of 65 feet; and, in this line, the pipes vary from 20 to 18 inches in diameter; their great width being intended to meet the exigency of having lateral branches extending to other remote springs on the north side of the Pentlands, as the increase of population may afterwards require. The next train has a diameter of only 15 inches diameter, but runs, with a fall of 286 feet, through an extent of 27,900 feet, being conducted by a tunnel of 360 fathoms length through Heriot's Ridge, and by another of 290 fathoms through the Castle Hill, till it reaches the level of Prince's Street. It may be computed, that the first train should deliver 442.93 cubic feet, and the second 416.71, or the compound system 426.31 every minute. Their discharge, however, has not yet exceeded 300 cubic feet in a minute, though, no doubt, they could easily convey one-third more.

It has been computed that, the quantity of rain which falls annually over any city, if carefully collected and deposited to purify in cisterns, would be

sufficient for the supply of the inhabitants, at least in all the essential domestic and culinary purposes. Venice has abundance of fine soft water procured in this way; and the store seldom fails, except in dry seasons, when it is recruited from the river Brenta. The roof of a lofty house at Paris, containing at an average 25 lodgers, might deliver annually 1800 cubic feet of rain-water, which would furnish each individual daily the fifth part of a cubic foot, or about thirteen pounds averdupois,—rather a scanty provision, to be sure, according to our modern ideas of comfort; yet Prony reckons ten *litres*, or the thousandth part of his modulus, as a sufficient supply, amounting only to about twenty-two pounds.

Since, from a pipe of the same diameter, the discharge in every case depends on the relation of the altitude of the source to the length of track, a lower elevation may frequently be preferred in conjunction with a shorter train. The diminished obstruction, in such instances, compensates for the inferior pressure. From any point of an inclined plane, the pipe would convey exactly an equal body of water. In the same train, the quantity of discharge, being as $d^{\frac{5}{2}}$, must increase in a faster ratio than the mere section of the pipe. Hence the manifest advantage of employing large pipes. For the same reason, aqueducts or open conduits are in many situations to be preferred. When these convey large streams of wa-

ter, the attrition of the sides and bottom is comparatively small, and they require very little descent. Such durable structures are common in the South of Europe, and often display much architectural symmetry in their extended and imposing ranges of arcades.

It is a very prevailing opinion, that the Romans, amidst all their magnificence, were ignorant of the simplest elements of Hydrostatics, and therefore entirely unacquainted with the method of conducting and raising water by a train of pipes. Nothing, however, can be worse founded than this notion. The ancient writers, who either treat of the subject, or incidentally mention it, are clear and explicit in their remarks, while many vestiges of art still attest the accuracy of those statements. Pliny, the natural historian, lays down the main principle, that "water will invariably rise to the height of its "source:" *Subit altitudinem exortus sui.* He subjoins, that leaden pipes must be employed, to carry water up to an eminence *. Palladius, in his treatise *De Re Rustica*, teaches how to find springs, by observing immediately before sunrise in the month of August, the vapours which hover above particular spots; and having there dug a well, he directs the water to be conducted to the farm or villa, either by

* Laminæ esse debebunt, per quas surgere in sublime opus fuerit, e plumbo.—PLIN. xxxvi. 7.

a channel constructed of masonry, or by means of pipes of lead, of wood, or even of earthenware *. He allows one foot in sixty, or in a hundred, for an uniform descent. But if the ground should afterwards rise, he says the conduit must be supported on piles or arches, or the water must be inclosed in leaden pipes, when it will mount just to the level of its head †. But Palladius testifies his aversion to the use of lead, as apt to become covered with ceruss, and thereby rendered unwholesome, or even poisonous. This consideration had no doubt served to restrain the general adoption of leaden pipes among the Romans. Still, however, we may infer, from the allusions of the Poets, that such pipes had come into very common use. They were not cast tubular as at present, but consisted of thin plates bent up into the form of a cylinder, and soldered along the edge. They must frequently have given way, therefore, at this seam. Horace asks, if the water which threatens in the streets to burst its *lead*, be purer than the rivulet that trembles and murmurs as it flows ‡? Ovid compares the gush of

* Cum vero ducenda est aqua, ducitur aut forma structili, aut canalibus ligneis, aut fictilibus tubis.—PALLAD. ix. 11.

† Sed si se vallis interserat, erectas pilas vel arcus usque ad aquæ justa vestigia construemus, aut plumbeis fistulis clausum dejici patiemur, et explicata valle consurgere.—IBIDEM.

‡ Purior in vicis aqua tendit rumpere plumbum,
Quam quæ per pronum trepidat cum murmure rivum?
EPIST. I. x. 20.

blood from the mortal wound which Pyramus, in the agony of despair, had inflicted upon himself, to the accidental rupture of a *leaden-pipe* *. Statius speaks, no doubt with poetical exaggeration, of whole rivers being discharged by such conduits †. Vitruvius describes the three principal modes of conveying water; but directs, as the previous operation, to trace a level *(libramentum)* on the ground. This *libration* was performed, by the *dioptron*, the *water-level*, or the *chorobates*. The dioptron seems to have been a sort of quadrant fitted with sights; the water-level consisted of a tube, probably of copper, five feet long and an inch wide, turned up an inch and half at both ends, and was adjusted till water rose equally in them; the chorobates, or perambulator, which he considered as the most accurate instru-

* Non aliter, quam cum vitiato fistula plumbo
Scinditur, et tenues stridente foramine longe
Ejaculatur aquas, atque ictibus aëra rumpit.

METAM. iv. 120.

Thus translated:

As out again the blade he, dying, drew,
Out spun the blood, and streaming upwards flew;
So if a conduit-pipe e'er burst you saw,
Swift spring the gushing waters through the flaw;
Then spouting in a bow they rise on high,
And a new fountain plays amid the sky.

† Terque per obliquum penitus quæ laberis amnem
Martia, et audaci transcurris flumina plumbo.

STATIUS, I. SYLV.

ment, was composed of a rod twenty feet long, having a square and plummet attached at each extremity. Vitruvius allows only half a foot in the hundred, for the slope of an aqueduct. After the water had reached the walls of a city, it was admitted into a reservoir or *castellum*, divided into three distinct and equal compartments, one to feed the pools and fountains, another to supply the public baths, and a third for the accommodation of palaces and private houses. The distribution of the water was effected commonly by means of leaden pipes. The smallest of these was called a *denaria*, being ten feet in length, the sixteenth part of this in breadth or girth, and weighing 120 Roman pounds. This gives, for the thickness of the lead, exactly the quarter of an English inch. In lower situations, where the stress against the sides was greater, the pipes appear to have been made proportionally stronger *.

* In the Physical Cabinet of the University of Edinburgh, is now deposited a specimen of ancient leaden pipe, lately brought from Rome, where it had been dug up among the ruins of the Palace of the Cæsars. It bears an inscription in raised letters, intimating the name of the plumber, and the year of the reign of the Emperor Domitian. Though only 16 inches long and 9½ in girth, it weighs 22½ lbs. averdupois; so that the lead must be very nearly half an inch thick. The pipe is slightly curved and rudely formed into merely a flattened oval, 2½ inches broad, and 1¼ wide; the joining at the edge being filled by a quantity of melted solder run along both inside and outside. The section corresponds to a circular orifice of 1¾ inches in diameter.

The quantity of water delivered from the cisterns was regulated by the dimensions of the spouts, termed *calices*. These formed a series of twenty-five different kinds, which served as *moduli*. Their diameters were sometimes reckoned by *ounces*, or the twelfth parts of a Roman foot, but more commonly by quarter digits, or the sixty-fourth part of a foot. The *quinaria* seems to have been considered as the standard, and its width must have hence corresponded to the .906 part of an English inch. The adjutage or length of all those spouts was the same, being twelve digits, or three-fourths of a Roman foot, and therefore equal to 8.7 English inches. Prony conjectures, from very probable grounds, that such was also the altitude of column of pressure above the middle of each orifice. This estimate gives 1979 cubic feet, for the quantity of discharge of a *denaria*, in the space of twenty-four hours.

Leaden pipes were likewise employed, to carry water across vales and other eminences. But it behoved to erect, at the several incurvations, *columnaria*, or chimneys, to give vent to the air which might collect and gorge up the passage of the water. Such funnels required to be raised to near the height of the fountain-head.

Vitruvius, however, joins with Palladius and Columella, in recommending pipes of earthenware, as not only cheaper, but more wholesome than those of lead. They could be formed thicker if necessary,

and might be farther strengthened and secured, they said, by an outer coating of lime worked up with oil. But such pipes, not being glazed, it became necessary, before using them, to fill up the pores by a sort of *puddling*, that is, to wash their inside with *favilla*, or fine wood-ashes.

No wonder, therefore, that leaden pipes were held in little estimation among the ancient Romans. They seem to have been seldom used indeed beyond the limits of the imperial city, except as auxiliaries in the smaller aqueducts. When such conduits happened to be interrupted by a deep narrow vale, instead of joining them by an arch thrown over the gap, the connexion was sometimes formed by an inverted syphon of lead, carried on the one side down to the bottom, and brought up on the other.

Rome was supplied by nine great aqueducts, according to Frontinus, who had been appointed curator of those magnificent works by the Emperor Nerva. He added five more; and the number was afterwards augmented, by successive emperors, to twenty. Of these, the most remarkable were, 1. The *Aqua Appia*, so named from its having been constructed by the Censor Appius Claudius in the 442d year of Rome, began between the 6th and 8th milestone, made a circuit of 880 paces, and then proceeded by a deep subterranean drain of more than 11 miles, delivering the main body of its water in the Campus Martius. 2. The *Old and New Anio*, con-

duits so called from their bringing into Rome the waters of that river. The former began above the Tiber at the 30th mile-stone, and consisted mostly of a winding drain carried through an extent of about 43 miles. The latter, constructed under Nero, took a higher level, running 7,543 paces above ground, and then pursuing a subterranean passage of 54,267 paces in length. 3. The *Aqua Martia*, which owed its erection to Quintus Martius, rose from a spring, distant 33 miles from Rome, made a circuit of 3 miles, and afterwards, forming a vault of 16 feet diameter, it ran 38 miles along a series of arcades at the elevation of 70 feet. It had vents perforated at certain intervals, for disgorging the collected air; and the conduit was occasionally interrupted by deep cisterns, in which the water settled and deposited its sediment. It was hence remarkable for its clear green colour *. The *Aqua Julia* and the *Aqua Tepula* were brought by the same aqueduct, only in two lower conduits. 4. The *Aqua Virginia*, conducted by Agrippa, the patriotic lieutenant of Augustus, who laboured to improve and beautify Rome, and who, according to Pliny, constructed in one year 70 pools, 105 fountains, and 130 reservoirs. It commenced at a very copious spring,

* Pliny celebrates particularly the coolness and salubrity of this water. "Clarissima aquarum omnium in toto orbe frigoris salubritatisque palma præconio urbis, Martia est reliqua deum munere urbi tributa," &c. Lib. xxx. 3.

in the midst of a marsh, at the distance of 8 miles from the city, and ran about 12 miles, passing through a tunnel of 800 paces in length. 5. The *Aqua Claudia*, begun by Nero and completed by Claudius, took its rise 38 miles from Rome; it formed a subterranean stream 36½ miles in length, run 10¾ miles along the surface of the ground, was vaulted for the space of 3 miles, and supported on arcades through the extent of 7 miles, being carried along so high a level as to supply all the hills of Rome. It was built of hewn stone, and still continues to furnish the modern city with water of the best quality, which has hence procured it the name of *Acqua Felice*.

The practice of tunnelling was begun under Augustus, who greatly extended the aqueducts. Other emperors likewise directed their attention to that important object. Trajan showed particular solicitude in improving the aqueducts. Those works were executed in the boldest manner; nothing could resist the skill and enterprise of the Romans; they drained whole lakes, drove mines through mountains, and raised up the level of valleys by rows of accumulated arcades. The water was kept cool by covering it with vaults, which were often so spacious, that, according to Procopius, who wrote in the time of Belisarius, a man on horseback might ride through them. So abundant indeed was the supply, as to induce Strabo to say, that whole rivers flowed through the streets of Rome. Contemplating the utility, the extent,

and grandeur of those aqueducts, Pliny justly regarded them as the wonder of the world *. The same admiration is expressed by the poet Rutilius †.

According to the enumeration of Frontinus, the nine earlier aqueducts delivered every day 14,018 quinaria. This corresponds to 27,748,100 cubic feet. We may therefore extend the supply, when all the aqueducts were in action, to the enormous quantity of 50,000,000 cubic feet of water. Reckoning the population of ancient Rome at a million, which it probably never exceeded, this would furnish no less than fifty cubic feet, for the daily consumption of each inhabitant.

In modern Rome, three aqueducts, the *Acqua Felice*, *Juliana* and *Paulina*, with some additional sources, deliver in twenty-four hours, according to the calculation of Prony, 5,305,000 cubic feet. This, shared among a population of 130,000, gives about

* Si quis diligentius æstimaverit aquarum abundantiam in Publico, Balneis, Piscinis, Domibus, Euripis, Hortis suburbanis, Villis, spatioque advenientes, exstructos arcus, montes perfossos, convalles æquatas; fatebitur, nihil magis mirandum fuisse in toto orbe terrarum. PLIN. xxxvi. 15.

† Quid loquar aerio pendente fornice rivos,
Qua vix imbriferas tollere Iris aquas,
Hos potius dicas crevisse in sidera montes:
Tale Giganteum Græcia laudat opus.
RUTILIUS IN ITIN.

forty cubic feet for each individual, being nearly the same comparative supply as in the period of Roman splendour.

Such profusion of water altogether transcends our conceptions. The supply of London in the year 1790 was only 2,626,560 cubic feet daily; but lately, when the rivalship of the several water-companies almost deluged the streets, it amounted to 3,888,000 cubic feet. It has again, by the mutual understanding of those associations, been reduced to about three millions of cubic feet; and this quantity may be sufficient for all the wants of a luxurious mass of inhabitants, equal certainly to the population of ancient Rome, where the consumption, however, was still sixteen times greater. How paltry then appears the actual supply of Paris, amounting only to 293,600 cubic feet of water in a day! It affords scarcely half a cubic foot, or thirty pounds averdupois, to each inhabitant, in a population of upwards of 600,000.

The Greeks of the Lower Empire had simplified the general mode of conducting water. This appears evident from the practice which now prevails in supplying the city of Constantinople. The ground is levelled by means of the *Terazi*, a sort of inverted mason's plummet, which hangs from the middle of a cord stretched between two rods divided into inches and parts, set upright and removed successively from one station to another. But the chief improvement consists in substituting, for the *colum-*

naria of the Romans, the *Souterazi* or *water-balance*, a sort of hydraulic obelisk or pyramid. By this ingenious contrivance, the expense of aqueducts is reduced to a fifth part. The water runs down with a gentle slope in covered drains, till it reaches an obelisk constructed of masonry; and rising up the one side, by a narrow channel, discharges itself into a basin at the top, from which again, at a level only 8 inches lower, it descends by a similar channel on the other side. The form of this hydraulic pyramid is shown in fig. 172, and the upper part of it is enlarged in fig. 173. Such auxiliary machines, which facilitate the escape of the air and allow the water to settle, are commonly erected at distances of about two hundred yards. The system is in fact only a repetition of conduits. From each separate basin, the water is distributed, by orifices of different diameters, but having their centres all in the same horizontal line, three inches beneath the brim.

The charge of the water-works at Constantinople is entrusted to a body of 300 Turks and 100 Albanese Greeks, who form almost an hereditary profession. According to the interesting work of General Andreossy on the Bosphorus, the whole supply of a population of 600,000 is only two-thirds of a cubic foot, or about forty pounds of water every day. There still remain at Constantinople two ancient cisterns: 1. The Subterranean Palace, built of hard brick, vaulted and resting on marble columns: and 2. The cistern of one hundred and one columns,

called anciently *Philoxene*; it consists of three tiers of columns, one above the other, and is capable of holding five days' supply, for the whole inhabitants of that spacious city.

The same principles which regulate the motion of water in pipes and along canals, are likewise applicable to the flow of rivers in their beds. Since the propelling power is proportional to the elevation of the main source, the celerity acquired by those descending streams would become enormous, if their force were not gradually absorbed by the operation of some constant impediments. Suppose such a river as the Rhone to receive its principal waters at the altitude of 900 feet above the level of the sea, and that no system of obstruction had intervened in its course, it would have shot into the Bay of Marseilles, with the tremendous velocity of 240 feet in a second, or at the rate of 164 miles every hour. Even an inferior stream, such as the Thames, fed at the height of only an hundred feet, would still, if not retarded by the attrition against its bottom and sides, have rushed into the sea with a velocity of 54½ miles in an hour.

The resistance of fluids, like the friction of solids, thus enters largely into the economy of nature. As the latter is the great principle of stability and consolidation, so the former serves most essentially to restrain the accumulation of celerity, and to moderate all violent motions. A current presses forwards with increasing rapidity, till the obstruction which

it encounters becomes at last equal to the inciting force; and having attained this limit, the water then continues to flow in a uniform stream. The maintaining power is proportional to the quantity of descent in a given space; but the impeding influence depends on the surface of the bed of the river compared with its volume of water. This obstruction must at first augment very fast, being as the square of the celerity.

Let n, as before, denote the measure of declivity, and a the mean hydraulic depth or the depth which a river would take if it stood upon an even base equal to the bottom and sides of its channel; then $100\sqrt{\frac{a}{n}}$ will express the resulting velocity in feet each second. Hence, if the rate of descent were only one part in ten thousand, the stream would acquire a velocity represented simply by the square root of the mean hydraulic depth. Let f denote the fall in feet each mile, and the formula will change into $v=\frac{100}{\sqrt{5280}}\sqrt{af}=\frac{11}{8}\sqrt{af}$. Hence the velocity, reckoning in miles every hour, is expressed by $\frac{11}{8}\cdot\frac{15}{22}\sqrt{af}=\frac{15}{16}\sqrt{af}$. This result is quite conformable to actual observation. The square root of the product of the hydraulic depth into the fall each mile in feet being diminished by one-sixteenth part, will hence represent the mean velocity of a river in

inches each hour. This gives the central velocity of any section of the stream; but the flow is the most rapid in the middle of the upper surface, and diminishes regularly as it approaches the bottom and sides of the channel, where the retardation originates. The extreme difference between the velocity at the top and near the bed of a river appears by observation to amount to half the square root of the computed mean velocity. Thus, the Ganges, in its ample tide, has only a fall of 4 inches in a mile, with an hydraulic depth of 30 feet; but $\sqrt{(30 \times \frac{1}{3})} = 3.16$, which diminished by one sixteenth part, is 2.96, or very nearly three miles an hour. Again, $\sqrt{2.96} = 1.70$, the half of which is .85; whence the celerity at the surface of the stream is 3.81, and at the bottom only 2.15.

These calculations proceed on the supposition that the river holds nearly a straight course. If it should wind considerably, the multiplied deflections which it suffers must still farther impede its motion. In every bend which it makes, part of its impulse will be spent against the concave side of the channel; the centrifugal effort will likewise raise the surface of the water in those sinuosities, and therefore augment the abrasion of the banks. Hence no stream can be long confined to a rectilineal channel. If an accidental swell should once effect a breach, the sweep of the current must necessarily tend to enlarge the concavity by an accelerating progression; the oppo-

site shore, from the accumulation of gravel and other deposits, gradually advancing into the channel. Rivers thus naturally form sinuosities; they seek to meander over the plains; and they would incessantly change their beds, if not restrained by sedulous attention and skilful hydraulic operations. In such a country as Italy, whose rich plains are swept by torrents from the Alps and Appenines, the superintendance of water-courses constitutes an important department of government.

If a flat surface be directly opposed to the action of a stream as it shoots from the side of a vessel, it must evidently sustain a pressure just equal to that which actually projected the fluid, or the load of the incumbent column. In every case, therefore, the impulsion of any current against a perpendicular plane, may be estimated by the weight of a body of the fluid standing upon that surface, and having the altitude due to the velocity. Resuming the former notation, since $v=8\sqrt{h}$, it follows that $h=\frac{v^2}{64}=\left(\frac{v}{8}\right)^2$. This expression corresponds, in the case of a stream of water, very nearly to one pound averdupois for every square foot of the obstacle, multiplied into the square of the velocity in feet each second. Let A denote the area of the opposing surface, and $\frac{62\frac{1}{2}}{64}.A.v^2$, or $\frac{250}{256}.A.v^2$, will express more accurately the mea-

sure of the shock. If the stream were to consist of sea-water, the fractional coefficient might be omitted, and $A.v^2$ simply will represent the impelling force.

Let V denote the velocity of the current in miles each hour, and the former expression must be multiplied by the square of the fraction $\frac{22}{15}$. This gives for the shock $\frac{250}{256} \cdot \frac{484}{225} \cdot A.V^2. = \frac{605}{288} \cdot A.V^2$, which is very nearly $\frac{21}{10} \cdot A.V^2$. In practice, it may be sufficiently accurate, to reckon, for every square foot of opposing surface, the product of two pounds averdupois into the square of the celerity of a stream of water expressed in miles an hour. The pressure of a river against the piers of a bridge may be hence computed. The shock becomes augmented in a high ratio during floods; for not only is a greater extent of surface then opposed to the current, but the effort on every given space follows also the square of the increased velocity.

The mighty rush of a torrent, carrying along with it fragments of rocks, stones, or gravel, depends on the same principle. Let $w = 62\frac{1}{2}$ lb., and a denote in feet the side of any cubical block, of which the specific gravity is g; its weight under water will evidently be expressed by $a^3\ (1-g)\ w^3$. The impulse of the stream would therefore be sufficient

to support the load, when $2a^2.V^2 = a^3(1-g)w^3$, or $V^2 = \frac{a}{2}(1-g)w$. In the case of stones, the value of g lies between $2\frac{1}{2}$ and 3, and consequently $2V^2 = 1.6 \times 62\frac{1}{2} \times a = 100a$, and $V = \sqrt{50.a}$. The force required to overcome the friction of a stone at the bottom of a current, or to urge it forward, will generally be less than this quantity, seldom perhaps exceeding the half of it. Hence we may assume $V = 4\sqrt{a}$. The effect will be nearly the same, if the block should approach to a round shape. A torrent with the celerity of eight miles an hour would therefore be capable of rolling a stone of four feet in diameter. But a stream gliding at the rate of two miles an hour would only be sufficient to carry along with it a pebble of three inches in diameter. With lower velocities, the current will scarcely move gravel. If the particles of sand were supposed to have a diameter equal to the twenty-fourth part of an inch, it would require a flow of a quarter of a mile in an hour to bear them along. A velocity of the tenth part of a mile in an hour would be sufficient to carry sandy particles of only the hundred and twenty-third part of an inch in diameter.

Hence, the theory of the washing of metallic ores, and the deposition of gold dust in the beds of rivers. The ores being broken into very small fragments by means of a stamper, these are laid upon an inclined plane, and exposed to the action of a descending stream of water, which sweeps away all the lighter

earthy particles. In like manner, the pellicles of gold, adhering commonly to minute portions of quartz, being at length detached by incessant rolling, are left in the little pools, while the sandy particles are carried still farther. Hence also the reason why the bottoms of rapid rivers are covered with large round stones, or at least with rolled pebbles. Where the celerity of the current becomes moderated, gravel and coarse sand begin to appear. But when the flow is sluggish, the bed of the river is always covered with fine sand or mud. Such deposits occur chiefly in the pools and near the influx into the sea. Hence likewise the gradual formation of banks, a process which is constantly going on over all the stagnant parts of water, and along the limits of opposite currents.

Water is the readiest and most powerful agent that can be directed by human skill. A mill-race, for example, three feet broad, and two feet deep, and running at the rate of four miles an hour, would communicate an impulsion equal to the fall through .538 parts of a foot; whence the action thus created, during the space of a minute, is expressed by the product $3 \times 2 \times 352 \times 62\frac{1}{2} \times .538 = 70{,}966$, which being incessant, amounts to the ordinary labour of an hundred men. If this current had then fallen $26\frac{1}{4}$ feet, its quantity of operation would have been augmented fifty times more. But such streams are easily collected and formed in nume-

rous situations over the undulating face of the country.

It will expand our conceptions, if we survey the great laboratory of Nature, and calculate the enormous extent of power, displayed in elevating the watery stores into the lofty regions of the atmosphere. Between the tropics, the annual fall of rain, and consequently the measure of evaporation which supplies it, amounts to about 10 feet; and estimating this in other countries, as nearly proportional to the cosine of the latitude, the quantity of moisture exhaled in the course of a year, over the whole surface of the globe, would form a shell of five feet deep. The number of cubic feet of water turned into vapour, and dispersed through the mass of atmosphere every minute, would hence be $5 \times 10,424,000,000$, or 52,120 millions. But this quantity is to be multiplied by 18,000, the mean height of the atmosphere in feet, and again by $62\frac{1}{2}$, the weight in pounds averdupois of a cubic foot of water; the final measure of effect is therefore expressed by 58,635,000,000 millions, and equal to the labour of about 80,000,000 millions of men. Now, the whole population of the globe being reckoned 800 millions, of which only the half is capable of labour; it follows, that the power exerted by Nature, in the mere formation of clouds, exceeds, by *two hundred thousand times* the whole accumulated toil of mortals.

A considerable portion of the power thus expended, might be directed to useful purposes, by inter-

cepting the water again in its descent towards the ocean. Suppose one-sixth of all the exhalations to return to this great gulf, and that half of the falls in the rivers and streams over the habitable earth, comprising the fifth part of the whole surface of the globe, are detained from an elevation of 600 feet; there would be drawn from those mighty stores a force *eleven times* greater than the aggregate of human labour.

It may be satisfactory, however, to take a more definite illustration. The surface of this island is computed at 67,243 square miles, or 1,874,627,000,000 square feet. But reckoning the annual measure of rain 36 inches, of which the sixth part may flow towards the sea, and supposing one half of this surplus, or three inches, to be intercepted at an elevation of 100 feet, it would require to multiply the former number by $100 \times \frac{1}{4} \times 62\frac{1}{2}$, or $1562\frac{1}{2}$, and divide the product by 525949, the number of minutes in a year, to obtain the quantity of performance. The result is 4,423,760,000, equivalent to the action of 6,703 steam-engines, of what is called twenty-horse power. It may hence be fairly estimated as not inferior to the ordinary labour of the whole of our male population. Such is the vast magazine of force, which a rigid economy might command.

The power exerted by the moon and sun to heave the tides of the ocean, is only about the eightieth part of the action of the atmosphere in producing the train of meteorological phænomena. Rec-

koning the swell at the equator to be 4 feet, this gives a mean elevation of 2 feet over the whole furface. Wherefore, a body of water, 2 feet deep, is raised to the intermediate height of one foot twice in the lunar day, or 706 times in the course of the year. The relation of the force thus employed to that of the general exhalation over sea and land, is hence as $2 \times 1 \times \frac{4}{5} \times 706$, or 1129.6 to 5×18000, or 90000, that is, as 1 to 80; and therefore still *two thousand five hundred times* greater than the aggregate labour of the human race.

But the rise and fall of the tide, along our extended shores, would be sufficient to drive numerous mills. Suppose a basin were inclosed only a chain or 66 feet in width and 10 chains in length, and containing therefore an acre of salt water, this would give an impulse equal to the flow of 43,560 cubic feet in 12 hours and 25 minutes, or about $58\frac{1}{2}$ feet every minute, with a fall of 5 feet. The expression for the force evolved, is hence the continued product of $5 \times 58\frac{1}{2} \times 64\frac{1}{2} = 18660$.

The performance of this tide-mill might hence be equal to that of 25 common labourers. But eight such basins could be included in each mile of coast; and therefore, estimating the circuit of the island at 1750 miles, there might be formed no fewer than 14,000 mills, by drawing a sea-wall 66 feet from the shore. In this way, a saving of power might be effected, equal to the labour of 350,000 men. The expence of erecting such a dam, would

probably defeat the object as a general scheme of improvement; but there occur very many creeks and bights along our indented coast, which could be most profitably inclosed as reservoirs for large tide-mills.

The float-board of a river-mill is impelled, not by the whole velocity of the stream, but only by the excess of this above the velocity of the board itself. Let A denote in square feet the surface immersed in the water, v the velocity of the current, and v' that of the middle of the float; then $2A\,(v-v')^2$ will express in pounds averdupois the pressure which turns the wheel. The momentum thus acquired must hence be proportional to $v'\,(v-v')^2$, an expression, which being assimilated to the property of a parabola, must become a maximum when $v' = \frac{1}{3}v$. The greatest impulsion communicated merely to a wheel is therefore only $\frac{1}{3}.\frac{4}{9}$ or $\frac{4}{27}$ of the whole power of the stream. But it would be more advantageous to make the float-boards turn slower, and to multiply their velocity afterwards by means of a train of internal machinery. The current might then strike with nearly its full celerity. In the case of undershot wheels, a great loss of power is occasioned by the accumulation of the *dead-water*, or of the water which, having impinged against a float-board, remains nearly stagnant, and therefore impedes the advance of the next float-board.

The shock will evidently be the same whether a current strikes against a fixed plane, or the plane itself moves with an equal and opposite velocity through a fluid at rest. The formula of impulsion already given must hence include likewise the case of resistance, which is therefore proportional to the square of the celerity. But this conclusion might be derived from direct considerations. As the plane advances, it receives the stroke of all the particles in its progress. The quantity of momentum thus consumed is evidently compounded of the number of particles encountered and the impetus of each; but the number and individual force of those particles being as the velocity, the combined effect must be proportional to the square of the velocity.

If a fluid strike a plane obliquely, its action is to be determined from the decomposition of forces. Let the oblique plane AB (fig. 174.) receive the shock of a current perpendicular to AC. If DB, assumed equal to AB, denote the direction and intensity of the stroke, this force may be resolved into BE, parallel to AB, and DE at right angles to it. It is the latter only that can have any effect upon the plane AB. But the force DE, again, is decomposed into DF, perpendicular to AC, and FE parallel to it. FE therefore urges the plane in the direction CA; but the breadth of the stream being likewise proportional to CA, the combined action must be represented by the rectangle FE, CA, or by

the solid AB.FE.CA, since AB is constant. Now the right angled triangle ABC, being obviously equal to DBE, the sides BC and CA are equal to BE and DE. But AB : AC : : BE or BC : EF, whence AB.EF = AC.BC, and the solid AB.AC.EF = AC^2.BC. The force which impels the plane AB in the direction CA is therefore exerted to the utmost when AC^2.BC is a *maximum*. But AC and BC are chords in a given semicircle, described on AB; and from what was shown in p. 258, the greatest solid requires that the square of AC should be double the square of BC. Whence the tangent of the angle BAC $= \sqrt{\frac{1}{2}} =$ tan. 35° 16′. Such is the angle that the rudder of a ship makes with a perpendicular to the keel, when it exerts the greatest power in turning the vessel.

Suppose an oblique plane AB (fig. 175.) or a wedge CAB to move in the direction CA against a quiescent fluid. Let BD be taken equal to AB and parallel to AC, draw DE and EF perpendicular to AB and BD. After repeated decomposition, the only portion of the impinging force DB opposed to the advance of the plane AB, is DF. The whole resistance is therefore expressed by BC.DF, or by BC.BD. But BD : DE : : DE : DF, or BG, if CG were let fall perpendicular to AB. Again, $BD^2 : DE^2$: : BD : DF; whence the resistance of the wedge CAB, while its base CB, or the transverse section of the fluid remains constant, is proportional to

the square of the sine of the angle BAC. But if AB, the side of the wedge, or the length of the oblique plane, should be the constant quantity, the resistance will follow the cube of the inclination.

These investigations, however, assume, for the sake of simplicity, that every particle, the instant it has impinged, retires into the general mass of the fluid, without consuming any force. But such a supposition deviates widely from the truth. The fluid continues, after collision, to press against the sides of the wedge, and therefore impede its advance. To overcome the inertia of this accumulation, will necessarily require the expenditure of a large share of the impulsion. The fluid particles which have met the shock of the oblique plane, must be pushed off laterally, by some force depending on the cosine of the angle of incidence.

The resistance which wedges experience in moving through water is accordingly much greater than what mere theory indicates. It diminishes indeed with their angle, but not in so fast a ratio as the square. The acuteness of that angle occasions a greater drag of the fluid, and a corresponding retardation. When the floating body is very long, the nature of the resistance becomes totally changed. Thus, a ship's mast, though it has a conical shape, is more easily drawn through the water with its broad, than with its narrow, end foremost. In this case, the primary obstruction is no doubt greater; but

the water heaped on the front, being made to stream off with a slight divergency, does not hang on the sides of the mast. For the same reason, whatever tends to weaken the adhesion, serves likewise to diminish resistance. A wedge which has its sides rubbed with grease, is found to move more freely through the water. Hence the great benefit derived from sheathing the bottom of a ship with copper.

To lessen the resistance, it is of more consequence that the prow of a vessel should be gently incurved than have a sharp outline. The roundness of the prow assists in gradually turning aside the current. Even the breadth of the bow of the vessel is sometimes advantageous; but the tapering of the hull is always an important requisite. The bilged form of the stern, which the Dutch have long adopted, appears at last to be judged preferable, not only strengthening the ship, but materially lessening its resistance in the water. Whenever an eddy occasioned by the abruptness of outline whirls under the stern, or a portion of fluid follows the vessel, its progress must evidently be retarded. In general the shape of the hinder part of any floating-body has a very considerable influence on the quantity of resistance which it must encounter. To determine all these circumstances from strict theory, would require an investigation of the most repulsive and hopeless intricacy. It would not only be necessary to calculate the intensity and direction of the shock of each

particle of fluid, but to trace the diverging streamlets with which they again recede, and to reduce into a single amount the effect of all the diversified elements of resistance. The higher calculus might compress those conditions under the powers of its notation; but the resulting equations would still admit of no complete integration. It is a much easier and a safer procedure, to resume the limited and imperfect deduction already given, and to correct it by a series of accurate experiments. Many of these have been made, but not on a scale, it must be confessed, at all suited to the importance of the subject. The reports of different observers are, besides, not very consistent. We may therefore assume, for an oblique plane whose inclination is the angle a, the expression $\sin a^2 + \frac{1}{4} \cos a$, as a tolerably near approximation to the resistance, that of its base being reckoned unit. The same result may be given geometrically for the wedge CAB (fig. 175.) by making BH $= \frac{1}{4}$ AC, when GH will indicate the resistance.

It may be worth while, however, to exhibit the gradation, by a tablet of the resistance for each five degrees, that of the base being unit.

Angle.	Resist.	Angle.	Resist.	Angle.	Resist.
5°	.255	30°	.466	55°	.814
10°	.276	35°	.534	60°	.875
15°	.308	40°	.605	65°	.927
20°	.362	45°	.677	70°	.968
25°	.385	50°	.748	75°	.998

If a stone be dropped into a deep pool, it will gradually accelerate its motion, till the increasing resistance it encounters becomes equal to its descending force, or the excess of its weight above that of the volume of water displaced; and it then continues to fall with an uniform velocity. This velocity, which serves as a limit, is hence called *terminal*; and it evidently depends on the form, the size, and the density of the plunging mass. To simplify the consideration, suppose the body were globular, its diameter in feet being denoted by d, its specific gravity by g, and the velocity by v. The excess of its weight above that of water will therefore be $\frac{\pi}{6}.d^3\,(g-1)$; while the resistance of a circular section is $\frac{\pi}{4}.d^2.\frac{v^2}{64}$, and that of the sphere itself the half of this, or $\frac{\pi}{8}.v^2.\frac{d^2}{64}$. Whence equating these expressions, we obtain $v=9\frac{1}{4}\sqrt{(d\,(g-1))}$. The terminal velocity is therefore in the subduplicate ratio of the diameter of the ball, and its preponderating density. Thus, an eighty pound iron shot, plunging into a fresh-water lake, would acquire a celerity of 20.4 feet in a second, or would descend through the space of 204 fathoms every minute; for reckoning the specific gravity at 7.5, the diameter of the ball would be about $8\frac{1}{2}$ inches, or .714 parts of a foot; whence $v=9\frac{1}{4}\sqrt{(.714\times 6.5)}$. In the ocean, the celerity

must, owing to the greater density, be somewhat less, or 201 fathoms. But a ball of a pound and a quarter, or 64 times lighter, and consequently having a diameter very little more than 2 inches, would sink twice as slow, or at the rate of 100½ fathoms every minute. Whether the balls drop softly into the water, or enter it with vehement impulsion, they will, in a very short time, acquire their terminal velocity. The same principle regulates the ascent of lighter bodies, the formula being only modified into $v=9\frac{1}{4}\sqrt{(d(1-g))}$. Thus, reckoning the specific gravity of cork to be .20, a ball of that substance, two inches in diameter, if not affected by compression, would rise from the bottom of a lake with a velocity of 34 fathoms in a minute.

In other fluids, it is only requisite to substitute the relative value of g. Thus, for atmospheric air, the formula is changed into $v = 9\frac{1}{4}\sqrt{(d(840g-1))}$. An eighty pound iron-shot, if fired upwards, would hence acquire, in falling back to the ground, the celerity of 608 feet in a second, which is sufficient to produce the most destructive effects.

In the case of drops of rain, the formula will be modified into $v=9\frac{1}{4}.29\sqrt{d}$; and for hailstones, we may assume $9\frac{1}{4}.30\sqrt{d}$. Thus, the largest drop of rain being only the fifth part of an inch diameter, the celerity of its descent cannot exceed $9\frac{1}{4}.29.\frac{4}{31}=34\frac{2}{5}$ feet in a second, or 2040 every minute; but the ordinary drops in this climate will seldom fall half as

fast. On the other hand, hailstones in the south of Europe have sometimes the enormous diameter of two inches. Here $v = 9\frac{1}{4} \cdot 30 \cdot \frac{20}{49} = 113\frac{1}{5}$ feet, or exceeding a mile and a quarter in a minute; a rapidity of stroke which destroys corn-fields, and ravages vineyards.

That the resistance which fluids oppose to penetration, increases with the square of the celerity, is easily shown by a series of glass-beads, of different densities, dropped into a very tall jar full of water; for these balls in a short space acquire a certain terminal velocity, with which, thenceforth, they pursue their uniform descent. While the standard of water is 1000, let the beads be adjusted to the specific gravities of 1001, 1004, 1009, 1016, proceeding by the square numbers as far, perhaps, as 1100, and have all of them an inch in diameter. Then, for the first bead, $v = 9\frac{1}{4}\sqrt{\frac{1}{12}}\sqrt{\frac{1}{1000}} = \frac{1}{12}$, or an inch; so that reckoning from the limit, or less than a foot below the surface, the velocities which they attain are expressed in inches each second, by the consecutive digits, 1, 2, 3, 4, &c. to 10. With the heavier beads, such as those marked 1009, 1016, 1025, &c. the correspondence between these numbers and the results of actual experiment is very satisfactory. But those beads near the beginning of the series descend more slowly than the theory indicates: Thus, the bead 1004, instead of falling

through two inches, scarcely describes one, and the first bead, instead of a single second, takes more than three seconds to pass through an inch. This sluggishness proceeds evidently from the imperfect fluidity of water, the effect of the tenacity, or mutual adhesion of its particles, which cramps their internal mobility. Hence the slow subsidence of earthy matters discharged into lakes; and hence, likewise, the suspension, in our atmosphere, of those very minute aqueous globules which compose the clouds.

If, instead of the tall jar, there be substituted tubes of the same height, but only $2\frac{1}{2}$, 2, and $1\frac{1}{2}$ inches in diameter, all the beads will descend much slower than before, especially in the narrowest one. The retardation is evidently caused by the difficulty with which the water in this case recedes, and gives way to the passage of the ball. This experiment illustrates the peculiar obstructionfound in drawing loaded boats through the narrow and shallow parts of a canal.

Since the resistance of water to the progress of a vessel is proportional to the square of the velocity, the most advantageous mode of applying force in navigation is to support a slow motion. If a steam engine of what is called a 20 horse power, give an impulsion of four miles an hour, it would require one 80 horse power to double this rate, and one of 180 to increase the celerity to twelve miles an hour. The same large engine would therefore be capable of drawing nine such boats only three times slower. But if ani-

mal exertion were employed, the effect would appear still more diminished, since the excess of muscular action would be proportionally reduced. Thus, if a horse were set to drag a boat, which required the force of 9 lb. to pull it along a canal at the rate of a mile in an hour, he would continually accelerate his pace till he reached the limit of three miles an hour, when the resistance, augmented to 81 lbs., becomes equal to his corresponding power of draught. If the force of 4 lbs. were sufficient to produce the motion of one mile in an hour, the horse would advance always faster, till he attained the pace of four miles, when his power of traction sinks to 64 lbs., the increased measure of resistance. The same horse would, therefore, be capable of dragging a train of 30 similar boats only four times slower, and might hence produce more than seven times as much effect in transporting goods.

In general, let P denote the force required to pull a boat along a canal, at the rate of a mile in an hour, and p the traction exerted at any other velocity, v when it becomes just equal to the accumulated resistance: Then, resuming a former expression, $(12-v)^2 = p = Pv^2$; whence $v = \frac{12}{\sqrt{P}+1}$, and $P = \left(\frac{12-v}{v}\right)^2$. Thus, on the Leeds and Liverpool canal, a horse, walking at the rate of $1\frac{1}{4}$ miles in an hour, draws a boat of 45 tons burthen, whence

the actual draught is $115\frac{1}{2}$ lbs., and P = 74 lbs. If the pace were quickened only to two miles, the traction would be increased to 296 lbs., and require 3 horses: were it accelerated to three miles, the force would amount to 666 lbs., requiring 8 horses: a traction of 1184 lbs., or the labour of 18 horses, would be necessary to bring the rate of travelling to four miles; and no fewer than 38 horses, pulling with the force of 1850 lbs., would have been required to bring the speed up to the rate of five miles in an hour.

The advantages resulting from the present mode of interior navigation are hence very conspicuous. The impulsion of locomotive engines along the track of a railway seems, to a certain extent, well calculated for the rapid conveyance of passengers and light goods; but the system of canals, with their slow regulated movements, which, at so little expense of labour, distribute or transport the ponderous materials and the staple productions of the country, must always be regarded as the grand feeder of our extended commerce.

An impulse may be conveyed through any fluid, as along a solid substance, by a chain of tremulous commotion, without any actual transfer of the particles which are effected in succession. A system of alternate contraction and dilatation pervades the whole extent. The celerity with which those vi-

brations are transmitted, it may be shown, is that due to half the altitude of the modulus of elasticity. The rapidity with which an impulse will shoot through water is hence equal to $8\sqrt{350,000}$, or 4730 feet every second. Suppose a train of pipes of a mile in length were filled with water, a violent blow, struck with a hammer at one end, would be transmitted through the fluid, and felt at the other end in little more than a second.

But if the water should extend along an open canal, or spread itself within a wide basin, the succession of alternating contractions and dilatations will produce a corresponding series of swells and concavities over the whole surface. These connected elevations and depressions must likewise modify and retard the tide of vibratory commotion which would rush impetuously below. The rise of the water above its level in any place will create a pressure acting on all sides, but more immediately exerted against the nearest portions of the fluid. Every swell over the surface must hence tend to subside, and every hollow again to mount upwards. A system of oscillations having been once exeited will therefore continue its operation, till the tenacity of the fluid particles extinguish the disturbing force.

Sir Isaac Newton, who first examined this intricate subject, was satisfied with comparing the motion of waves on the surface of a pool to the undulation of water in an inverted syphon. Suppose a hori-

zontal glass tube AB (fig. 176.) turned up at each end, were filled with any liquid, whether alcohol, or water, or mercury; disturbed by raising the column on the one side to E, it will sink equally in the other to F. Abandoned in this situation, the fluid would again retreat towards its former position, urged by the pressure of EG on the double of CE. Bisect AB in O, and the inciting force will be expressed by $\frac{CE}{CAO}$; and since CAO remains constant, this force will be proportional to EG, the space of derangement. Hence the fluid, after it has gained the level CD, will yet shoot just as far beyond it to G and H. In this new position, it will be pressed back by an inverted force, and made to return to E and F. But these vibratory motions, whether wide or narrow, must all be performed in the same time. They correspond exactly with the oscillation of a pendulum, whose length is OC, or half the extent of the recurved tube from C to D. If the undulations of water be therefore assimilated to these movements, the interval between each swell and the consequent subsidence will be equal to the time of the vibration of a pendulum, which has for its length the distance of the top of a wave from the middle of the subjacent hollow.

But waves never emerge and sink again in the same place. They seem to take their origin from some agitated spot, and appear thence to advance in

expanding concentric circles. The subaqueous propulsion accompanying them decides no doubt the direction of their progress, and prevents them from remaining in a pendulous state. If we examine attentively the motion of a wave, we shall find that the fore-part is always in the act of rising, while the hinder-part is constantly sinking. The whole system hence appears to roll onwards, though there is actually no translocation of any portion of the mass, and each particle in succession merely oscillates with nearly a vertical ascent and descent.

The motions of waves are perfectly imitated on the stage, by turning slowly an open helical screw applied round an horizontal axis. This effect is produced by the varying rate of the vertical elevation and depression, which must be as the versed sine of the distance of each point from the summit. Its celerity is greatest in emerging from the level of the water; it becomes stationary when it has gained its utmost ascent; but it again acquires an equal and opposite celerity as it sinks under that level, till it comes to pause at the limit of depression. The figure and apparent motion of a wave hence result from this unequal and reciprocating vertical play of each particle, combined with the continued and uniform advance of the inciting energy.

The vertical motions of the particles which compose a wave are thus exactly similar to the oscillations of a pendulum. Let *a* OA (fig. 177.) repre-

sent a level surface of the water, describe a circle *a*PAQ, erect the perpendicular POQ, and having divided each quadrant into equal arcs at the points A, B, C, &c. draw the parallels BE, CD, &c. If those arcs AB, AC, AP, AD, AE, &c., denote the times elapsed, the corresponding altitudes OF, OG, OP, and again OG, OF, &c. will express the successive positions of a point which emerged from O. In like manner, *f*, *g*, Q, and again *g*, *f* will indicate the series of positions of O, in its subsequent descent, answering to the times elapsed, AP*b*, AP*c*, APQ, APQ*d*, and APQ*e*. The particles which succeed to this will gain the same positions at equal intervals of time, and must consequently, by their combination, mark the outline of a curve, which has the arcs of a circle for its abscissæ, and the corresponding series for its ordinates. This is called the *Curve of Sines*. (See Geometry of Curve Lines, pp. 406-8.) When its ordinates are all diminished or augmented in a given ratio, it becomes changed into the *Harmonic Curve*. Such then is the sinuous curve which represents the concatenation of a system of waves. Let the horizontal line AN (fig. 178.) be distinguished into successive portions at the points B, C, D, E, equal to the arcs of the circle APQ, (fig. 177.) which measure the intervals of time, and consequently the progress of the swell; and from those points, erect perpendiculars equal to the corresponding sines OF, OG, OP, &c.; the

summits of these being joined, will indicate the contour of the undulating surface of the water. The front of the wave AVG rises in the first instant to R, and the whole appears to advance into the position BXRH. In the next instant, the same portion mounts to S, and the wave comes into the situation CYSI. In another instant, the fore-part of the wave attains the elevation T, and the outline changes into DTK. Again, the hinder-part of this wave, by an inverted or reciprocating motion, descends first from V to X, then to Y, and next sinks to D. Each wave thus appears to roll onward, although none of its component particles really change their places. They merely vibrate in the same vertical lines, rising or falling with a variable celerity.

It is evident that the relative elevations of waves may differ in every proportion. Fig. 179. exhibits the contour of a very high wave or billow; and fig. 180. represents the outline of a gentle swell or undulation.

Waves are always seen rolling towards the shore; but an obstacle opposed to them becomes the centre of a new series, which spread in circles. One set of waves, however, does not interfere with the motion of another, and they will cross each other without occasioning the smallest interruption. Sometimes the ordinary undulations are combined with a distant swell, called the *bore*, which rises impetuous after certain considerable intervals.

NOTES.

Note I. P. 25.

This capital experiment was first devised and performed on a small scale by Mr Canton in 1760. It established incontestably the compression of water, but seems to have been generally overlooked by succeeding popular writers, many of whom still continue to repeat the erroneous conclusion of the Academicians *del Cimento*, which represents that fluid as absolutely incompressible. The instrument alluded to in the text holds about 12 pounds of water, which was introduced with great care and patience: The contraction and subsequent dilatation in the stem, from abstracting and restoring the pressure of nine-tenths of an atmosphere, amounts to 3 or 4 inches, and is rendered visible at the distance of several benches in a large class-room, by help of a drop of quicksilver resting on the top of the aqueous column.

I have likewise had constructed, by our ingenious young optician Mr John Adie, a large and delicate instrument, suggested by the plan of Oerstedt, and capable of extensive application. It bears safely a pressure of 12 or 15 atmospheres, and not only measures easily the contraction of different fluids, but serves to indicate the various compressibility of solid substances. From a series of experiments which I have instituted, I may venture to anticipate the detection of some interesting and important facts in the economy of Nature.

The theory of the compression of bodies, carried to its full extent, might give rise to several bold but striking speculations regarding

the internal constitution of our globe. Let the density of any substance, at a depth corresponding to the distance x from the centre in miles, be denoted by d, (that at the surface being assumed the unit,) and the radius and the modulus of elasticity expressed by r and m. Since the power of internal gravitation is directly as the distance from the centre, it will be demonstrated in the Second Volume of this Work, that Hyp. Log. $d = \frac{r^2-x^2}{2\,mr}$, or, adopting common logarithms, and inserting the numerical values,

$$\text{Log. } d = \frac{3956^2-x^2}{18218\ m}.$$

For Atmospheric Air, this *formula* becomes,

$$\text{Log. } d = \frac{3956^2-x^2}{91090}.$$

For pure Water, it passes into,

$$\text{Log. } d = \frac{3956^2-x^2}{2415707}.$$

And for white Marble, the *formula* is,

$$\text{Log. } d = \frac{3956^2-x^2}{7287200}.$$

Hence it may be computed, that if the same law of condensation continued, Air would become as dense as Water at the depth of 33¼ miles; it would even acquire the density of Quicksilver at a farther depth of 163½ miles.

The idea which I formerly threw out in the article Meteorology, of the Supplement to the Encyclopædia Britannica, that the ocean may rest on a subaqueous bed of compressed air, is therefore not devoid of probability. Supposing the rate of contraction were to proceed more slowly than at first, still the required measure of condensation would be attained at a depth which forms a very small part of the radius of the globe.

But Water, under the weight of an enormous column, must likewise largely contract. At the depth of 93 miles, it would be compressed into half its former bulk; and at the depth of 362½ miles, it would acquire the ordinary density of quicksilver. Even Mar-

ble itself, subjected to its own pressure, would become twice as dense as before at the enormous depth of $287\frac{2}{3}$ miles.

It is curious to remark, that, from its rapid compressibility, Air would sooner acquire the same density with Water, than this fluid would reach the condensation of Marble. For the coincidence of Air and Water, the *formula* becomes

$$x^2 = 15649936 - \frac{2415707-91090}{2320617} \text{Log. } 840\text{; whence the depth}$$

is $35\frac{1}{3}$ miles.

For equal densities of Water and Marble, the *formula* is

$$x^2 = 15649936 - \frac{7287200.2415707}{4871493} \text{ Log. } 2.34\text{; and the depth}$$

descends to $172\frac{9}{10}$ miles.

If we calculate for a depth of $395\frac{3}{5}$ miles, which is only the tenth part of the radius of the earth, we shall find that Air would attain the enormous density of 101960 billions; while, at the same depth, Water would acquire but a density of 4.3492, and Marble only 3.8095.

At the centre of the earth, the several *formulæ* will become simpler. The logarithm of the final augmented density would be for Air $\frac{15649936}{91090}$, for Water $\frac{15649936}{2415707}$, and for Marble $\frac{15649936}{7287200}$. Air would hence reach the inconceivable density expressed by 764 with 166 ciphers annexed, while Water would be condensed 3009000 times, and Marble acquire the density of 119.

Such are the prodigious results deduced from the law of gravitation, even supposing the structure of the globe were uniform. But if we take into the estimate the augmented power from condensation, the numbers would become still more stupendous. It follows, therefore, that if the great body of our earth consisted of any such materials as we are acquainted with, its mean density would very far surpass the limits assigned by the most accurate investigations. The astronomical observation by Dr Maskelyne on the deflection of a pendulum, caused by the attraction of the sides of Mount Schehallien, and the nice experiments made with the *Balance of Torsion* by Mr Cavendish, on the mutual action

Uranus, the most distant of the planets yet discovered. Only such surpassing powers of repulsion would appear at all adequate to balance the cumulative mass of compression, and restrain the condensation of our globe within moderate limits.

We are thus led, by a close train of induction, to the most important and striking conclusion. The great central concavity is not that dark and dreary abyss which the fancy of Poets had pictured. On the contrary, this spacious internal vault must contain the purest ethereal essence, *Light* in its most concentrated state, shining with intense refulgence and overpowering splendour.

Note II. P. 96.

The problem of the *Swiftest Descent*, or the investigation of the *Brachystochronous Curve*, was solved only by Mathematicians of the first order about the close of the seventeenth and beginning of the eighteenth century. It particularly exercised the ingenuity, but it likewise unhappily excited the contentious spirit, of the Bernoullis. The last solution which James the elder brother produced in 1718 seems uncommonly elegant, and deserves the more attention, as it may be considered as the germ of the *Method of Variations* with which Lagrange has enriched the *Higher Calculus*. I have endeavoured in the text to give the substance of that solution without involving it in symbolical notation; but perhaps the reader will not be averse to see the demonstration remoulded.

If a body descend by its own gravity in the shortest time possible from one point to another in the same vertical plane, it must describe every element of its curvilineal path likewise in the shortest time. Conceive this element to be composed of two minute portions AC and BC, viewed as straight lines; or let the extreme points A and B stand equidistant on both sides of the horizontal line CD, and a body descend with a certain velocity from A towards this line at C, and next with a different velocity from C to B, so as to complete the track ACB in the shortest time. As this is only a

limiting position, assume the proximate path AcB, and from A and B describe the minute arcs C*a* and *cb*, the track AC is thus lengthened by *ca*, while BC is shortened by C*b*; and consequently *ca* must be described in the same instant of time as C*b*, or *ca* is to C*b* as the velocity of description above the line CD is to the velocity below it. But the elemental triangles being viewed as right angled, *ca* and C*b* are the sines of the angles *c*C*a* and *c*C*b*, which are equal to the angles which AC and CB make with a vertical line. Wherefore the Brachystochronous Curve must be such, that the sine of the declination of every portion of it from the perpendicular shall be proportional to the corresponding celerity of descent. Now this property belongs to the Cycloid. For in fig. 57, the tangent at E being parallel to the cord GD of the generating circle, makes with a vertical the angle GDC, of which the sine CG is proportional to the velocity acquired in falling directly from C to H, or along the cycloidal arc from A to E.

Note III. P. 101.

Huygens strongly recommended the adoption of the length of a second's pendulum as an universal standard measure, it being divided into three equal parts, which he proposed to call *horary feet*. The project was afterwards frequently resumed and abandoned. The great difficulty consists in determining experimentally the position of the centre of oscillation. In 1778, Mr Hatton proposed to avoid this, by applying a moveable point of suspension, and taking for the standard the difference between the lengths of the two pendulums thus formed. The ingenious Mr Whitehurst, in 1787, improved the plan, by making his pendulums to vibrate 84 and 42 times in a minute, their lengths being 20 and 80 inches, and leaving, consequently, a difference of 5 feet as the standard. In performing the experiment, he found the second's pendulum in London to have the length of 39⅛ inches.

After a long interval of time, the question of a standard pendu-

lum was again revived. The very skilful artist, Mr Troughton, proposed a cylinder of brass, suspended alternately from either end, and two-thirds of the intermediate distance to be assumed, according to theory, for the length of the pendulum. It was soon found, however, liable to much inaccuracy from the unequal density of the metal. Captain Kater, in 1818, happily availed himself of the beautiful property demonstrated by Huygens, that the point of suspension and the centre of oscillation are interchangeable. He employed a bar of plate brass, with two moveable knife edges for suspension, and a heavy weight attached. A small intermediate slider was annexed, and shifted along, till the vibrations from both points of suspension became isochronous. The number of those vibrations was reckoned from their successive coincidences with the oscillations of the pendulum of a chronometer. In this way was formed a standard pendulum, the interval between the two points of suspension marking its true length. After applying several delicate corrections, Captain Kater fixed the length of a pendulum vibrating seconds in London, at the temperature of 62° *in vacuo*, at 39.1386 inches. In 1819, this scientific observer rectified his conclusion, and determined the length of the pendulum at different remote stations. For London, he finally gives 39.13929, and for Leith Fort 39.15554 inches.

Note IV. P. 110.

On the principle of Centrifugal Force, might be constructed a sort of *Revolving Battery*. Suppose a wide vertical cylinder holding a supply of balls, and carrying a very long horizontal branch, to be whirled rapidly about its axis: it would evidently discharge from the mouth of the pipe a torrent of shot with considerable force. Each ball, driven along the tube by centrifugal action, and describing at the same time a circle, must be projected at an oblique angle. To find this deviation from the tangent, or a perpendicular to the tube, is a curious problem. Resuming the notation of the text, and putting x for the distance of the ball from the centre of mo-

tion, $f=\frac{\pi^2 x}{8t^2}$, and consequently, $vdv=\frac{\pi^2 xdx}{8t^2}$, and the integral $v^2=\frac{\pi^2 x^2}{8t^2}$, or $\frac{\pi}{\sqrt{8}}\cdot\frac{x}{t}=v$, the radiating velocity. But the velocity of revolution being $\frac{2\pi x}{t}$, is evidently to v as $2:\frac{1}{\sqrt{8}}$ or $\sqrt{32}:1$, which is the ratio of radius to the tangent of declination, or 10° 1′ 35″. Hence the obliquity of discharge remains in all cases precisely the same.

In Holland and Flanders, the bleachers use a very long scoop for watering their linen. The stream must therefore issue from the groove always nearly at the same angle.

Note V. P. 168.

If the Concentrator of Force receive its impulsion from a cord wound about the circumference of a cylinder, the moving power will evidently travel with an accelerated velocity. Such a mode of action is easily obtained from a descending weight or an expanding spring. But where animal power is employed, or the pressure of a regular engine, the primary motion must necessarily be uniform. This condition is very nearly, if not absolutely, obtained, by substituting a sort of tapered *fusee* instead of the cylinder, the pressure beginning at a considerable distance from the axis, and then gradually approaching it. It is a curious, though not a difficult problem, to determine the nature of the curve, which is a species of hyperbola. The reversed incurvation serves to soften and equalize the discharge of the *momentum* acquired by the Concentrator.

Note VI. P. 209.

The common atmospheric engine, having a counterpoise at the farther end of the beam, acts merely by *pulling*. The perpendi-

cular motion of its pump rod is therefore produced by a flexible or jointed chain (like that of a watch) bending on a sector. But in Mr Watt's engine, the power, being communicated by alternately *drawing and pushing*, it became necessary to direct the rod of the piston in a vertical and rectilineal path. This is effected, at least approximately, by a simple and ingenious contrivance called the *Parallel Motion*. Fig. 113. represents it: AC is one half of the beam turning on its centre C, ADEF is a parallelogram jointed at its four angles, and carrying the rod from F, which is guided by the bar BE, fastened to a firm beam at B, and working on the joint E. It is easy to see that E must describe a small arc, and that the deviation from the tangent and towards B will be inversely as the radius BE. But, for the same reason, the point D deviates towards C by a quantity as $\frac{1}{CD}$; and EF being kept parallel to CD, the point F will decline proportionally from the vertical towards C. Thus, the deviation of the top of the piston-rod to the right and to the left will be denoted by $\frac{1}{BE}$, and by $\frac{1}{CD} \cdot \frac{EF}{CD}$ or $\frac{EF}{CD^2}$; and consequently, to correct those opposite effects, it is requisite that $\frac{1}{BE} = \frac{EF}{CD^2}$, or $BE \,.\, EF = CD^2$. Wherefore the beam CA must be so divided, that the part CD shall be a mean proportional between the remainder DA and the length of the stay BE.

This mode of rectification is sufficient for practice, though strictly applicable only to small arcs of vibration. The extended path of E is a complex curve line.

Note VII. P. 259.

This property may be demonstrated geometrically. For let the parabola BAC (fig. 134.) have its parameter and its axis AB equal to the diameter of the circle; if AD be taken the third part of AB,

then DE, in the parabola, will be equal to the corresponding chord of the circle, DB will be proportional to the square of the chord AE. The strength of the beam will hence be represented by the rectangle BDEF. But this space is a *maximum*; for the tangent EF being drawn, AD=AT, and consequently DT=DB. But the elementary rectangle KD is equal to KG or EL. The rectangle BDET has thus its increments balanced by the decrement, and is therefore the greatest that can be inscribed in the given parabola.

Note VIII. P. 281.

The power of traction exerted by a horse, being evidently compounded of the velocity and the strain, is hence denoted by $v\,(12-v)^2$, which becomes a maximum when v is equal to four miles, or the third part of 12. But the same conclusion may be derived geometrically. Let OAC (fig. 133.), be a parabola, of which AO is the axis, and AB a vertical tangent: If the part DB denote the actual velocity, the perpendicular DE, being as the square of AD, will express the strain. Wherefore the exterior rectangle DEF will represent the compound effect. Contiguous to E assume the point K, and draw the secant EKT; then AT will approximate to an equality with AG. The elementary rectangle GK, being equal to KH, must therefore be double of KD. But the rectangle EB in the state of a *maximum*, will have its increment KD on the one side equal to its decrement EL on the other. Consequently GH is double of EL, and the part AD double of DB, which is therefore the third part of the whole AB.

Note IX. P. 399.

When air is projected from a small orifice, either into free space or into a close vessel with a wider exit, it will evidently spread out in diverging streamlets, and hence suffer rarefaction. The radiating discharge of a fluid thus involves a principle, as I noticed in

the article Meteorology of the Supplement to the Encyclopædia Britannica, which explains a number of curious facts. Hence the suspension and play of a little ball above a jet of air from a condensing engine; and hence, too, the lowered temperature of such a stream from a condensing engine, or of a current of the High Pressure Steam.

The same principle elucidates the seeming paradox in the action of fluids, lately considered by the ingenious M. Hachette. If a pipe bent directly downwards expand at its lower extremity into a cone, the base of which is horizontal, encircled by a narrow brim, and pierced with a little central hole, but having a circular plate fitting loosely with a weight appended: a strong current of air issuing through that narrow orifice, so far from blowing away the cover, will draw it forcibly and support the load. The sheet of air between the opposite surfaces being kept rarefied by its diverging streamlets, the external atmosphere presses strongly upwards. Hence arises the uncertainty in some cases of safety valves.

A similar effect is produced in the expanding discharge of water from a narrow outlet. The streamlets dividing from the centre, draw in air and keep it rarefied by their rapid divergence. The play of a ball over a *jet d'eau* depends on a modification of the same principle. A like explanation may be given of another fact, mentioned to me by a friend, that the common bellows work with great difficulty when the valve is much larger than the hole which it covers.

NOTE X. P. 407.

The latest and best set of experiments on the motion of fluids, was performed between the years 1811 and 1815 at Fahlun, by Lagerhielm, Forselles, and Kallstenius, at the expense of the Mining Society, and published in 1818 and 1822 at Stockholm in the Swedish language, under the title of *Hydrauliska Försök.* The profound mathematical knowledge displayed in that work is highly creditable to the practicable engineers of Sweden. I regret that I cannot afford space for any detail of their experiments. It may

suffice to remark, that they found the discharge of water, from pressure through a circular hole, to be almost exactly 3–5ths, and the shock of a current against a plane, 9–10ths of the quantities assigned by theory; and that the resistance which a wedge encounters in water, consists of two distinct portions, as already stated in the text, the one depending on the square of sine of the angle, and the other on the cosine of that angle.

Note XI. P. 432.

I find that, adopting round numbers, what is called a single horse power in mechanical performance may be estimated at one thousand cubic feet of water, raised to the height of one foot in a minute. This might be exemplified in the small river Leven, which issues from a Lake of that name, and runs mostly through Fifeshire about twelve miles, till it discharges itself into the Firth of Forth. The mean flow being reckoned 4000 cubic feet for every minute, the descent of 306 feet would certainly leave a fall of 200 available for the purpose of driving Machinery. The whole impulsion of the stream of the Leven is hence represented by 4×200, or 800 horse power; or what is equivalent to the operation of 40 steam engines, each of 20 horse power.

Note XII. P. 437.

A ship must evidently sail faster in proportion to the extent of canvas exposed to the wind, and the smallness of its resistance in ploughing the water. The rate of going will hence depend as much on the stability of the vessel as on its tapered form. The late Admiral Chapman of the Swedish Navy, the ablest and most scientific shipbuilder of his day, gave, as the result of very long experience, that the square of the effectual velocity of a ship may be expressed by $\frac{B^{\frac{1}{2}}L^3}{D^3}$, where B denotes the breadth, and D the

depth at the bilge, and L the length of the hull. Fast sailing is thus the least affected by the breadth, but depends chiefly on the length and on the smallness of the draught of water. It hence follows, that the absolute efficiency of navigation, or the quantity of goods transported in a given time, will be represented by $\frac{B^{\frac{5}{4}}L^{\frac{5}{6}}}{D^{\frac{1}{2}}}$.

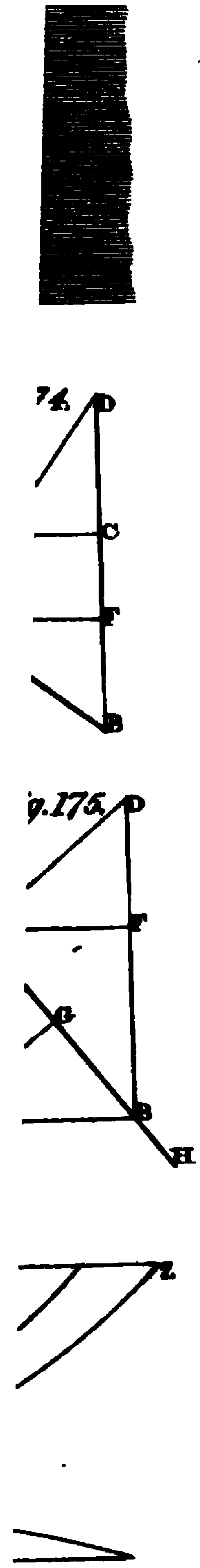
Our merchant ships, in their construction, are notoriously inferior to those of most other nations, and particularly the Swedes and Americans. They sail worse, have less stability, and suffer more from stress of weather. These defects have been chiefly caused by the operation of the absurd rule for the admeasurement of tonnage, which takes into account only the length and breadth of the ship, assuming the depth as always the same portion of the latter. The builders are therefore tempted to increase the depth at the expense of the breadth, and the vessel is thus made to draw more water and carry less sail. Another evil arises from this construction, the necessity of deepening our harbours. Let us hope, that so gross a mode of ascertaining the tonnage will be speedily abolished.

NOTE XIII. P. 448.

I have endeavoured, in a popular way, to reconcile the Newtonian theory of waves with the actual appearances. But to investigate the subject thoroughly would prove a most arduous task. The profound researches of Lagrange and Poisson on the vibrations of fluids, are only fine speculations, which yet seem to bring out no definite or practical results.

END OF VOLUME FIRST.

EDINBURGH:
PRINTED BY JAMES WALKER.


74.
D
C
F
B
g.175.
D
F
G
B
H
Z